Pairwise S**G - Connectedness in Bitopological Spaces

V.Subha[*1], N. Seenivasagan[#2], A. Edward Samuel[#1]

[#1]Department of Mathematics, Govt Arts College, Kumbakonam, INDIA
[#2]Department of Mathematics, Govt Arts College for Women, Nilakottai, INDIA

Abstract — *In this paper, we introduce the new type of connected and disconnected spaces called pairwise s**g - connected spaces, pairwise s**g - disconnected spaces.*

Mathematics Subject Classification 54E55
Keywords —*pairwise s**g - connected spaces and pairwise s**g - disconnected spaces.*

1. INTRODUCTION

Connectedness and compactness are powerful tools in topology but they have many dissimilar properties. The concept of Hausdorff spaces is almost an integral part of compactness. Investigations into the properties of cut points of topological spaces which are connected, compact and Hausdorff date back to the 1920's. Connectedness together with compactness with the assumption of Hausdorff has been studied in from the view point of cut points. A cover (or covering) of a space X is a collection μ of subsets of X whose union is X. The axioms that involve the notion of coverings are known as covering axioms. Compactness, one of the oldest covering axiom, plays almost the same role in General Topology as the closed and bounded intervals play in Classical Analysis. Pervin [4] was first to define connectedness and components in a bitopological spaces, whereas the concept of quasi components in bitopological spaces was introduced by Reilly and Young [6]. Recently, the notions of pairwise S*GO - connected spaces was introduced by K.Kannan [1] in bitopological spaces in 2009. In this section we introduce the new type of connected and disconnected spaces called pairwise *s**g* - connected spaces, pairwise *s**g* - disconnected spaces.

2. PRELIMINARIES

Let (X, τ_1, τ_2) be a bitopological space or simply X. For any subset A is contained in X, the interior of A is denoted by τ_i– int(A) and the closure of A is denoted by τ_i– cl(A), respectively.A^c or X – A denotes the complement of A in X unless explicitly stated.We shall now require the following known definitions.

Definition 2.1 Let (X, τ_1, τ_2) be a bitopological space. Then (X, τ_1, τ_2) is said to be pairwise connected if X cannot be expressed as the union of two non-empty disjoint sets A and B such that $(A \cap \tau_1 \text{cl}(B)) \cup (\tau_2 \text{cl}(A) \cap B) = \phi$ … (1). If (1) is satisfied, we call A and B as pairwise separated sets. If $X = A \cup B$, where A, B satisfy (1), then X is called a pairwise disconnected space. In this case, we write X = A \B, a pairwise separation of X.

Definition 2.2 A function $f : (X, \tau_1, \tau_2) \to (Y, \mu_1, \mu_2)$ is said to be pairwise continuous if $f^{-1}(U)$ is τ_j- closed in X for every μ_j closed set U in Y.

Definition 2.3 A function $f : (X, \tau_1, \tau_2) \to (Y, \mu_1, \mu_2)$ is said to be pairwise pre semi closed if f(U) is μ_j – semi closed in Y for every τ_j- semi closed set U in X.

Definition 2.4 A subset A of a bitopological space (X, τ_1, τ_2) is called pairwise semi star generalized closed if τ_2– cl (A) $\subseteq$ U whenever A$\subseteq$ U and U is τ_1- semi open in X.

Definition 2.5 A subset A of a bitopological space (X, τ_1, τ_2) is called pairwise semi star generalized open if X-A is pairwise semi star generalized closed in X.

Definition 2.6 A function $f :(X, \tau_1, \tau_2) \to (Y, \mu_1, \mu_2)$ is said to be a pairwise continuous bijective and pairwise pre semi closed function if the inverse image of each μ_i-s*g closed set in Y is τ_i -s*g closed set in X.

3. PAIRWISE S**G - CONNECTED SPACES

Definition 3.1. Let (X, τ_1, τ_2) be a bitopological space. Then (X, τ_1, τ_2) is said to be pairwise s**g - connected if X cannot be expressed as the union of 2 non empty disjoint sets A and B such that $(A \cap \tau_1\text{- s**g cl}(B)) \cup (\tau_2\text{- s**g cl}(A) \cap B) = \phi$ … (1). If (1) is satisfied, we call A and B as pairwise s**g - separated sets. If $X = A \cup B$, where A, B satisfy (1), then X is called a pairwise s**g - disconnected space. In this case, we write X = A \ B a pairwise s**g - separation of X.

Example 3.1. Let X = {a, b, c}, τ_1 = {ϕ, X, {a}} and τ_2 = {ϕ, X, {b,c}}.Then (X, τ_1, τ_2) is pairwise s**g - disconnected space.

Example 3.2. Let X = {a, b, c, d}, τ_1 = {ϕ, X, {a}} and τ_2 = {ϕ, X, {a,b}, {a, b, c}}. Then (X, τ_1, τ_2) is pairwise s**g - connected.

Theorem 3.1 The following conditions are equivalent for any bitopological space X.

(i) X is pairwise *s**g* – connected.

(ii) X cannot be expressed as the union of 2 non empty disjoint sets A and B such that A is τ_1 - s**g open and B is τ_2 - s**g open.

(iii) X contains no nonempty proper subset which is both τ_1 - s**g open and τ_2 - s**g closed

Proof. (i)⇒ (ii): Let X be pairwise *s**g* – connected…(1).Assume that X can be expressed as the union of two non empty disjoint sets A and B such that A is τ_1-s**g open and B is τ_2-s**g open. Then A ∩ B = ϕ.⇒ A ⊆B^c⇒ τ_2-s**gcl(A) ⊆ τ_2-s**gcl(B^c) = B^c⇒ τ_2-s**gcl(A) ∩B)=ϕ. ..(2). Similarly, B ⊆A^c⇒ τ_1-s**gcl(B) ⊆ τ_1 - s**gcl(A^c) = A^c.⇒ τ_1-s**gcl(B) ∩ A = ϕ…(3). From (2) and (3) we have (τ_1-s**gcl(B) ∩ A) ∪(τ_2-s**gcl(A) ∩ B)= ϕ. This is contradiction to (1). Hence X can not be expressed as the union of two nonempty disjoint sets A and B such that A is τ_1-s**g open and B is τ_2-s**g open.

(ii)⇒ (iii): Let X can not be expressed as the union of two nonempty disjoint sets A and B such that A is τ_1-s**g open and B is τ_2-s**g open…(1).Assume that X contains a nonempty proper subset A which is both τ_1-s**g open and τ_2-s**g closed. ⇒ X = A∪A^c where A, A^c are disjoint, A is τ_1-s**g open and A^c τ_2-s**g open. This is contradiction to (1).Hence X contains no nonempty proper subset which is both τ_1-s**g open and τ_2-s**g closed.

(iii) ⇒ (i) Let X contains no nonempty proper subset which is both τ_1-s**g open and τ_2-s**g closed.

Assume that X is pairwise s**g – disconnected. ⇒ X can be expressed as the union of two nonempty disjoint sets A and B such that (τ_1-s**gcl(B) ∩ A) ∪(τ_2-s**gcl(A) ∩ B) = ϕ. Since A ∩ B = ϕ, we have A = B^c…(1) and B = A^c…(2). Since τ_2-s**gcl(A) ∩ B = ϕ, we have τ_2-s**gcl(A) ⊆B^c.⇒ τ_2-s**gcl(A) ⊆ A. .⇒A is τ_2-s**g closed. Similarly, we have τ_1-s**gcl(B) ∩ A = ϕ. ⇒ τ_1-s**gcl(B) ⊆A^c⇒ τ_1-s**gcl(B) ⊆ B [by (2)] ⇒ B is τ_1-s**g closed. ⇒B^c is τ_1-s**g open. ⇒ A is τ_1-s**g open [by (1)]. Therefore, there exists a nonempty proper subset A which is both τ_1-s**g open and τ_2-s**g closed. This is contradiction to (1). Hence X is pairwise s**g – connected.

Theorem 3.2. If C is a pairwise *s**g* - connected subset of a bitopological space (X, τ_1, τ_2) which has the pairwises**g - separation X = A \ B then C ⊆ A or C ⊆ B.

Proof. Suppose that (X, τ_1, τ_2) has the pairwise s**g - separation X = A \ B. Then X = A ∪ B, where A and B are nonempty disjoint sets such that A ∩ (τ_1 - s**g cl(B)) ∪ (τ_2 - s**g cl(A) ∩ B) = ϕ … (1). Since A ∩ B = ϕ, we have A = B^c and B = A^c… (2). Now, ((C ∩ A) ∩ τ_1 - s**g cl(C ∩ B)) ∪ (τ_2 - s**g cl(C ∩ A) ∩ (C ∩ B)) ⊆ A ∩ τ_1 - s**g cl(B)) ∪ (τ_2 - s**g cl(A) ∩ B) = ϕ [by (1)]. ⇒ C ∩ A = ϕ or C ∩ B = ϕ. ⇒ C ⊆A^C(or) C ⊆B^C.⇒ C ⊆ B (or) C ⊆ A [by (2)].

Theorem 3.3. If A is a pairwise *s**g* - connected and A ⊆ B ⊆ τ_1 - s**g cl(A) ∩ τ_2 - s**g cl(A) then B is pairwise *s**g* - connected.

Proof . Suppose that B is not pairwise s**g - connected . Then B = C ∪ D, where C and D are 2 non empty disjoint sets such that (C ∩ τ_1 - s**g cl(D)) ∪ (τ_2 - s**g cl(C) ∩ D) = ϕ. Since A is pairwise s**g - connected, we have A ⊆ C or A ⊆ D. Suppose A ⊆ C. Then D ⊆D∩ B ⊆ D ∩ τ_2 - s**g cl(A) ⊆ D ∩ τ_2 - s**g cl(C) = ϕ. Therefore, ϕ⊆ D ⊆ϕ. Consequently, D = ϕ. Similarly, we can prove C =ϕ if A ⊆ D {by theorem 3.2}. This is the contradiction to the fact that C and D are nonempty. Therefore, B is pairwise *s**g* - connected.

Theorem 3.4. Let (X, τ_1, τ_2) be a bitopological space. If every 2 points of X are contained in some pairwise *s**g* - connected space of X then X is pairwise *s**g* - connected.

Proof. Assume that X is not pairwise *s**g* - connected. Then X = A\B. ⇒ X = A ∪ B, where A is τ_1 - s**g - open and τ_2 - s**g - open set with A ∩ B = ϕ. Let x∈ A and y∈ B. By hypothesis, there exists a pairwise s**g - connected subset C of X such that x ∈ C and y ∈ C, by the above theorem 3.2 , C ⊆ A or C ⊆ B. ⇒ x, y ∈ A or x, y ∈ B, a contradiction. Hence X is pairwise s**g - connected.

Theorem 3.5 Let f : (X, τ_1, τ_2) → (Y, μ_1, μ_2) be a pairwise continuous bijective and pairwise pre semi closed function then the image of a pairwise *s**g* connected space is pairwise *s**g* connected under f..

Proof: Let f : X → Y be a pairwise continuous surjection and pairwise pre semi closed. Let X be pairwise s**g connected…(1). Assume that Y is pairwise s**g -disconnected.Then Y = C ∪ D where C is μ_1 – s**g open and D is μ_2- s**g open in Y. Since f is pairwise continuous and pairwise pre semi closed, then we have f^{-1}(C) is τ_1 – s**g open and f^{-1}(D) is τ_2 –s**g open in X. ⇒ X = f^{-1}(A) ∪f^{-1}(B), f^{-1}(A) and f^{-1}(B) aretwo non empty disjoint sets. Hence X is pairwise s**g- disconnected, and this leads to a contradiction which proves that Y is pairwise s**g connected.

References

[1] K.Kannan, Contribution to the study of some generalized closed sets in bitopological spaces , March 2009 , (Ph.D Thesis) .

[2] K.Kannan, $\tau_1\tau_2$ - semi star generalized closed sets, International Journal of Pure and Applied Mathematics, Volume 76(2012), No. 2 , 277 - 294.

[3] K.Kannan and R.Aarthi , More on semi star generalized closed sets, Natl. Acad. Sci. Lett. 35 (2012), No. 6, 525 - 529.
[4] W.J.Pervin, Connectedness in bitopological spaces, Indag. Math., 29(1967), 369 – 372.
[5] Ivan L .Reily , On pairwise connected bitopological spaces, Kyungpook Math. J., 11(1971), 25 - 28.
[6] Youngs, J.W.T, A note on separation axioms & their application in the theory of a locally connected & topological space, Bull Amer. Math. Soc. 49 (1943), 383 - 385.
[7] Yogesh kumar , " A study of some separation and covering axioms in topological and bitopological spaces " , Ph. D. Thesis, Meerut Univ. Dec.1990.