Multiple integrals involving the S generalized Gauss's hypergeometric function, class of polynomials, multivariable Aleph-function and multivariable I-function I

$F.Y.\ AYANT^1$

1 Teacher in High School , France

ABSTRACT
In the present paper we evaluate a generalized multiple integrals involving the product of the S generalized Gauss hypergeometric function, multivariable Aleph-function, the multivariable I-function and general class of polynomials of several variables. The importance of the result established in this paper lies in the fact they involve the Aleph-function of several variables which is sufficiently general in nature and capable to yielding a large of results merely by specializating the parameters their in.

Keywords:Multivariable Aleph-function, general class of polynomials, multiple integral, S generalized hypergeometric function, multivariable I-function, multivariable H-function

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1.Introduction and preliminaries.

The function Aleph of several variables is an extension the multivariable I-function recently studied by C.K. Sharma and Ahmad [3] , itself is an a generalisation of G and H-functions of multiple variables. The multiple Mellin-Barnes integral occurring in this paper will be referred to as the multivariables Aleph-function throughout our present study and will be defined and represented as follows.

We define : $\aleph(z_1,\cdots,z_r)=\aleph^{0,N:M_1,N_1,\cdots,M_r,N_r}_{P_i,Q_i,\tau_i;R:P_{i^{(1)}},Q_{i^{(1)}},\tau_{i^{(1)}};R^{(1)};\cdots;P_{i^{(r)}},Q_{i^{(r)}};\tau_{i^{(r)}};R^{(r)}}\left(\begin{array}{c|}\mathrm{y}_1\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{y}_r\end{array}\right.$

$$[(\mathrm{a}_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)})_{1,N}]\ ,[\tau_i(a_{ji};\alpha_{ji}^{(1)},\cdots,\alpha_{ji}^{(r)})_{N+1,P_i}]:$$
$$\cdots\cdots\cdots\cdots\ ,[\tau_i(b_{ji};\beta_{ji}^{(1)},\cdots,\beta_{ji}^{(r)})_{M+1,Q_i}]:$$

$$\left.\begin{array}{l}[(\mathrm{c}_j^{(1)});\gamma_j^{(1)})_{1,N_1}],[\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)};\gamma_{ji^{(1)}}^{(1)})_{N_1+1,P_i^{(1)}}];\cdots;[(\mathrm{c}_j^{(r)});\gamma_j^{(r)})_{1,N_r}],[\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)};\gamma_{ji^{(r)}}^{(r)})_{N_r+1,P_i^{(r)}}]\\ [(\mathrm{d}_j^{(1)});\delta_j^{(1)})_{1,M_1}],[\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)};\delta_{ji^{(1)}}^{(1)})_{M_1+1,Q_i^{(1)}}];\cdots;[(\mathrm{d}_j^{(r)});\delta_j^{(r)})_{1,M_r}],[\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)};\delta_{ji^{(r)}}^{(r)})_{M_r+1,Q_i^{(r)}}]\end{array}\right)$$

$$=\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^r\theta_k(s_k)y_k^{s_k}\,\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{1.1}$$

with $\omega=\sqrt{-1}$

$$\psi(s_1,\cdots,s_r)=\frac{\prod_{j=1}^N\Gamma(1-a_j+\sum_{k=1}^r\alpha_j^{(k)}s_k)}{\sum_{i=1}^R[\tau_i\prod_{j=N+1}^{P_i}\Gamma(a_{ji}-\sum_{k=1}^r\alpha_{ji}^{(k)}s_k)\prod_{j=1}^{Q_i}\Gamma(1-b_{ji}+\sum_{k=1}^r\beta_{ji}^{(k)}s_k)]} \tag{1.2}$$

and

$$\theta_k(s_k) = \frac{\prod_{j=1}^{M_k}\Gamma(d_j^{(k)}-\delta_j^{(k)}s_k)\prod_{j=1}^{N_k}\Gamma(1-c_j^{(k)}+\gamma_j^{(k)}s_k)}{\sum_{i^{(k)}=1}^{R^{(k)}}[\tau_{i^{(k)}}\prod_{j=M_k+1}^{Q_{i^{(k)}}}\Gamma(1-d_{ji^{(k)}}^{(k)}+\delta_{ji^{(k)}}^{(k)}s_k)\prod_{j=N_k+1}^{P_{i^{(k)}}}\Gamma(c_{ji^{(k)}}^{(k)}-\gamma_{ji^{(k)}}^{(k)}s_k)]} \tag{1.3}$$

Suppose , as usual , that the parameters

$b_j, j=1,\cdots,Q; a_j, j=1,\cdots,P;$

$c_{ji^{(k)}}^{(k)}, j=n_k+1,\cdots,P_{i^{(k)}}; c_j^{(k)}, j=1,\cdots,N_k;$

$d_{ji^{(k)}}^{(k)}, j=M_k+1,\cdots,Q_{i^{(k)}}; d_j^{(k)}, j=1,\cdots,M_k;$

with $k=1\cdots,r, i=1,\cdots,R$, $i^{(k)}=1,\cdots,R^{(k)}$

are complex numbers , and the $\alpha's, \beta's, \gamma's$ and $\delta's$ are assumed to be positive real numbers for standardization purpose such that

$$U_i^{(k)} = \tau_i\sum_{j=N+1}^{P_i}\alpha_{ji}^{(k)} - \tau_{i^{(k)}}\sum_{j=N_k+1}^{P_{i^{(k)}}}\gamma_{ji^{(k)}}^{(k)} + \tau_i\sum_{j=1}^{Q_i}\beta_{ji}^{(k)} - \tau_{i^{(k)}}\sum_{j=M_k+1}^{Q_{i^{(k)}}}\delta_{ji^{(k)}}^{(k)} \leqslant 0 \tag{1.4}$$

The reals numbers τ_i are positives for $i=1$ to R , $\tau_{i^{(k)}}$ are positives for $i^{(k)}=1$ to $R^{(k)}$

The contour L_k is in the s_k-p lane and run from $\sigma - i\infty$ to $\sigma + i\infty$ where σ is a real number with loop , if necessary ,ensure that the poles of $\Gamma(d_j^{(k)}-\delta_j^{(k)}s_k)$ with $j=1$ to m_k are separated from those of $\Gamma(1-a_j+\sum_{i=1}^{r}\alpha_j^{(k)}s_k)$ with $j=1$ to N and $\Gamma(1-c_j^{(k)}+\gamma_j^{(k)}s_k)$ with $j=1$ to N_k to the left of the contour L_k . The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$|argz_k| < \frac{1}{2}A_i^{(k)}\pi$, where

$$A_i^{(k)} = \sum_{j=1}^{N}\alpha_j^{(k)} - \tau_i\sum_{j=N+1}^{P_i}\alpha_{ji}^{(k)} - \tau_i\sum_{j=1}^{Q_i}\beta_{ji}^{(k)} + \sum_{j=1}^{N_k}\gamma_j^{(k)} - \tau_{i^{(k)}}\sum_{j=N_k+1}^{P_{i^{(k)}}}\gamma_{ji^{(k)}}^{(k)}$$

$$+\sum_{j=1}^{M_k}\delta_j^{(k)} - \tau_{i^{(k)}}\sum_{j=M_k+1}^{Q_{i^{(k)}}}\delta_{ji^{(k)}}^{(k)} > 0, \quad \text{with } k=1\cdots,r, i=1,\cdots,R, i^{(k)}=1,\cdots,R^{(k)} \tag{1.5}$$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable Aleph-function.

We may establish the the asymptotic expansion in the following convenient form :

$\aleph(y_1,\cdots,y_r) = 0(|y_1|^{\alpha_1},\cdots,|y_r|^{\alpha_r}), max(|y_1|,\cdots,|y_r|) \rightarrow 0$

$\aleph(y_1,\cdots,y_r) = 0(|y_1|^{\beta_1},\cdots,|y_r|^{\beta_r}), min(|y_1|,\cdots,|y_r|) \rightarrow \infty$

where, with $k=1,\cdots,r : \alpha_k = min[Re(d_j^{(k)}/\delta_j^{(k)})], j=1,\cdots,M_k$ and

$\beta_k = max[Re((c_j^{(k)}-1)/\gamma_j^{(k)})], j=1,\cdots,N_k$

Serie representation of Aleph-function of several variables is given by

$$\aleph(y_1,\cdots,y_r)=\sum_{G_1,\cdots,G_r=0}^{\infty}\sum_{g_1=0}^{M_1}\cdots\sum_{g_r=0}^{M_r}\frac{(-)^{G_1+\cdots+G_r}}{\delta_{g_1}G_1!\cdots\delta_{g_r}G_r!}\psi(\eta_{G_1,g_1},\cdots,\eta_{G_r,g_r})$$

$$\times\ \theta_1(\eta_{G_1,g_1})\cdots\theta_r(\eta_{G_r,g_r})y_1^{-\eta_{G_1,g_1}}\cdots y_r^{-\eta_{G_r,g_r}} \tag{1.6}$$

Where $\psi(.,\cdots,.)$, $\theta_i(.)$,$i=1,\cdots,r$ are given respectively in (1.2), (1.3) and

$$\eta_{G_1,g_1}=\frac{d_{g_1}^{(1)}+G_1}{\delta_{g_1}^{(1)}},\cdots,\ \eta_{G_r,g_r}=\frac{d_{g_r}^{(r)}+G_r}{\delta_{g_r}^{(r)}}$$

which is valid under the conditions $\delta_{g_i}^{(i)}[d_j^i+p_i]\neq\delta_j^{(i)}[d_{g_i}^i+G_i]$ (1.7)

for $j\neq M_i, M_i=1,\cdots\eta_{G_i,g_i};P_i,N_i=0,1,2,\cdots,;y_i\neq 0,i=1,\cdots,r$ (1.8)

In the document , we will note :

$$G(\eta_{G_1,g_1},\cdots,\eta_{G_r,g_r})=\phi(\eta_{G_1,g_1},\cdots,\eta_{G_r,g_r})\theta_1(\eta_{G_1,g_1})\cdots\theta_r(\eta_{G_r,g_r}) \tag{1.9}$$

where $\phi(\eta_{G_1,g_1},\cdots,\eta_{G_r,g_r}),\theta_1(\eta_{G_1,g_1}),\cdots,\theta_r(\eta_{G_r,g_r})$ are given respectively in (1.2) and (1.3)

We shall note the Aleph-function of r variables $\aleph_{u:w}^{0,N:v}\begin{pmatrix}\mathrm{z}_1\\ \cdot\cdot\cdot\\ \mathrm{z}_r\end{pmatrix}$ (1.10)

The multivariable I-function is defined in term of multiple Mellin-Barnes type integral :

$$I(z_1,\cdots,z_s)=I_{p_2,q_2,p_3,q_3;\cdots;p_s,q_s:p',q';\cdots;p^{(s)},q^{(s)}}^{0,n_2;0,n_3;\cdots;0,n_r:m',n';\cdots;m^{(s)},n^{(s)}}\left(\begin{array}{c|c}\mathrm{z}_1 & (\mathrm{a}_{2j};\alpha'_{2j},\alpha''_{2j})_{1,p_2};\cdots;\\ \cdot & \\ \cdot & \\ \cdot & \\ \mathrm{z}_s & (\mathrm{b}_{2j};\beta'_{2j},\beta''_{2j})_{1,q_2};\cdots;\end{array}\right.$$

$$\left.\begin{array}{c}(\mathrm{a}_{sj};\alpha'_{sj},\cdots,\alpha_{sj}^{(s)})_{1,p_s}:(a'_j,\alpha'_j)_{1,p'};\cdots;(a_j^{(s)},\alpha_j^{(s)})_{1,p^{(s)}}\\ \\ (\mathrm{b}_{sj};\beta'_{sj},\cdots,\beta_{sj}^{(s)})_{1,q_s}:(b'_j,\beta'_j)_{1,q'};\cdots;(b_j^{(s)},\beta_j^{(s)})_{1,q^{(s)}}\end{array}\right) \tag{1.11}$$

$$=\frac{1}{(2\pi\omega)^s}\int_{L'_1}\cdots\int_{L'_s}\xi(t_1,\cdots,t_s)\prod_{i=1}^{s}\phi_i(t_i)z_i^{t_i}\mathrm{d}t_1\cdots\mathrm{d}t_s \tag{1.12}$$

The defined integral of the above function, the existence and convergence conditions, see Y,N Prasad [2]. Throughout the present document, we assume that the existence and convergence conditions of the multivariable I-function.
The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of

the corresponding conditions for multivariable H-function given by as :

$|argz_k| < \frac{1}{2}\Omega_i^{(k)}\pi$, where

$$\Omega_i^{(k)} = \sum_{k=1}^{n^{(i)}} \alpha_k^{(i)} - \sum_{k=n^{(i)}+1}^{p^{(i)}} \alpha_k^{(i)} + \sum_{k=1}^{m^{(i)}} \beta_k^{(i)} - \sum_{k=m^{(i)}+1}^{q^{(i)}} \beta_k^{(i)} + \left(\sum_{k=1}^{n_2} \alpha_{2k}^{(i)} - \sum_{k=n_2+1}^{p_2} \alpha_{2k}^{(i)}\right) + \cdots +$$

$$\left(\sum_{k=1}^{n_s} \alpha_{sk}^{(i)} - \sum_{k=n_s+1}^{p_s} \alpha_{sk}^{(i)}\right) - \left(\sum_{k=1}^{q_2} \beta_{2k}^{(i)} + \sum_{k=1}^{q_3} \beta_{3k}^{(i)} + \cdots + \sum_{k=1}^{q_s} \beta_{sk}^{(i)}\right) \tag{1.13}$$

where $i = 1, \cdots, s$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable I-function.

We may establish the the asymptotic expansion in the following convenient form :

$$\aleph(z_1, \cdots, z_s) = 0(\,|z_1|^{\alpha_1'}, \cdots, |z_s|^{\alpha_s'}\,), max(\,|z_1|, \cdots, |z_s|\,) \rightarrow 0$$

$$\aleph(z_1, \cdots, z_s) \; = 0(\,|z_1|^{\beta_1'}, \cdots, |z_s|^{\beta_s'}\,), min(\,|z_1|, \cdots, |z_s|\,) \rightarrow \infty$$

where, with $k = 1, \cdots, z : \alpha_k' = min[Re(b_j^{(k)}/\beta_j^{(k)})], j = 1, \cdots, m_k$ and

$$\beta_k' = max[Re((a_j^{(k)} - 1)/\alpha_j^{(k)})], j = 1, \cdots, n_k$$

We will use these following notations in this paper :

$$U = p_2, q_2; p_3, q_3; \cdots; p_{s-1}, q_{s-1}; V = 0, n_2; 0, n_3; \cdots; 0, n_{s-1} \tag{1.14}$$

$$W = (p', q'); \cdots; (p^{(s)}, q^{(s)}); X = (m', n'); \cdots; (m^{(s)}, n^{(s)}) \tag{1.15}$$

$$A = (a_{2k}, \alpha_{2k}', \alpha_{2k}'')_{1,p_2}; \cdots; (a_{(s-1)k}), \alpha'_{(s-1)k}, \alpha''_{(s-1)k}, \cdots, \alpha_{(s-1)k}^{(s-1)})_{1,p_{s-1}} \tag{1.16}$$

$$B = (b_{2k}, \beta_{2k}', \beta_{2k}'')_{1,q_2}; \cdots; (b_{(s-1)k}), \beta'_{(s-1)k}, \beta''_{(s-1)k}, \cdots, \beta_{(s-1)k}^{(s-1)})_{1,q_{s-1}} \tag{1.17}$$

$$\mathfrak{A} = (a_{sk}; \alpha'_{sk}, \alpha_{sk}'', \cdots, \alpha_{sk}^{(s)})_{p_s,q_s} : \mathfrak{B} = (b_{sk}; \beta'_{sk}, \beta_{sk}'', \cdots, \beta_{sk}^{(s)})_{1,q_s} \tag{1.18}$$

$$A' = (a_k', \alpha_k')_{1,p'}; \cdots; (a_k^{(s)}, \alpha_k^{(s)})_{1,p^{(s)}}; B' = (b_k', \beta_k')_{1,q'}; \cdots; (b_k^{(s)}, \beta_k^{(s)})_{1,q^{(s)}} \tag{1.19}$$

The multivariable I-function writes :

$$I(z_1, \cdots, z_s) = I_{U:p_s,q_s;W}^{V;0,n_s;X}\left(\begin{array}{c|c} z_1 & A\,;\mathfrak{A};A' \\ \cdot & \\ \cdot & \\ \cdot & \\ z_s & B;\,\mathfrak{B};\,B' \end{array}\right) \tag{1.20}$$

The generalized polynomials defined by Srivastava [4], is given in the following manner :

$$S_{N_1',\cdots,N_t'}^{M_1',\cdots,M_t'}[y_1,\cdots,y_t] = \sum_{K_1=0}^{[N_1'/M_1']}\cdots\sum_{K_t=0}^{[N_t'/M_t']}\frac{(-N_1')_{M_1'K_1}}{K_1!}\cdots\frac{(-N_t')_{M_t'K_t}}{K_t!}$$

$$A[N_1',K_1;\cdots;N_t',K_t]y_1^{K_1}\cdots y_t^{K_t} \tag{1.21}$$

where $M_1',\cdots,M_s'$ are arbitrary positive integers and the coefficients $A[N_1',K_1;\cdots;N_t',K_t]$ are arbitrary constants, real or complex. In the present paper, we use the following notation

$$a_1 = \frac{(-N_1')_{M_1'K_1}}{K_1!}\cdots\frac{(-N_t')_{M_t'K_t}}{K_t!}A[N_1',K_1;\cdots;N_t',K_t] \tag{1.22}$$

2. S Generalized Gauss's hypergeometric function

The S generalized Gauss hypergeometric function $F_p^{(\alpha,\beta;\tau,\mu)}(a,b,c;z)$ introduced and defined by Srivastava et al [5, page 350 , Eq.(1.12)] is represented in the following manner :

$$F_p^{(\alpha,\beta;\tau,\mu)}(a,b,c;z)=\sum_{n=0}^{\infty}(a)_n\frac{B_p^{(\alpha,\beta;\tau,\mu)}(b+n,c-b)}{B(b,c-b)}\frac{z^n}{n!} \quad (|z|<1) \tag{2.1}$$

provided that $(Re(p)\geqslant 0, minRe(\alpha,\beta,\tau,\mu)>0; Re(c)>Re(b)>0)$

where the S generalized Beta function $B_p^{(\alpha,\beta;\tau,\mu)}(x,y)$ was introduced and defined by Srivastava et al [5, page 350, Eq(1.13)]

$$B_p^{(\alpha,\beta;\tau,\mu)}(x,y) = \int_1^1 t^{x-1}(1-t)^{y-1}{}_1F_1\left(\alpha;\beta;\frac{-p}{t^{\tau}(1-t)^{\mu}}\right)\mathrm{d}t \tag{2.2}$$

provided that $(Re(p)\geqslant 0, minRe(x,y,\alpha,\beta)>0; Min\{Re(\tau),Re(\mu\}>0)$

3. Required integral

We have the following integral, see (Marichev et [2] ,3.3.3, Eq.5, page 589)

Lemma.

$$\int_{x_1\geqslant 0}\cdots\int_{x_n\geqslant 0}\frac{x_1^{\upsilon_1-1}\cdots x_n^{\upsilon_n-1}}{(1-x_1^{\alpha_1}-\cdots-x_n^{\alpha_n})^{\mu}}\mathrm{d}x_1\cdots\mathrm{d}x_n = \Gamma\left[\begin{array}{c}\frac{\upsilon_1}{\mu_1},\cdots,\frac{\upsilon_n}{\mu_n},1-\mu\\ \cdots\\ 1\text{-}\mu+\frac{\upsilon_1}{\mu_1}+\cdots+\frac{\upsilon_n}{\mu_n}\end{array}\right] \tag{3.1}$$

where $x_1^{\alpha_1}+\cdots+x_n^{\alpha_n}\leqslant 1, \alpha_i>0, \upsilon_i\in\mathbb{R}^+, i=1,\cdots,n$ and $Re(\mu)<1$

4. Main integral

We note $X_{\upsilon_1,\cdots,\upsilon_n,\mu} = \dfrac{x_1^{\upsilon_1}\cdots x_n^{\upsilon_n}}{(1-x_1^{\alpha_1}-\cdots-x_n^{\alpha_n})^{\mu}}$ (4.1)

We have the following formula

Theorem

$$\int_{x_1\geqslant 0}\cdots\int_{x_n\geqslant 0}\frac{x_1^{v_1-1}\cdots x_n^{v_n-1}}{(1-x_1^{\alpha_1}-\cdots-x_n^{\alpha_n})^{\mu'}}F_p^{(\alpha,\beta;\tau,\mu)}(a,b,c;yX_{a_1,\cdots,a_n;b'})S_{N_1,\cdots,N_t}^{M_1,\cdots,M_t}\left(\begin{array}{c}\mathrm{y}_1X_{\gamma_1^1,\cdots\gamma_1^n,\mu_1}\\ \cdots\\ \mathrm{y}_tX_{\gamma_t^1\cdots\gamma_t^n,\mu_t}\end{array}\right)$$

$$\aleph_{u:w}^{0,\mathfrak{n}:v}\left(\begin{array}{c}\mathrm{z}_1X_{\alpha_1^1,\cdots\alpha_1^n,\beta_1}\\ \cdots\\ \mathrm{z}_rX_{\alpha_r^1\cdots\alpha_r^n,\beta_r}\end{array}\right)I_{U:p_s,q_s;W}^{V;0,n_s;X}\left(\begin{array}{c}\mathrm{Z}_1X_{\eta_1^1\cdots\eta_1^n,\epsilon_1}\\ \cdots\\ \mathrm{Z}_RX_{\eta_R^1\cdots\eta_R^n,\epsilon_R}\end{array}\right)\mathrm{d}x_1\cdots\mathrm{d}x_n=\frac{1}{\alpha_1\cdots\alpha_n}\sum_{G_1,\cdots,G_r=0}^{\infty}$$

$$\sum_{g_1=0}^{m_1}\cdots\sum_{g_r=0}^{m_r}\sum_{K_1=0}^{[N_1/M_1']}\cdots\sum_{K_t=0}^{[N_t/M_t']}\sum_{n'=0}^{\infty}(a)_{n'}\frac{B_p^{(\alpha,\beta;\tau,\mu)}(b+n',c-b)}{B(b,c-b)}\frac{(-)^{G_1+\cdots+G_r}}{\delta_{g_1}G_1!\cdots\delta_{g_r}G_r!}\;G(\eta_{G_1,g_1},\cdots,\eta_{G_r,g_r})$$

$$a_1\frac{y^{n'}}{n'!}z_1^{\eta_{G_1,g_1}}\cdots z_r^{\eta_{G_r,g_r}}y_1^{K_1}\cdots y_t^{K_t}\,I_{U:p_s+n+1,q_s+1;W}^{V;0,n_s+n+1;X}\left(\begin{array}{c|l}\mathrm{Z}_1 & \mathrm{A}\ ;\\ \cdots & \\ \cdots & \\ \mathrm{Z}_R & \mathrm{B};\end{array}\right.$$

$$\left[1-\frac{v_i+n'a_i+\sum_{j=1}^t K_j\gamma_j^i+\sum_{j=1}^r\eta_{G_j,g_j}\alpha_j^i}{\alpha_i};\frac{\eta_1^i}{\alpha_i},\cdots,\frac{\eta_s^i}{\alpha_i}\right]_{1,n},$$
$$\cdots$$
$$\mathrm{B}_1,$$

$$\left.\begin{array}{c}(\mu'+n'b'+\sum_{j=1}^t K_j\mu_j+\sum_{j=1}^r\eta_{G_j,g_j}\beta_j;-\eta_1,\cdots,-\eta_R),\mathfrak{A}:A'\\ \cdots\\ \mathfrak{B}:\mathrm{B}'\end{array}\right)\tag{4.2}$$

Where

$$B_1=\left[\mu'+n'b'+\sum_{j=1}^t K_j\mu_j+\sum_{j=1}^r\beta_j\eta_{G_j,g_j}\text{-}\sum_{i=1}^n\left[\frac{v_i+n'a_i+\sum_{j=1}^t K_j\gamma_j^i+\sum_{j=1}^r\eta_{G_j,g_j}\alpha_j^i}{\alpha_i}\right]\right.$$

$$\left.;\frac{\eta_1^1}{\alpha_1}\cdots\frac{\eta_1^n}{\alpha_n}-\epsilon_1,;\frac{\eta_R^1}{\alpha_1}\cdots\frac{\eta_R^n}{\alpha_n}-\epsilon_R,\right]\tag{4.3}$$

and $U,V,X,Y,A,B,\mathfrak{A},\mathfrak{B},A'$ and B' are defined respectively by (1.14), (1.15), (1.16), (1.17), (1.18) and (1.19)

Provided that

a) $min\{a_i, b, \gamma_j^i, \mu_j, \alpha_k^i, \beta_k, \eta_l^i, \epsilon_l\} > 0, i = 1, \cdots, n; j = 1, \cdots, t; k = 1, \cdots, r; l = 1, \cdots, R$

and $j = 1, \cdots, t$

b) $1 > Re(\mu' + n'b') + \sum_{j=1}^{r} \beta_j \min_{1 \leqslant k \leqslant M_j} Re\left(\frac{d_k^{(j)}}{\delta_k^{(j)}}\right) + \sum_{j=1}^{R} \epsilon_j \min_{1 \leqslant k \leqslant m^{(j)}} Re\left(\frac{b_k^{(j)}}{\beta_k^{(j)}}\right) > 0$

c) $Re(\upsilon_i + n'a_i) + \sum_{j=1}^{r} \alpha_j^i \min_{1 \leqslant k \leqslant M_j} Re\left(\frac{d_k^{(j)}}{\delta_k^{(j)}}\right) + \sum_{j=1}^{R} \eta_j^i \min_{1 \leqslant k \leqslant m^{(j)}} Re\left(\frac{b_k^{(j)}}{\beta_k^{(j)}}\right) > 0$, $i = 1, \cdots, n$

d) $|argz_k| < \frac{1}{2} A_i^{(k)} \pi$, where $A_i^{(k)}$ is defined by (1.5) ; $i = 1, \cdots, r$

e) $|argZ_k| < \frac{1}{2} B_i^{(k)} \pi$, where $B_i^{(k)}$ is defined by (1.13) ; $i = 1, \cdots, R$

f) $x_1^{\alpha_1} + \cdots + x_n^{\alpha_n} \leqslant 1, \alpha_i > 0, \upsilon_i \in \mathbb{R}, i = 1, \cdots, n$

g$(Re(p) \geqslant 0, minRe(\alpha, \beta, \tau, \mu) > 0; Re(c) > Re(b) > 0)$

Proof

First, expressing the generalized the S generalized Gauss hypergeometric function $F_p^{(\alpha,\beta;\tau,\mu)}(a, b, c; z)$ in serie with the help of equation (2.1), the Aleph-function of r variables in series with the help of equation (1.6), the general class of polynomial of several variables $S_{N_1',\cdots,N_t'}^{M_1',\cdots,M_t'}[y_1, \cdots, y_t]$ with the help of equation (1.21) and the I-function of s variables in defined by Prasad [2] in Mellin-Barnes contour integrals with the help of equation (1.12), changing the order of integrations ans summations (which is easily seen to be justified due to the absolute convergence of the integrals and the summations involved in the process) and then evaluating the resulting $(x_1, \cdots, x_n)$-integrals with the help of equation (3.1). Now interpreting the result obtained in the Mellin-barnes contour integral, we arrive at the desired result.

5. Particular case

If $U = V = A = B = 0$, the multivariable I-function defined by Prasad [2] reduces in multivariable H-function defined by Srivastava et al [6]. We have the following result.

Corollary

$$\int_{x_1 \geqslant 0} \cdots \int_{x_n \geqslant 0} \frac{x_1^{\upsilon_1 - 1} \cdots x_n^{\upsilon_n - 1}}{(1 - x_1^{\alpha_1} - \cdots - x_n^{\alpha_n})^{\mu'}} F_p^{(\alpha,\beta;\tau,\mu)}(a, b, c; yX_{a_1,\cdots,a_n;b'}) S_{N_1,\cdots,N_t}^{M_1,\cdots,M_t}\begin{pmatrix} \mathrm{y}_1 X_{\gamma_1^1,\cdots\gamma_1^n,\mu_1} \\ \cdots \\ \mathrm{y}_t X_{\gamma_t^1\cdots\gamma_t^n,\mu_t} \end{pmatrix}$$

$$\aleph_{u:w}^{0,\mathfrak{n}:v}\begin{pmatrix} \mathrm{z}_1 X_{\alpha_1^1,\cdots\alpha_1^n,\beta_1} \\ \cdots \\ \mathrm{z}_r X_{\alpha_r^1\cdots\alpha_r^n,\beta_r} \end{pmatrix} H_{p_s,q_s;W}^{0,n_s;X}\begin{pmatrix} \mathrm{Z}_1 X_{\eta_1^1\cdots\eta_1^n,\epsilon_1} \\ \cdots \\ \mathrm{Z}_R X_{\eta_R^1\cdots\eta_R^n,\epsilon_R} \end{pmatrix} \mathrm{d}x_1 \cdots \mathrm{d}x_n = \frac{1}{\alpha_1 \cdots \alpha_n} \sum_{G_1,\cdots,G_r=0}^{\infty}$$

$$\sum_{g_1=0}^{m_1} \cdots \sum_{g_r=0}^{m_r} \sum_{K_1=0}^{[N_1/M_1']} \cdots \sum_{K_t=0}^{[N_t/M_t']} \sum_{n'=0}^{\infty} (a)_{n'} \frac{B_p^{(\alpha,\beta;\tau,\mu)}(b + n', c - b)}{B(b, c - b)} \frac{(-)^{G_1 + \cdots + G_r}}{\delta_{g_1} G_1! \cdots \delta_{g_r} G_r!} G(\eta_{G_1,g_1}, \cdots, \eta_{G_r,g_r})$$

$$a_1 \frac{y^{n'}}{n'!} z_1^{\eta_{G_1,g_1}} \cdots z_r^{\eta_{G_r,g_r}} y_1^{K_1} \cdots y_t^{K_t} H^{0,n_s+n+1;X}_{p_s+n+1,q_s+1;W} \left(\begin{array}{c} Z_1 \\ \cdots \\ \cdots \\ Z_R \end{array} \right|$$

$$\left[1 - \frac{v_i + n' a_i + \sum_{j=1}^t K_j \gamma_j^i + \sum_{j=1}^r \eta_{G_j,g_j} \alpha_j^i}{\alpha_i}; \frac{\eta_1^i}{\alpha_i}, \cdots, \frac{\eta_s^i}{\alpha_i}\right]_{1,n},$$

$$\cdots$$

$$B_1,$$

$$\left. \begin{array}{c} (\mu' + n'b' + \sum_{j=1}^t K_j \mu_j + \sum_{j=1}^r \eta_{G_j,g_j} \beta_j; -\eta_1, \cdots, -\eta_R), \mathfrak{A} : A' \\ \cdots \\ \mathfrak{B} : B' \end{array} \right) \tag{5.1}$$

under the same notations and conditions that (4.1) with $U = V = A = B = 0$. B_1 are defined by (4.2).

6. Conclusion

In this paper we have evaluated a generalized multiple integrals involving the multivariable Aleph-function, a class of polynomials of several variables, the S generalized Gauss hypergeometric function and the multivariable I-function defined by Prasad [2]. The integral established in this paper is of very general nature as it contains multivariable Aleph-function, which is a general function of several variables studied so far. Thus, the integral established in this research work would serve as a key formula from which, upon specializing the parameters, as many as desired results involving the special functions of one and several variables can be obtained.

REFERENCES

[1] Marichev O.I. Prudnikov A.P. And Brychkow Y.A. Elementay functions. Integrals and series Vol 1. USSR Academy of sciences . Moscow 1986

[2] Y.N. Prasad , Multivariable I-function , Vijnana Parishad Anusandhan Patrika 29 (1986) , page 231-237

[3] Sharma C.K.and Ahmad S.S.: On the multivariable I-function. Acta ciencia Indica Math , 1994 vol 20,no2, p 113-116.

[4] Srivastava H.M. A multilinear generating function for the Konhauser set of biorthogonal polynomials suggested by Laguerre polynomial, Pacific. J. Math. 177(1985), page183-191.

[5] Srivastava H.M. Agarwal P. and Jain S. Generating functions for the generalized Gauss hypergeometric functions. Applied Mathematics and computations 247 (2014), page 358-352

[6] H.M. Srivastava And R.Panda. Some expansion theorems and generating relations for the H-function of several complex variables. Comment. Math. Univ. St. Paul. 24(1975), p.119-137.