

Heat Transfer in a Viscoelastic Fluid Over a Stretching Sheet

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Abstract: *An analysis is carried out to study the flow and heat transfer characteristics in the laminar boundary layer flow of visco elastic fluid over a non-isothermal stretching sheet with internal heat generation. A numerical method, Quasilinearization technique is used to study velocity and temperature profiles of the fluid. Heat transfer analysis is carried out for two types of thermal boundary conditions namely, (i) Prescribed Surface temperature (PST) and (ii) Prescribed wall Heat Flux (PHF). The effects of various parameters such as Prandtl number, suction, visco-elasticity and temperature parameter on flow and heat transfer are presented through graphs and discussed.*

Keywords: *Boundary layer flow, Heat transfer, Visco elastic fluid, Stretching sheet, Heat generation or absorption*

I. INTRODUCTION

Flow induced by a stretching sheet of Newtonian or Non-Newtonian fluid is a classical problem in Fluid mechanics. Study of momentum and heat transfer in laminar boundary layer flow of a viscoelastic fluid over a linearly stretching sheet is of great importance in polymer processing industries e.g., in manufacturing artificial fibres and artificial films etc., Sakiadis [1] initiated the study of these applications by considering the boundary layer flow over a continuous solid surface moving with constant speed. Erickson et al [2] extended this problem to the case in which the transverse velocity at the moving surface is non zero, with heat and mass transfer in the boundary layer being taken into account.

The above investigations have a definite bearing on the problem of a polymer sheet extruded continuously from a die. It is usually assumed that the sheet is inextensible, but situations may arise in the polymer industry in which it is necessary to deal with a stretching plastic sheet, as noted by Crane [3]. Danberg and Fansler [4] investigated the non-similar solution for the flow in the boundary layer past a wall that is stretched with a velocity proportional to the distance along the wall.

Dutta, Roy and Gupta [5] have analyzed the temperature distribution in the flow over a stretching sheet with uniform wall heat flux. Vajravelu and Rollins [6] investigated the heat transfer characteristics in a visco-elastic fluid over a continuous impermeable, linearly stretching sheet with PST and PHF cases. They studied the heat transfer characteristics in fluid initially at rest and at uniform temperature. As there is an appreciable temperature difference between the surface and the ambient fluid, one needs to consider the temperature dependent heat sources or sinks which may exert strong influence on the heat transfer characteristics.

Foraboschi and Federico [7] studied heat transfer in a laminar flow of non-Newtonian heat generation fluids. Vajravelu and Hadjinicolaou [8] have studied the heat transfer characteristics in a viscous fluid over a stretching sheet with viscous dissipation and internal heat generation. Vajravelu and Roper [9] analyzed heat transfer characteristics in a second grade fluid over stretching sheet with viscous dissipation, internal heat generation, and work due to deformation. Rollins and Vajravelu[10] studied flow and heat transfer characteristics in a second order fluid over a stretching sheet and obtained analytical solutions for flow and heat transfer Kummer's function.

In present paper, a numerical solution for flow and heat transfer of an incompressible second order fluid past stretching sheet with internal heat generation or absorption is obtained. A numerical approach, Quasilinearization technique is used to study flow and heat transfer characteristics of the fluid. This approach is easily adoptable and it is observed that results are in good agreement with the available literature. The effects of various parameters on flow and heat transfer are analyzed through numerical calculations.

II. MATHEMATICAL FORMULATION:

Following the postulates of gradually fading memory, Coleman and Noll [11] derived the constitutive equation of second order fluid flow in the form

$$T = -pI + \mu A_1 + \alpha_1 A_2 + \alpha_2 A_1^2 \tag{1}$$

where T is the Cauchy stress tensor, $-pI$ is the spherical stress due to constraint of incompressibility, μ is the dynamics viscosity, α_1, α_2 are the material constants and A_1 and A_2 are the first two Rivlin–Ericksen tensors [12] defined as

$$A_1 = (\text{grad } v) + (\text{grad } v)^T \tag{2}$$

$$A_2 = \frac{dA_1}{dt} + A_1(\text{grad } v) + (\text{grad } v)^T A_1 \tag{3}$$

Here, v denotes the velocity field and d/dt is the material time derivative. If the fluid of second grade modelled by (1) is to be compatible with thermodynamics and is to satisfy the Clausius-Duhem inequality for all motions and the assumption that the specific Helmholtz free energy of the fluid is a minimum when it is locally at rest, Dunn and Fosdick [13] found that the material moduli must satisfy

$$\mu \geq 0, \alpha_1 \geq 0, \alpha_1 + \alpha_2 = 0 \tag{4}$$

But later on Fosdick and Rajagopal [14] have reported, by using the data reduction from experiments, that in the case of a second order fluid the material constants μ, α_1, α_2 should satisfy the relation

$$\mu \geq 0, \alpha_1 \leq 0, \alpha_1 + \alpha_2 \neq 0 \tag{5}$$

They also reported that the fluids modeled by (1) with the relationship (5) exhibit some anomalous behavior. A critical review on this controversial issue can be found in the work of Dunn and Rajagopal [15]. Second-order fluid, obeying model equation (1) with $\alpha_1 < \alpha_2, \alpha_1 < 0$ although exhibits some undesirable instability characteristics, its approximations are valid at low shear rate. Now in literature the fluid satisfying the model equation (1) with $\alpha_1 < 0$ is termed as second-order fluid and with $\alpha_1 < 0$ is termed as second grade fluid.

A laminar steady flow of an incompressible viscoelastic (Walters’ liquid B model) fluid over a wall coinciding with the plane $y = 0$ is considered, the flow being confined to $y > 0$. Two equal and opposite forces are applied along the x-axis, so that a sheet is stretched with a velocity proportional to the distance from the origin. The resulting motion of the quiescent fluid is thus caused solely by the moving surface. The flow satisfies the rheological equation of state derived by Beard and Walters [16]. The steady two-dimensional boundary layer equations for this fluid, in the usual form are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{6}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial v}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - k_0 \left\{ \frac{\partial}{\partial x} \left(u \frac{\partial^2 u}{\partial y^2} \right) + \frac{\partial u}{\partial y} \frac{\partial^2 v}{\partial y^2} + v \frac{\partial^3 u}{\partial y^3} \right\} \tag{7}$$

where $\nu = \frac{\mu}{\rho}, k_0 > 0$

Where u and v are the velocity components along the x and y directions respectively, ν is the kinematic viscosity $k_0 = -\alpha_1 / \rho$ is the co-efficient of elasticity, and ρ is the density. Hence, in the case of second order fluid flow, k_0 takes positive value, as α_1 takes negative value and other quantities have their usual meanings. In deriving the equation (7), it is assumed that the normal stress is of the same order of magnitude as that of the shear stress, in addition to usual boundary layer approximations.

The boundary conditions for the velocity field are

$$u = u_w = bx, v = 0 \quad \text{at } y = 0, b > 0$$

$$u \rightarrow 0, \frac{\partial u}{\partial y} \rightarrow 0 \quad \text{as } y \rightarrow \infty \tag{8}$$

The condition $\partial u / \partial y \rightarrow 0$ as $y \rightarrow \infty$ is the augmented condition, since the flow is in an unbounded domain, which has been discussed by Rajgopal [17]. In this case, the flow is caused solely by the stretching of the sheet, since the free stream velocity is zero.

Defining new variables:

$$u = bx f_{\eta}(\eta), v = -\sqrt{b\nu} f(\eta), \eta = \sqrt{b/\nu} y \tag{9}$$

where $f_{\eta}(\eta)$ denotes differentiation with respect to η .

Clearly u and v defined above satisfy the continuity equation (6), and the equation (7) reduces to

$$f_{\eta}^2 - ff_{\eta\eta} = f_{\eta\eta\eta} - k_1 \{2f_{\eta} f_{\eta\eta\eta} - ff_{\eta\eta\eta\eta} - f_{\eta\eta}^2\} \tag{10}$$

where $k_1 = k_0 b / \nu$, is the visco-elastic parameter.

The boundary conditions (8) become

$$f(0) = 0, f_{\eta}(0) = 1 \tag{11a}$$

$$f_{\eta}(\eta) \rightarrow 0, f_{\eta\eta}(\eta) \rightarrow 0 \text{ as } \eta \rightarrow \infty \tag{11b}$$

1. Heat Transfer Analysis:

The governing boundary layer equation with internal heat generation or absorption is given by

$$\rho C_p \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \frac{\partial^2 T}{\partial y^2} + Q(T - T_{\infty}) \tag{12}$$

Where k is the thermal diffusivity and c_p is the specific heat of a fluid at constant pressure and Q is the volumetric rate of heat generation.

Since there is an appreciable temperature difference between the surface and the ambient fluid, temperature dependent heat sources or sinks exert strong influence on the heat transfer characteristics so above equation (12) includes internal heat generation or absorption

Here the thermal boundary condition depends on the type of heating process under consideration. Heat transfer analysis is carried out for two types of thermal boundary conditions namely, (i) Prescribed Surface temperature (PST) and (ii) Prescribed wall Heat Flux (PHF), which are given below.

A. Prescribed Surface Temperature (PST case):

For this circumstance, the boundary conditions are

$$T = T_w \left[= T_{\infty} + A \left(\frac{x}{l} \right)^r \right] \text{ at } y=0 \tag{13a}$$

$$T \rightarrow T_{\infty} \text{ as } y \rightarrow \infty \tag{13b}$$

Here l is the characteristic length and r is the temperature parameter. Define non-dimensional temperature as

$$\theta(\eta) = \frac{T - T_{\infty}}{T_w - T_{\infty}} \tag{14}$$

Using (14), equation (12) reduces to

$$\theta'' + Pr f\theta' - (r Pr f' - \alpha)\theta = 0 \tag{15}$$

where $Pr = \mu c_p / k$, Prandtl number

$\alpha = Q / (b\rho c_p)$, heat source/sink parameter.

with boundary conditions

$$\theta(0) = 1, \theta(\infty) \rightarrow 0 \tag{16}$$

B. Prescribed surface Heat Flux (PHF Case):

Prescribed power law surface heat flux (PHF), where surface is subjected to a power law heat flux q_w on the wall surface is considered to be a quadratic power of x in the form

$$-k \frac{\partial T}{\partial y} = q_w = D \left(\frac{x}{l} \right)^r \text{ at } y = 0 \tag{17}$$

$$T \rightarrow T_\infty \text{ as } y \rightarrow \infty$$

Where D is a constant, k is the thermal conductivity

Define a dimensionless, scaled temperature as

$$g(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \tag{18}$$

where $T_w - T_\infty = \frac{D}{k} \left(\frac{x}{l} \right)^r \sqrt{\frac{\nu}{b}}$

Using (18), equation (12) reduces to

$$g_{\eta\eta} + Pr f g_\eta - (r Pr f_\eta - \alpha)g = 0 \tag{19}$$

boundary conditions are:

$$\begin{aligned} g_\eta(\eta) &= -1 \text{ at } \eta = 0 \\ g(\eta) &\rightarrow 0 \text{ as } \eta \rightarrow \infty \end{aligned} \tag{20}$$

III. NUMERICAL SOLUTION OF THE PROBLEM

The flow equation (10) coupled with energy equation (15) or (19) constitute a set of highly nonlinear differential equations in each thermal boundary condition. So obtaining closed form solution for this set is cumbersome and time consuming. Hence quasilinearization method, given by Bellman & Kalaba [18] is used to solve this system. This method is quadratically convergent, starting from the initial guess value and the solution is valid for a large range of parameters. Even when the required number of initial conditions is not given, this method converges very fast.

For convenience equations (10), (15) and (19) are rearranged as

$$f^{iv} = \frac{1}{k_1 f} \left[(f')^2 - ff'' - f''' + 2k_1 f f''' - k_1 (f'')^2 \right] \tag{20}$$

$$\theta'' = -Pr f\theta' + (r Pr f' - \alpha)\theta \quad \text{(PST case)} \tag{21a}$$

$$g'' = -Pr fg' + (r Pr f' - \alpha)g \quad \text{(PHF case)} \tag{21b}$$

In order to implement the quasilinearization method, the equations (20) and (21) are converted to a system of first order differential equations by substituting

$$(f, f', f'', f''', \theta, \theta') = (x_1, x_2, x_3, x_4, x_5, x_6) \quad \text{(PST case)}$$

(OR)

$$(f, f', f'', f''', g, g') = (x_1, x_2, x_3, x_4, x_5, x_6) \quad \text{(PHF case)}$$

Then equations (20) and (21) reduce to:

$$\frac{dx_1}{d\eta} = x_2$$

$$\frac{dx_2}{d\eta} = x_3$$

$$\begin{aligned} \frac{dx_3}{d\eta} &= x_4 \\ \frac{dx_4}{d\eta} &= \frac{1}{k_1 x_1} \{x_2^2 - x_1 x_3 - x_4 + 2k_1 x_2 x_4 - k_1 x_3^2\} \\ \frac{dx_5}{d\eta} &= x_6 \\ \frac{dx_6}{d\eta} &= -\text{Pr } x_1 x_6 + (r \text{Pr } x_2 - \alpha)x_5 \quad (\text{PST \&PHF cases}) \end{aligned} \tag{22}$$

Using Quasilinearization technique, the system (22) can be linearized as

$$\begin{aligned} \frac{dx_1^{r+1}}{d\eta} &= x_2^{r+1} \\ \frac{dx_2^{r+1}}{d\eta} &= x_3^{r+1} \\ \frac{dx_3^{r+1}}{d\eta} &= x_4^{r+1} \\ \frac{dx_4^{r+1}}{d\eta} &= \left(\frac{-1}{k_1 (x_1^r)^2} \left((x_2^r)^2 - x_4^r + 2k_1 x_2^r x_4^r - k_1 (x_3^r)^2 \right) \right) x_1^{r+1} \\ &\quad + \left(\frac{1}{k_1 x_1^r} (2x_2^r + 2k_1 x_4^r) \right) x_2^{r+1} \\ &\quad + \left(\frac{1}{k_1 x_1^r} (-x_1^r - 2k_1 x_3^r) \right) x_3^{r+1} \\ &\quad + \left(\frac{1}{k_1 x_1^r} (-1 + 2k_1 x_2^r) \right) x_4^{r+1} + \left(\frac{-x_4^r}{k_1 x_1^r} \right) \\ \frac{dx_5^{r+1}}{d\eta} &= x_6^{r+1} \\ \frac{dx_6^{r+1}}{d\eta} &= (-\text{Pr } x_6^r)x_1^{r+1} + (r \text{Pr } x_5^r)x_2^{r+1} + (r \text{Pr } x_2^r - \alpha)x_5^{r+1} \\ &\quad - \text{Pr } x_1^r x_6^{r+1} + (\text{Pr } x_1^r x_6^r - r \text{Pr } x_2^r x_5^r) \end{aligned} \tag{23}$$

The above system of equations (23) is linear in $x_i^{r+1} (i = 1, 2, \dots, 6)$ and general solution can be obtained by using the principle of superposition.

The boundary conditions given by (11), (16) and (20) reduce to

$$x_1^{r+1}(0) = 0, \quad x_2^{r+1}(0) = 1, \quad x_5^{r+1}(0) = 1 \quad \text{for PST case} \tag{24a}$$

$$x_1^{r+1}(0) = 0, \quad x_2^{r+1}(0) = 1, \quad x_6^{r+1}(0) = -1 \quad \text{for PHF case} \tag{24b}$$

$$x_2^{r+1}(\eta) \rightarrow 0, \quad x_3^{r+1}(\eta) \rightarrow 0, \quad x_5^{r+1}(\eta) \rightarrow 0 \quad \text{as } \eta \rightarrow \infty$$

The initial values are chosen as follows:

For homogeneous solution:

$$\begin{aligned} x_i^{h_1}(\eta) &= [0 \ 0 \ 1 \ 0 \ 0 \ 0] \\ x_i^{h_2}(\eta) &= [0 \ 0 \ 0 \ 1 \ 0 \ 0] \\ x_i^{h_3}(\eta) &= [0 \ 0 \ 0 \ 0 \ 0 \ 1] \end{aligned} \quad \text{(PST case)} \tag{25}$$

(OR)

$$x_i^{h_3}(\eta) = [0 \ 0 \ 0 \ 0 \ 1 \ 0] \quad \text{(PHF case)}$$

For particular solution:

$$x_i^p(\eta) = [0 \ 1 \ 0 \ 0 \ 1 \ 0] \quad \text{(PST case)} \tag{26a}$$

(OR)

$$x_i^p(\eta) = [0 \ 1 \ 0 \ 0 \ 0 \ -1] \quad \text{(PHF case)} \tag{26b}$$

The general solution for the system (23) is given by

$$x_i^{r+1}(\eta) = C_1 x_i^{h_1}(\eta) + C_2 x_i^{h_2}(\eta) + C_3 x_i^{h_3}(\eta) + x_i^p(\eta) \tag{27}$$

where C_1, C_2, C_3 are the unknown constants, which are to be determined by considering the boundary conditions as $\eta \rightarrow \infty$. This solution ($x_i^{r+1}, i = 1, 2, \dots, 6$) is then compared with solution at the previous step ($x_i^r, i = 1, 2, \dots, 6$) and next iteration is performed, if the convergence has not been achieved or greater accuracy is desired.

IV. RESULTS AND DISCUSSIONS

The flow and heat transfer characteristics of an incompressible second order fluid past a stretching sheet is studied for two types of thermal boundary conditions i) Prescribed Surface Temperature (PST) and ii) Prescribed Heat Flux (PHF), in the presence of internal heat generation or absorption, have been examined. The governing boundary layer equations are solved using Quasilinearization Technique. The computational results of flow and heat transfer characteristics for various parameters are presented in graphs and discussed.

Fig 1(a) and Fig. 1(b) depict the effect of viscoelastic parameter k_1 on longitudinal and transverse velocity components. It can be seen, for a fixed value of η , both $f'(\eta)$ and $f(\eta)$ decrease with increasing values of viscoelastic parameter k_1 . This can be explained by the fact that, as the viscoelastic parameter k_1 increases, the boundary layer adheres strongly to the surface, which in turn retards the flow in longitudinal and transverse directions. In Fig 2(a) and 2(b), non-dimensional temperature $\theta(\eta)$ is plotted for various values internal heat source/sink parameter (α). It shows that $\theta(\eta)$ increases with increasing values α . This is due to the fact that heat is generated inside the boundary layer for increasing values of heat source/sink parameter (α).

Fig 3(a) and 3(b) reveal the effect of Prandtl number (Pr) on non-dimensional temperature $\theta(\eta)$ profiles are shown. Temperature $\theta(\eta)$ decreases with increase in the Prandtl number Pr. This is consistent with the fact that the thermal boundary layer thickness decreases with increasing values Prandtl number Pr. In Fig 4(a) and 4(b) display the values of temperature $\theta(\eta)$ for different values of viscoelastic parameter (k_1). It can be observed that, at a given point η , $\theta(\eta)$ increases with increasing values of k_1 . This is due to the fact that viscoelastic normal stress gives rise to thickening of thermal boundary layer.

In Fig 5, the effect of temperature parameter(r) on the temperature distribution is depicted. When $r > 0$, the heat flows from the stretching sheet to the ambient medium. When $r < 0$, the temperature gradient is now positive and heat flows into the stretching sheet from the ambient medium.

The heat transfer phenomena is usually analyzed from the numerical values of two physical parameters namely, wall temperature gradient $\theta'(0)$ and wall temperature $g(0)$. The values of wall temperature gradient – $\theta'(0)$ and wall temperature $g(0)$ as a function of all the parameters of the thermal boundary layer are analyzed

and tabulated in Table 1, which reveals that the increase in visco elastic parameter (k_1) is to increase the wall temperature gradient $\theta'(0)$ in PST case and wall temperature $g(0)$ in PHF case. The increase in value of heat source/sink parameter (α) is to increase both $\theta'(0)$ and $g(0)$. Effect of Prandtl number (Pr) is to decrease the magnitude in both the cases.

From our numerical results, it can be concluded that, temperature of the fluid

- Increases with increasing values of viscoelastic parameter k_1 . Hence viscoelastic liquids having low viscous dissipation must be chosen.
- Increases with increasing values of heat source/sink parameter (α). Hence heat source/sink parameter (α) is better suited for cooling purposes.
- Decreases with increasing values of Prandtl number Pr .
- The results of the PHF cases are qualitatively similar to that of the PST case, but quantitatively in the reduced magnitude.
- The Power Law Heat Flux (PHF) boundary condition is better suited for effective cooling of the stretching sheet.

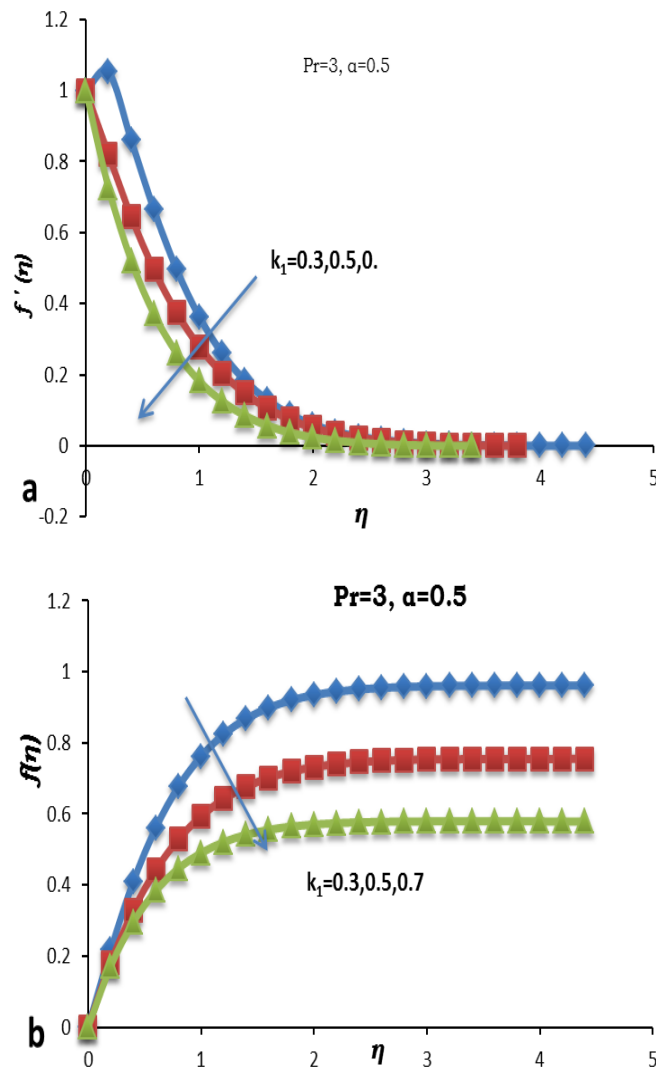


Fig.1: Effect of viscoelasticity (k_1) on (a) transverse velocity component, (b) longitudinal component

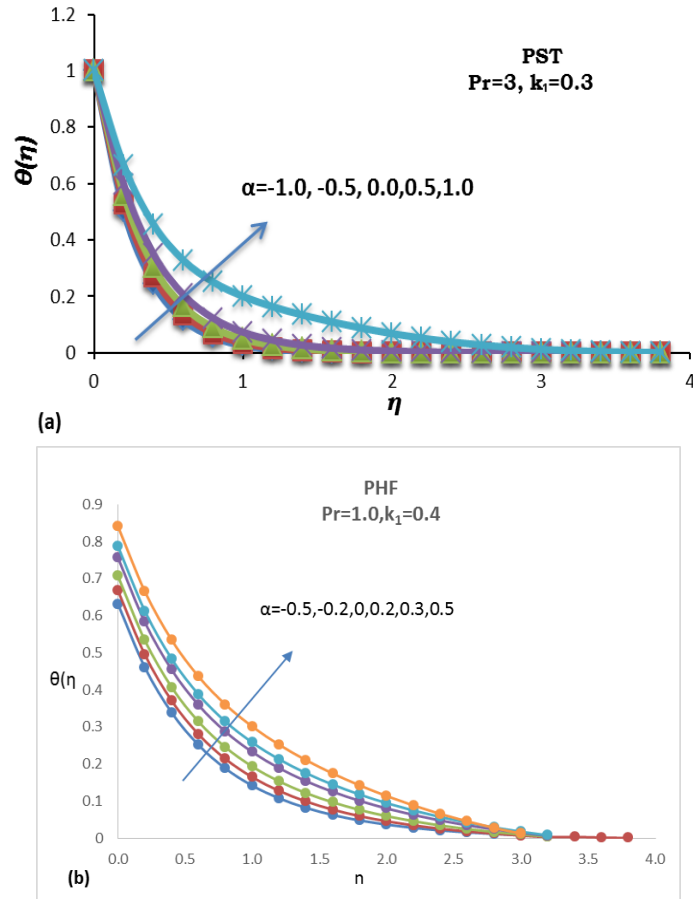
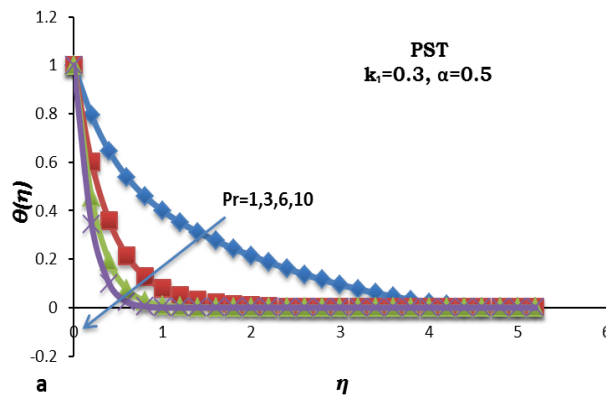


Fig. 2: Effect of heat source/sink parameter (α) on temperature distribution $\theta(\eta)$ in **a)** PST case **b)** PHF case



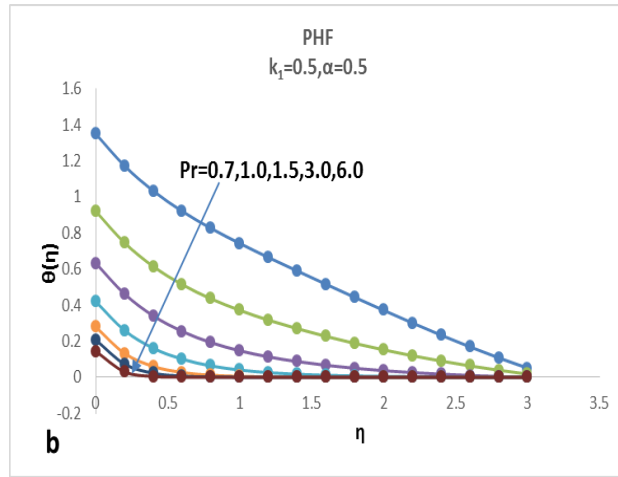


Fig3: Effect of Prandtl number (Pr) on temperature distribution $\theta(\eta)$ in (a) PST case (b) PHF case

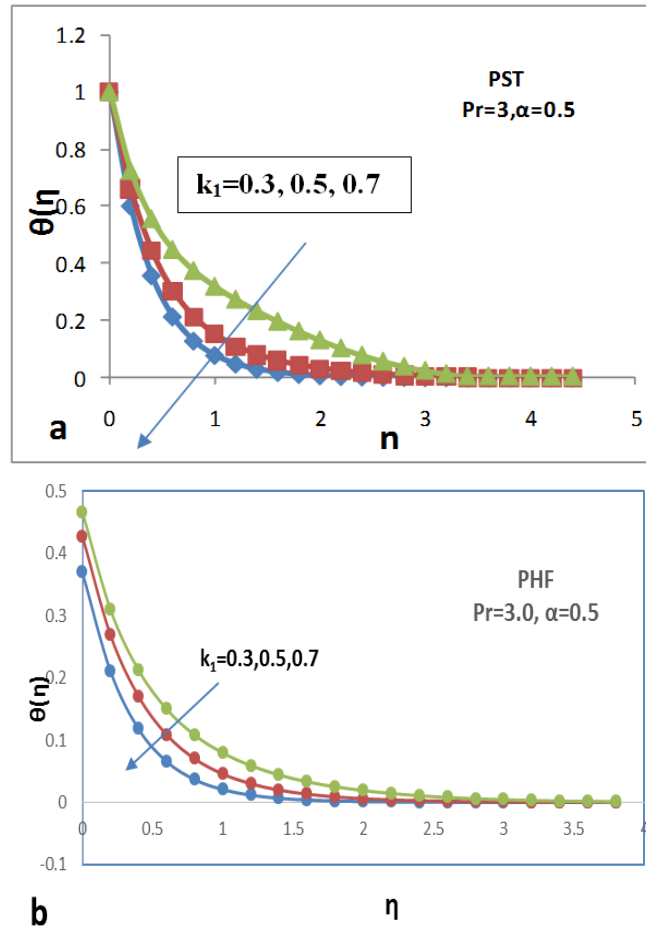


Fig. 4. Effect of viscoelastic parameter (k_1) on $\theta(\eta)$ in a) PST case and b) PHF case

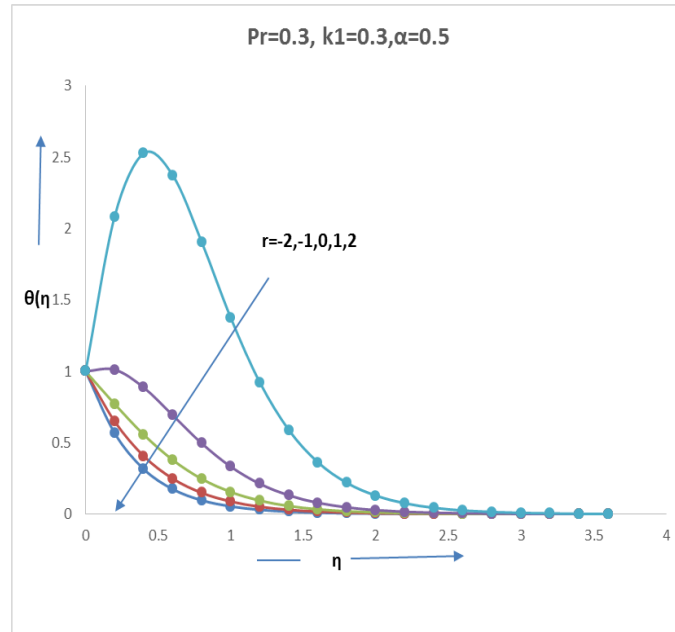


Fig. 5: Variation of $\theta(\eta)$ vs η for different values of wall temperature parameter(r)

TABLE I: VALUES OF TEMPERATURE GRADIENTS - $\theta'(0)$ AND WALL TEMPERATURE $G(0)$

Visco-elastic parameter (k_1)	Prandtl number (Pr)	Heat source/sink parameter (α)	Temperature parameter (r)	PST $\theta'(0)$	PHF $g(0)$
0.3	3	0.5	2	-2.46416	0.36954
0.5				-2.09070	0.42805
0.7				-1.75928	0.46702
0.4	1	-0.5	2	-3.07090	0.63148
		0		-2.80242	0.70688
		0.5		-2.49414	0.84066
0.5	1	0.5	2	-1.19491	0.91972
	3			-2.46416	0.41900
	6			-3.63958	0.27815
	10			-4.74637	0.20677

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