Japanese vs Vedic Methods for Multiplication

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Abstract – *Japanese multiplication method is very useful for a person who has only skill of counting ability without having knowledge of addition and multiplication. Vertically and crosswise Vedic multiplication method is useful as a speedy calculator. In the present study, we have described both methods in our own manner by keeping Sole idea unchanged. Also it has been given the logic for the procedures. Comparative Study of both methods has been given which reflects the merit and demerit of both Methods.*

Keywords: *Vedic method, Japanese method, Multiplication methods.*

1. Introduction

Professor Fujisawa Rikitaro (1900) of imperial University of Tokyo created the Japanese multiplication method. His method is geometric based Multiplication method.

After deep study and analysis Jagadguru Sankaracarya Sri Bharati Krsna Tirthaji Maharaja (1884–1960) of Govardhana Matha, Puri, India obtained sixteen simple Mathematical Sutras (formulae) from veda. V. S. Agrawala (1965) edited the book 'Vedic Mathematics' by taking ideas and concept of sixteen formulae of Sri Bharati Krsna Tirthaji Maharaja. The third sutra (formula) is known as Urdhva – Triyagbhyam (Vertically and crosswise). With this formula, Multiplication of two numbers may be obtained very easily and quickly. Here, we shall describe both methods in our own manner keeping sole idea Unchanged by giving the logic for procedure and then compare their merits and demerits.

2. Geometrical Construction of Fujisawa's Concept of multiplication

Fujisawa gave a concept for multiplying two numbers in his own way which is known as Japanese multiplication method. For this, he took help of two kinds of parallel lines – Horizontal and Vertical. Let mn and pqr are two numbers such that m, n, p, q, r $\in N \cup \{0\}$, N is the set of natural number, $mn = 1 \times n + 10 \times m$ and $pqr = 1 \times r + 10 \times q + 100 \times p$.

Let us draw m number of closed parallel lines in horizontal direction and after leaving some space below, let us draw n number of closed parallel lines along the same direction (Figure – 1).

Now, from the left side, let us draw p number of closed parallel lines along the vertical direction such that these parallel lines intersect horizontal parallel lines. Similar to above process, leaving some spaces on the right, let us draw q and r numbers closed parallel lines respectively along vertical direction.

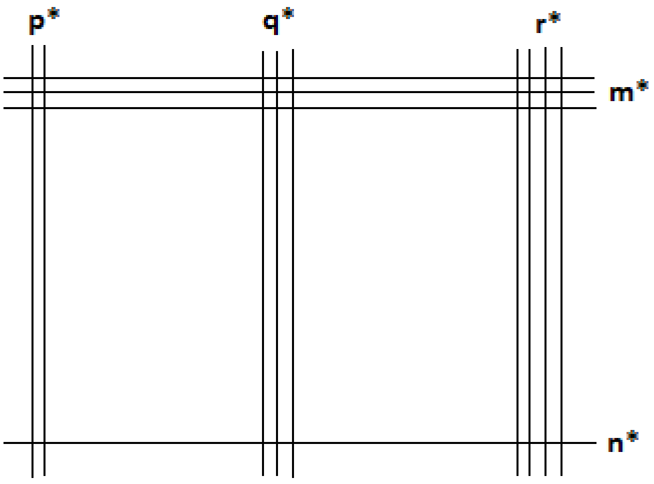


Fig.–1

Here, m* means m number of parallel lines.

3. Process of Calculation of multiplication

Let us draw the same figure as mentioned in figure – 1. We start our work by counting the number of points of intersection at different position according to the order of different arrow lines.

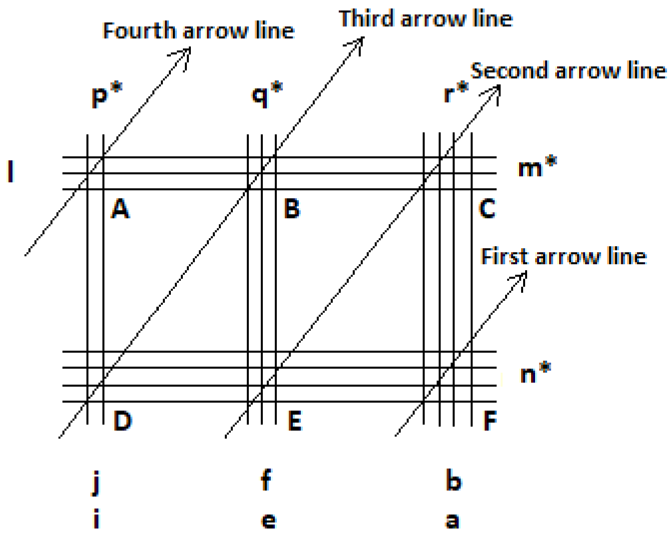


Fig.–2

Now, we count number of point of intersection made by two kinds of parallel lines at different position A, B, C, D, E and F by following arrow lines in order. At the position F, The number of points of intersection is ab (let), a, b $\in N \cup \{0\}$, Where ab = 1 × b + 10 ×a. Here, b is written in position F and a is written below b as carry number and b is included in the result.

Let, total number of points of intersection of parallel lines at E and C which is equal to cd (let), c, d $\in N \cup \{0\}$. Let cd and the carry number a make the number ef, f is written at the position E and carry number e is written below f. f is included in the result left to b.

Let total number of points of intersection of parallel lines at D and B which is equal to gh (let). Let gh and e make number ij, j is written at the position D and carry number i is written below j. j is included in the result left to f.

The number of points of intersection at A is kl (let). kl and i make number st. Let us write st at the position A. st is included in the result left to j.

Then,

mn × pqr = stjfb

Example – 1. Evaluate: 45 ×123

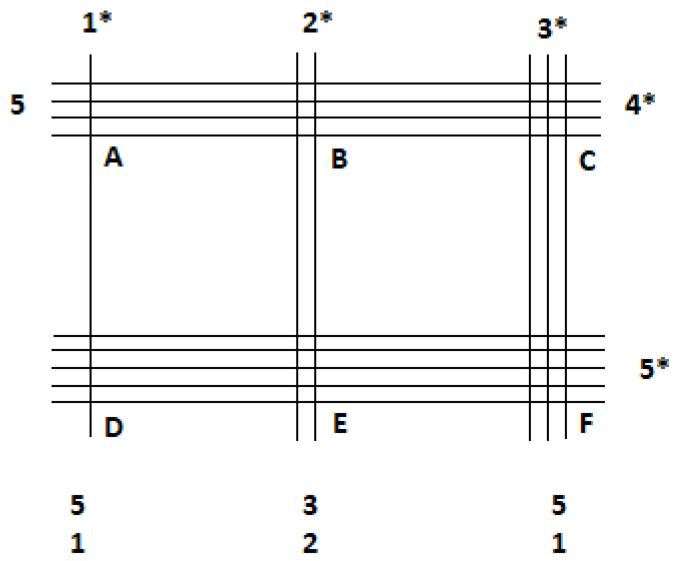


Fig.–3

Number of points of intersection at position F is 15. Here, carry number 1 is written below 5. 5 is included in the result.

Total number of points of intersection at position E and C is 22 which makes 23 with the carry number 1 and hence 3 is written at E and 2 is written below 3 as carry number. 3 is included in the result left to 5.

Total number of points of intersection at position D and B is13 which makes 15 with the carry number 2 and hence 5 is written at D and 1 is written below 5 as carry number. 5 is included in the result left to 3.

Number of points of intersection at position A is 4 which make 5 with the carry number 1. 5 is included in the result left to 5.

Then,

∴ 45 × 123 = 5535.

Example – 2. Evaluate: 23 × 14

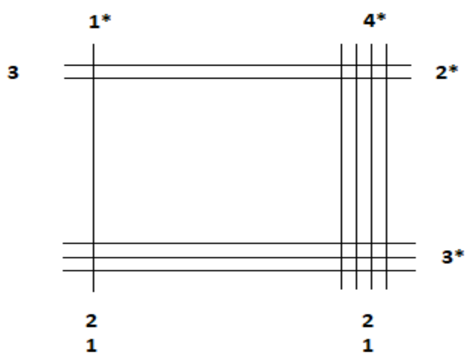


Fig.–4

Then,

$\therefore$ 23 × 14 = 322.

Example – 3. Evaluate: 132 × 141

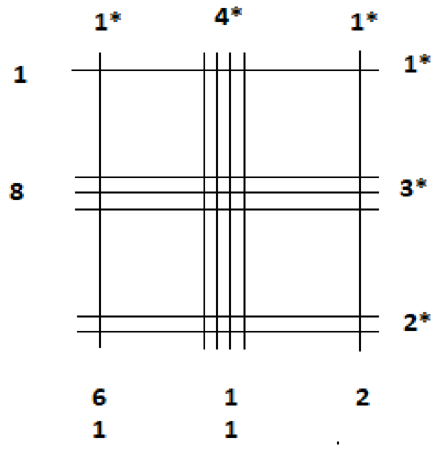


Fig.-5

Then ,

$\therefore$ 132 ×141 = 18612.

4. Geometry and process of counting when 0 is one of the digits

When 0 is one of the digits in a number then we draw a dotted line as parallel line for zero digit and we don't count the points of intersection of a parallel line with this dotted line.

Example – 4. Evaluate: 102 × 141

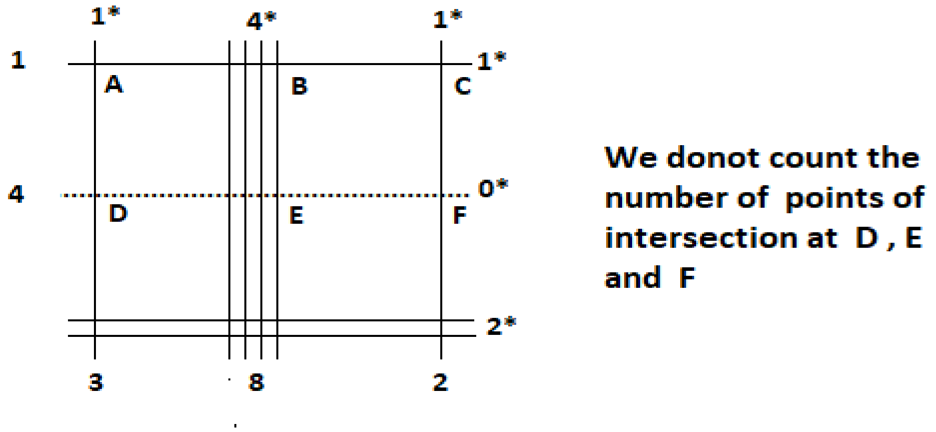


Fig.–6

Then,

$\therefore$ 102 × 141 = 14382.

Example – 5. Evaluate: 34 × 203

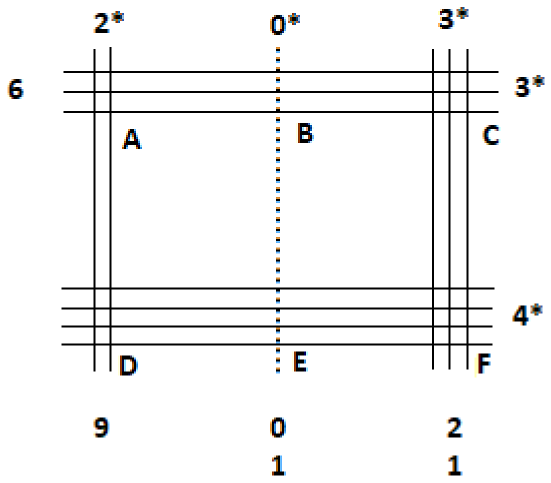


Fig.–7

Then,

$\therefore$ 34 × 203 = 6902

5. Logic for the Procedure

The following figure shows the position of different points.

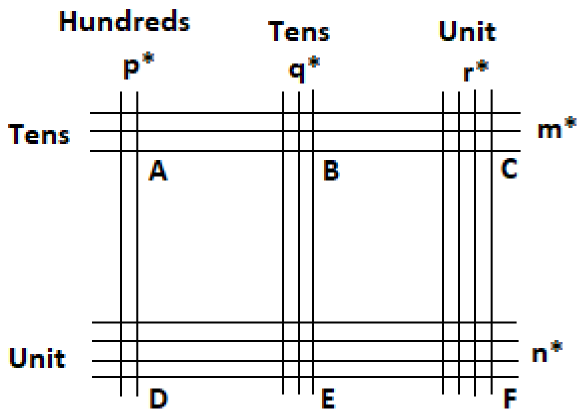


Fig.–8

Place value at the position F is Unit × Unit = Unit

Place value at the position E is Unit × Tens = Tens

Place value at the position C is Tens × Unit = Tens

Place value at the position D is Unit × Hundreds = Hundreds

Place value at the position B is Tens × Tens = Hundreds

Place value at the position A is Tens × Hundreds = Thousands

The number of points of intersection at the position E and C are obtained by adding them for their same place values.

Similarly, points of intersection at the position D and B are added for their same place values.

At the position F, Number of points of intersection = $n \times r$

$$= ab \text{ (let)}, a,b \in N \cup \{0\},$$

Where, $ab = 1 \times b + 10 \times a$. Here, a be the carry number and b is included in the result.

Total number of points of intersection at the position E and C be $n \times q + m \times r = cd$ (let); $c, d \in N \cup \{0\}$. The carry number a is added with cd which gives cd + a = ef, Where e is the carry number. f is included in the result left to b.

Total number of points of intersection at the position D and B be $n \times p + m \times q = gh$ (let); $g, h \in N \cup \{0\}$. The carry number e is added with gh which gives gh + e=ij, Where i is the carry number. j is included in the result left to f.

Number of points of intersection at A be $m \times p = kl$ (let). The carry number i is added with kl which gives kl + i = st. st is included in the result left to j.

Finally, we get the result of the multiplication by arranging the numbers st, j, f and b at their place value.

$$\therefore \quad mn \times pqr = stjfb$$

We may describe example – 1 with this logic as follow -

Number of points of intersection at F = $5 \times 3 = 15$; Here, 1 is the carry number.

Total number of points of intersection at E and C be $5 \times 2 + 4 \times 3 = 22$.

Adding the carry number 1, we get

$$22 + 1 = 23; \text{ here, 2 is the carry number.}$$

Total number of points of intersection at D and B be $5 \times 1 + 4 \times 2 = 13$.

Adding the carry number 2, we get

$$13 + 2 = 15; \text{ here, 1 is the carry number.}$$

Number of points of intersection at A be $4 \times 1 = 4$. Adding the carry number 1, we get

$$4 + 1 = 5$$

Hence,

$$\therefore\ 45 \times 125 = 5535.$$

6. Commutativity character of Japanese multiplication method

As the number of points of intersection at different positions is obtained by multiplying number of horizontal parallel lines with number of vertical parallel lines which must be equal to the multiplication of number of vertical parallel lines with number of horizontal parallel lines. So, it shows the commutativity character.

7. Vertically and Crosswise Vedic method for multiplication

For vertically and crosswise product of mn and pqr, we have to make number of digits of two number to be same. For this we include 0 (Zero) at hundreds place of mn and mn is written as 0mn.

Multiplication is written as -

$$\begin{array}{rccc} & p & q & r \\ \times & 0 & m & n \\ \hline \end{array}$$

Multiplication process is as follow

(i) Step – 1. $$\begin{matrix} p & q \\ 0 & m \end{matrix}\begin{pmatrix} r \\ n \end{pmatrix} = r \times n = ab$$

$$a, b \in N \cup \{0\}, ab = 1 \times b + 10 \times a$$

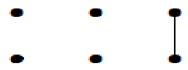


Fig.–9

Here, a be the carry number and b is included in the result.

(ii) Step – 2. $$\begin{matrix} p \\ 0 \end{matrix}\begin{pmatrix} q & r \\ m & n \end{pmatrix} = n \times q + r \times m = cd$$

and $cd + a = ef$

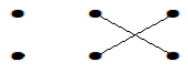


Fig.–10

Here, e be the carry number and f is included in the result left to b of step – 1.

(iii) Step – 3. $$\begin{pmatrix} p & q & r \\ 0 & m & n \end{pmatrix} = n \times p + r \times 0 + q \times n = gh$$

and $gh + e = ij$

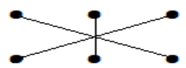


Fig.–11

Here, i be the carry number and j is included in the result left to f of step – 2.

(iv) Step – 4. $$\begin{pmatrix} p & q \\ 0 & m \end{pmatrix}\begin{matrix} r \\ n \end{matrix} = m \times p + q \times 0 = kl$$

and $kl + i = st$

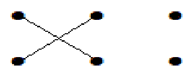


Fig.–12

Here, s be the carry number and t is included in the result left to j of step – 3.

(v) Step – 5. $$\begin{pmatrix} p \\ 0 \end{pmatrix}\begin{matrix} q & r \\ m & n \end{matrix} = p \times 0 = 0$$

and $o + s = s$

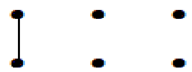


Fig.–13

Here, s is included in the result left to t of step – 4.

Hence,

$$\therefore \; mn \times pqr = stifb$$

Example – 7. Evaluate: 45×123

We write

$$\begin{array}{rrrr} & 1 & 2 & 3 \\ \times & 0 & 4 & 5 \\ \hline \end{array}$$

(i) Step – 1. $$\begin{matrix} 1 & 2 \\ 0 & 4 \end{matrix}\begin{pmatrix} 3 \\ 5 \end{pmatrix} = 3 \times 5 = 15$$

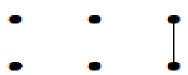


Fig.–14

Here, 1 be the carry number and 5 is included in the result.

(ii) Step – 2. $\begin{matrix} 1 \\ 0 \end{matrix}\begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix} = 5 \times 2 + 3 \times 4 = 22$

and $22 + 1 = 23$

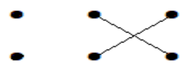


Fig.–15

Here, 2 be the carry number and 3 is included in the result left to 5 of step – 1.

(iii) Step – 3. $\begin{pmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \end{pmatrix} = 5 \times 1 + 3 \times 0 + 2 \times 4 = 13$

and $13 + 2 = 15$

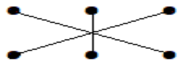


Fig.–16

Here, 1 be the carry number and 5 is included in the result left to 3 of step – 2.

(iv) Step – 4. $\begin{pmatrix} 1 & 2 \\ 0 & 4 \end{pmatrix}\begin{matrix} 3 \\ 5 \end{matrix} = 4 \times 1 + 2 \times 0 = 4$

and $4 + 1 = 5$

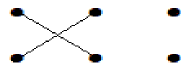


Fig.–17

Here, 5 is included in the result left to 5 of step – 3.

(v) Step – 5. $\begin{pmatrix} 1 \\ 0 \end{pmatrix}\begin{matrix} 2 & 3 \\ 4 & 5 \end{matrix} = 1 \times 0 = 0$

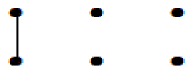


Fig.–18

Here, 0 is included in the result left to 5 of step – 4.

Hence,

$\therefore\ 45 \times 123 = 5535.$

In short we may calculate this multiplication with the help of above process as follow

$$\begin{array}{r} 1\ \ 2\ \ 3 \\ \times\ \ 0\ \ 4\ \ 5 \\ \hline 0\ 5_1 5_2 3_1 5 \end{array}$$

$\therefore$ Answer = 5535.

Example – 8. Evaluate: 23×14

Multiplication process is done by following steps -

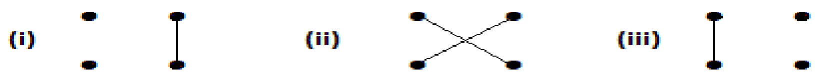


Fig.–19

$$\begin{array}{r} 2\ \ 3 \\ \times\ \ 1\ \ 4 \\ \hline 3\ {}_1 2\ {}_1 2 \end{array}$$

$\therefore$ Answer = 322.

Example – 9. Evaluate: 132 × 141

$$\begin{array}{r} 1\ \ 3\ \ 2 \\ \times\ 1\ \ 4\ \ 1 \\ \hline 1\,8\,{}_{1}6\,{}_{1}1\,2 \end{array}$$

∴ Answer = 18612.

Example – 10. Evaluate: 102 × 141

$$\begin{array}{r} 1\ \ 0\ \ 2 \\ \times\ 1\ \ 4\ \ 1 \\ \hline 1\,4\,3\,8\,2 \end{array}$$

∴ Answer = 14382.

8. Logic for the Procedure

Since the multiplication process is done by following steps –

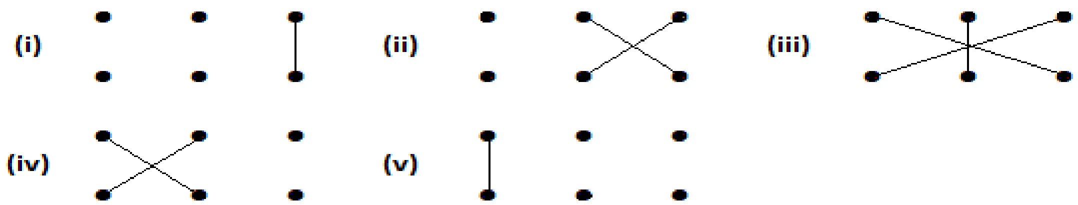
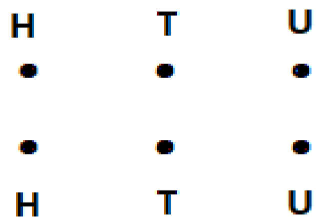


Fig.–20

Place values of dots are shown in table-1

Table-1

(Place value at different position of dots)



Where, U = Unit, T = Tens, H = Hundreds.

Step – (i) is Unit × Unit = Unit

Step – (ii) is Unit × Tens + Unit × Tens = Tens

Step – (iii) is Unit × Hundreds + Unit × Hundreds + Tens × Tens = Hundreds

Step – (iv) is Tens × Hundreds + Tens × Hundreds = Thousands

Step – (v) is Hundreds × Hundreds = Ten Thousands

9. Comparision of two methods

(i) Japanese method of multiplication is performed with the help of geometrical figure where as in Vedic method it is evaluate mentally. So, Japanese method is more effective for a person having less knowledge.

(ii) Without knowledge of addition and multiplication table but with the help of counting ability, the Japanese method of multiplication can be performed where as during the course of Vedic method, the knowledge of addition and multiplication is necessary.

(iii) Japanese method is more time consuming and Vedic method is less time consuming. So, Vedic method acts as a speedy calculator.

(iv) In Japanese method, more space is necessary but in Vedic method very less space is required i.e., in a single line it can be evaluated.

(v) Numbers containing small digits gives quick result in Japanese method since number of lines drawn is less. In Vedic method, it will take less time than the same case of Japanese method.

(vi) Number containing large digits will take more time to draw more number of parallel lines. So, in this case Japanese method is more time consuming. But Vedic method is easy and less time consuming with a little practice.

References

[1] Fugisawa Rikitarou, Sugaku Kyoujyuhou Kougi Hikki, Dainihonn Tosyo, 1900.

[2] Jagatguru Sankaracarya Sri Bharati Krsna Tirtha Maharaja and, V.S.Agrawala, *Vedic mathematics*, Motilal Banarsidass Publishers Private Limited, Delhi, pp. 33 – 44, 1965.

[3] A.P. Nicholas, K.R. William, and j. Pickles, *Vertically and Crosswise,* Motilal Banarsidass Publishers Private Limited, Delhi, pp.1 – 3, 1999.