Fuzzification of Generalized Petersen Graphs

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Abstract - *Generalized Pertersen graphs are cubiv connected graphswith inner star polygon and outer regular polygon.Evidence labeling is a special kind of labeling for the fuzzification of graphs. In this paper , we discuss about generalized Petersen graphs and fuzzication in generalized Petersen graphs by Evidence Labeling.*

Keywords— *Evidence Labeling, Generalized Petersen Graph .*

I. INTRODUCTION

All graphs we discuss in this paper are simple and undirected.

Graphs are the the convenient way of representing relationship between objects..

The most remarkable invention of this century is the invention of fuzzy set by Lofti.A.Zadeh in 1965.Fuzzy graphs were introduced by A.Rosenfeld in 1975 to deal with relations involving uncertainty. A labeling of a graph G is an assignment of labels to edges , vertices or both edges and vertices of a graph.

In [1] , we introduced evidence labeling and showed the admissibility of evidence labeling in path related graphs.

In this paper we show the admissibility of evidence labeling in some Generalized Petersen graphs.

Definition

A fuzzy graph G = (V,μ ,ρ) is a non empty set V together with a pair of functions

$\mu : V \to [0,1]$ and $\rho : V \times V \to [0,1]$ such that for all x, y in V , $\rho(x, y) \leq \mu(x) \wedge \mu(y)$.

We call μ the fuzzy vertex set of G and ρ , the fuzzy edge set of G respectively.

We denote the underlying graph of the fuzzy graph G = (μ , ρ) by G* = (μ*, ρ*), where

$\mu^* = \{x \in V : \mu(x) > 0\}$ and $\rho^* = \{(x,y) \in V \times V : \rho(x,y) > 0\}$.

II. MAIN RESULTS

Theorem 1

The 3-prism graph GP(3,1) admits evidence labeling.

Proof

Consider the 3-prism graph GP(3,1) =G(V,E) , where $|V(G)| = 6$, $|E(G)| = 9$.

Let $V^* = (v_1,v_2, v_3, v_4, v_5,v_6)$.

Define $m : V^* \to [0,1]$ as $m(v_i) = \frac{i}{6}$, $1 \leq i \leq 6$ and

$\rho: E \to [0,1]$ as $\rho(v_i, v_{i+1}) = \frac{i(i+2)}{6^3}$, $1 \leq i \leq 4$.

$\rho(v_i, v_6) = \frac{i}{6^4}$, i=2,4,5.

$\rho(v_1, v_5) = \frac{1}{6^2}$, $\rho(v_1, v_3) = \frac{4}{6^3}$

If possible, suppose that m is not injective.

Then $m(v_i)=m(v_j)$ for some $i \neq j$

$\implies i = j$.

Suppose that $\rho(v_i, v_{i+1}) = \rho(v_j, v_{j+1})$ for some $i \neq j$

$$\implies \frac{i(i+2)}{6^3} = \frac{j(j+2)}{6^3}$$

$$\implies i^2 - j^2 = 2(j-i)$$

$$\implies i + j = -2, \text{ contradiction.}$$

Suppose that $\rho(v_i, v_{i+1}) = \rho(v_j, v_6)$, j =2,4,5.

Then $\frac{i(i+2)}{6^3} = \frac{j}{6^4}$, j =2,4,5.

If j =2, $6(i^2+2i) = 2$, has no integer solution.

If j =4, $6(i^2+2i) = 4$, has no integer solution.

If j =5, $6(i^2+2i) = 5$, has no integer solution.

Suppose that $\rho(v_i, v_6) = \rho(v_j, v_6)$ for i, j =2,4,5.

Then $\frac{i}{6^4} = \frac{j}{6^4} \Rightarrow i = j$.

Suppose that $\rho(v_i, v_6) = \rho(v_1, v_5)$ for some i =2,4,5.

Then $\frac{i}{6^4} = \frac{1}{6^2}$, which has no solution.

Suppose that $\rho(v_i, v_6) = \rho(v_1, v_3)$ for some i =2,4,5.

Then $\frac{i}{6^4} = \frac{4}{6^3}$, which has no integer solution.

Suppose that $\rho(v_i, v_{i+1}) = \rho(v_1, v_5)$, $1 \leq i \leq 6$.

Then $\frac{i(i+2)}{6^3} = \frac{1}{6^2}$ for $1 \leq i \leq 6$, which has no integer solution.

Suppose that $\rho(v_i, v_{i+1}) = \rho(v_1, v_3)$, $1 \leq i \leq 6$.

Then $\frac{i(i+2)}{6^3} = \frac{4}{6^3}$ for $1 \leq i \leq 6$, which has no integer solution.

So m is injective.

Theorem 2

The 4-prism graph GP(4,1) admits evidence labeling.

Proof

Consider the graph GP(4,1) =G (V,E) where $|V| = 8$, $|E|=12$

Let $V^* =(v_1,v_2,....,v_8)$.

Define $m : V^* \to [0,1]$ as $m(v_i) = \frac{i}{8}$, $1 \le i \le 8$

$\rho: E \to [0,1]$ as $\rho(v_i, v_{i+1}) = \frac{i(i+2)}{8^3}$, $1 \le i \le 6.$

$$\rho(v_i,v_8) = \frac{i}{8^4} , \quad i = 3,5,7.$$

$$\rho(v_i, v_{i+5}) = \frac{i(i+6)}{8^3} , \quad i =1,2 .$$

$$\rho(v_1, v_4) = \frac{5}{8^4} .$$

Clearly m and ρ are injective.

Theorem 3

The 5-prism graph GP(5,1) admits evidence labeling.

Proof

Consider the 5-prism graph GP(5,1) =G(V,E) , where $|V| = 10$,$|E|= 15$.
Let $V^* =(v_1,v_2,....,v_{10})$

Define $m : V^* \to [0,1]$ as $m(v_i) = \frac{i}{10}$, $1 \le i \le 10$ and

$$\rho(v_i, v_{i+1}) = \frac{i(i+2)}{10^3}, \quad 1 \le i \le 8$$

$$\rho(v_i, v_{10}) = \frac{i}{10^4} , \ i =4,6,9$$

$$\rho(v_1, v_5) = \frac{6}{10^4},$$

$$\rho(v_i, v_{i+6}) = \frac{i(i+7)}{10^3} , \ 1 \le i \le 3$$

Clearly m and ρ are injective.

Theorem 4

The Petersen graph GP(5,2) admits evidence labeling.

Proof

Consider the Petersen graph GP(5,2) = G(V,E) ,where $|V| = 10$,$|E|= 15$

Let $V^* =(v_1,v_2,....v_{10})$.

Define $m : V^* \to [0,1]$ as $m(v_i) = \frac{i}{10}$, $1 \le i \le 10$ and

$\rho: E \to [0,1]$ as $\rho(v_i, v_{i+1}) = \frac{i(i+2)}{10^3}$, $1 \le i \le 8$

$$\rho(v_i, v_{10}) = \frac{i}{10^4}, \quad i =2,6,9.$$

$$\rho(v_4, v_9) = \frac{40}{10^3}.$$

$\rho(v_3, v_7) = \frac{8}{(10)^8}$,

$\rho(v_1, v_5) = \frac{6}{10^4}$.

$\rho(v_1, v_8) = \frac{9}{10^8}$.

Clearly m and ρ are injective.

Theorem 5

The 6-prism graph GP(6,1) admits evidence labeling.

Proof

Consider the 6-prism graph GP(6,1) =G (V,E) where $|V| = 12$,

$|E(G)| = 18$.

Let $V^* = (v_1, v_2, \ldots, v_{12})$.

Define $m : V^* \rightarrow [0,1]$ as $m(v_i) = 12$, $1 \leq i \leq 12$

$\rho: E \rightarrow [0,1]$ as $\rho(v_i, v_{i+1}) = \frac{i(i+2)}{12^8}$, $1 \leq i \leq 10$.

$\rho(v_i, v_{12}) = \frac{i}{(12)^4}$, i =5,7,11.

$\rho(v_1, v_6) = \frac{7}{(12)^8}$,

$\rho(v_i, v_{i+7}) = \frac{i(i+8)}{(12)^8}$, $1 \leq i \leq 4$

Clearly m and ρ are injective.

Theorem 6

The graph GP(6,2) admits evidence labeling.

Proof

Consider the graph GP(6,2) , where $|V(G)| = 12$,

$|E(G)| = 18$.

Let $V^* = (v_1, v_2, \ldots, v_{12})$.

Define $m : V^* \rightarrow [0,1]$ as $m(v_i) = \frac{i}{12}$, $1 \leq i \leq 12$.

$\rho: E \rightarrow [0,1]$ as $\rho(v_i, v_{i+1}) = \frac{i(i+2)}{12^8}$, i =1, $4 \leq i \leq 10$

$\rho(v_i, v_{12}) = \frac{i}{(12)^4}$, i =3,7,11.

$\rho(v_2, v_3) = \frac{24}{(12)^8}$,

$\rho(v_1, v_3) = \frac{12}{(12)^8}$,

$\rho(v_4, v_9) = \frac{40}{(12)^8}$,

$\rho(v_4, v_6) = \frac{28}{(12)^8}$,

$\rho(v_2, v_{10}) = \frac{22}{12^8}$,

$\rho(v_5, v_{11}) = \frac{60}{12^8}$,

$\rho(v_1, v_8) = \frac{9}{(12)^8}$.

Clearly m and ρ are injective.

Theorem 7

The 7-prism graph GP(7,1) admits evidence labeling.

Proof

Consider the 7-prism graph GP(7,1) = G(V,E) ,where $|V(G)| = 14$,

$|E(G)| = 21$.

Let $V^* = (v_1, v_2, \ldots, v_{14})$.

Define $m : V^* \to [0,1]$ as $m(v_i) = \frac{i}{14}$, $1 \leq i \leq 14$

$\rho: E \to [0,1]$ as $\rho(v_i, v_{i+1}) = \frac{i(i+2)}{14^8}$, $1 \leq i \leq 12$

$\rho(v_i, v_{14}) = \frac{i}{(14)^4}$, i = 6,8,13.

$\rho(v_i, v_{i+8}) = \frac{i(i+9)}{14^8}$, $1 \leq i \leq 5$

$\rho(v_1, v_7) = \frac{8}{14^8}$.

Clearly m and ρ are injective.

III. CONCLUSIONS

In this paper we discussed about Generalized Petersen Graphs and the admissibility of Evidence labeling in these graphs. Evidence labeling helps to fuzzify Generalized Petersen Graphs .

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