Heronian Mean Labeling In The Context of Duplication Of Graph Elements

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ABSTRACT

Here we look into some graphs on Heronian Mean Labeling of graphs. In this paper we investigate Heronian mean labeling for various graphs resulted from the duplication of graph elements.

Key words:

Graph, Heronian mean graph, Duplication.

1. INTRODUCTION:

The concept of graph labeling was introduced by Rosa in 1967.A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions.If the domain of the mapping is the set of vertices (or edges),then the labeling is called a vertex labeling (or an edge labeling).Here we consider simple,finite,undirected and connected graph G = (V,E). For all standard terminology and notation we follow Chartrand and Lesniak [1].

DEFINITION:1.1

A graph G=(V,E) with p vertices and q edges is said to be a Heronian Mean Graph if it is possible to label the vertices xϵV with distinct labels f(x) from 1,2,3,. . . ,q+1 in such a way that each edge e=uv is labeled with f(e=uv)= $\left\lfloor \frac{f(u)+\sqrt{f(u)f(v)}+f(v)}{3} \right\rfloor$ (OR) $\left\lceil \frac{f(u)+\sqrt{f(u)f(v)}+f(v)}{3} \right\rceil$ then the edge labels are distinct. In this case f is called a **Heronian Mean Labeling** of G.

DEFINITION:1.2

Two vertices of a graph which are adjacent are said to be neighbours. The set of all neighbours of a vertex v is called the neighbourhood set denoted as N(v).

DEFINITION:1.3

Duplication of a vertex v_k of graph G produces a new graph G', by adding a vertex v'_k with $N(v_k) = N(v'_k)$.

In other words a vertex v'_k is said to be duplication of v_k if all the vertices which are adjacent to v_k are now adjacent to v'_k also.

DEFINITION:1.4

Duplication of an edge e = uv of graph G produces a new graph G' by adding an edge e' = u'v' such that N(u') = N(u) ∪ {v'} – {v} and N(v') = N(v) ∪ {u'} – {u}

DEFINITION:1.5

Duplication of a vertex v_k by a new edge e' = u'v' in a graph G produces a new graph G' such that N(v') = $\{v_k,u'\}$ and N(u') = $\{v_k,v'\}$.

DEFINITION:1.6

Duplication of an edge e = uv by a vertex v' is a graph G produces a new graph G' such that N(v') = {u,v}.

2. MAIN RESULTS:

THEOREM: 2.1

The graph obtained by duplication of an arbitrary vertex v_k in cycle C_n is a heronian mean graph.

Proof:

Let $v_1, v_2 \ldots v_n$ be the vertices of cycle C_n. Without loss of generality we duplicate the vertex v_1 thus added vertex is v'.

Now the resultant graph G will have n+1 vertices and n+2 edges.

To define f: $V(G) \rightarrow \{1,2,\ldots,n+3\}$ we consider following two cases.

Case (i): When n = 3

The graph obtained by duplication of a vertex is cycle C_3 and its heronian mean labeling is shown in figure:1

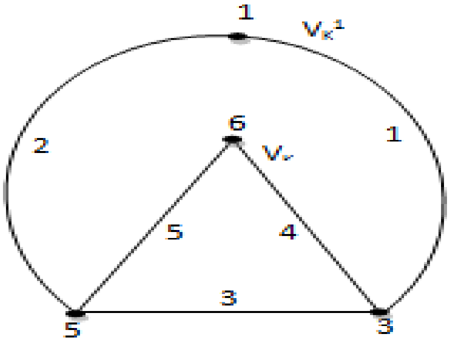

Figure :1

Case (ii): When $n \neq 3$

$f(v') = 1$

$f(v_i) = i + 1 \; ; \; 1 \leq i \leq 2$

$f(v_i) = n + 5 - i \; ; \; 3 \leq i \leq n$

In view of the above labeling pattern we have distinct edge labels from $\{1,2,\ldots, n + 2\}$.

Hence from case (i) and case (ii) we have the graph obtained by duplication of an arbitrary vertex V_x is cycle C_n is a heronian mean graph.

Illustration: 2.2

The graph obtained by duplication of a vertex in C_5 and its heronian mean labeling is shown in Figure: 2

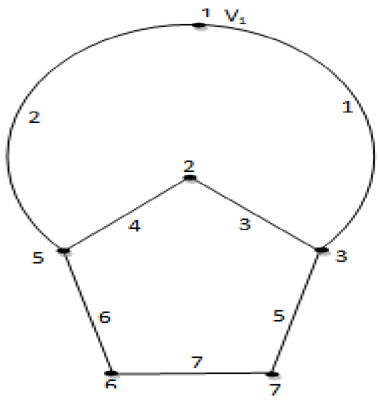

Figure :2

THEOREM: 2.3

The graph obtained by duplication of an arbitrary edge e_k in cycle C_n is a heronian mean graph.

Proof:

Let $e_1, e_2, \ldots e_n$ be the edges of cycle C_n. Without loss of generality we duplicate the edge e_1 thus added vertices are v'_1 and v'_2.Now the resultant graph G will have n + 2 vertices and n + 3 edges.

To define f : $V(G) \rightarrow \{1,2,\ldots,n+4\}$ we consider following two cases.

Case (i): When n = 3

The graph obtained by duplication of an edge is cycle C_3 and its heronian mean labeling is shown in figure:3

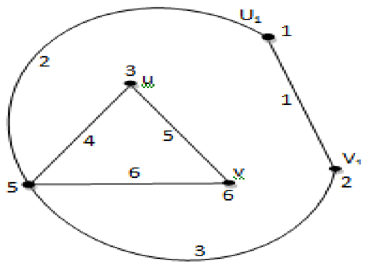

Figure :3

Case (ii): When $n \neq 3$

$f(v'_1) = 1$; $f(v'_2) = 3$

$f(v_i) = 2i$; $1 \leq i \leq 2$

$f(v_i) = n + 6 - i$; $3 \leq i \leq n$

In view of the above defined labeling pattern we have distinct edge labels.

Hence the graph obtained by duplication of an arbitrary edge e_k in cycle C_n is a Heronian mean graph.

Illustration: 2.4

The graph obtained by duplication of an edge in C_6 and its heronian mean labeling is shown in Figure: 4

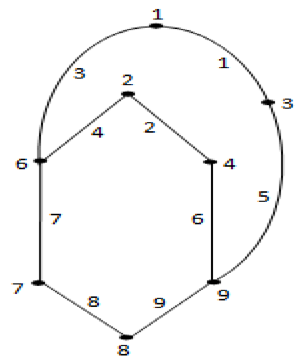

Figure :4

THEOREM: 2.5

The graph obtained by duplication of an arbitrary vertex by a new edge in cycle C_n is a heronian mean graph.

Proof:

Let $v_1, v_2, \ldots v_n$ be the vertices and $e_1, e_2, \ldots e_n$ be the edges of cycle C_n. Without loss of generality we duplicate the vertex v_n by an edge e_{n+1} with end vertices as v'_1 and v'_2. The resultant graph G will have n + 2 vertices and n + 3 edges.

We define $f : V(G) \rightarrow \{1,2,\ldots,n+4\}$ as follows

$f(v'_i) = i$; $1 \leq i \leq 2$

$f(v_1) = 3$

$f(v_i) = i + 3$; $2 \leq i \leq n$

In view of the above defined labeling pattern we have distinct edge labels from $\{1,2, \ldots, n+3\}$.

Hence the graph obtained by duplication of an arbitrary vertex by a new edge in cycle C_n is heronian mean graph.

Illustration:2.6

The graph obtained by duplication of a vertex by a new edge in cycle C_8 and its heronian mean labeling is shown in Figure 5

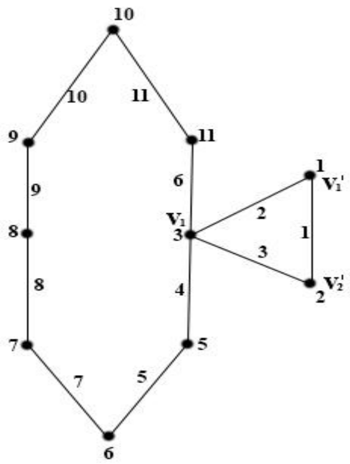

Figure :5

THEOREM: 2.7

The graph obtained by duplication of an arbitrary edge by a new vertex in cycle C_n is a heronian mean graph.

Proof:

Let $v_1, v_2, \ldots v_n$ be the vertices and $e_1, e_2, \ldots e_n$ be the edges of cycle C_n. Without loss of generality we duplicate the edge $v_n v_1$ by a vertex v'. The resultant graph G will have n + 1 vertices and n + 2 edges.

To define $f : V(G) \rightarrow \{1,2,\ldots,n+3\}$ we consider following two cases.

Case (i): When n = 3

The graph and harmonic mean labeling is shown in figure.6

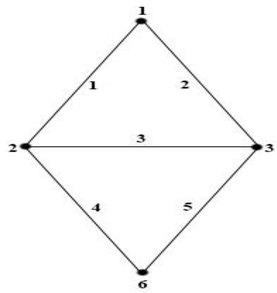

Figure:6

Case (ii): When $n \neq 3$

$f(v') = 1$

$f(v_1) = 2$, $f(V_2) = 4$

$f(v_i) = i + 3$; $3 \leq i \leq n$

In view of the above labeling pattern we have distinct edge labels from {1,2, …, n+2}.

Hence from case(i) and case (ii) we have the graph obtained by duplication of an arbitrary edge by a new vertex in cycle C_n is heronian mean graph.

Illustration:-2.8

The graph obtained by duplication of an edge by a new vertex in cycle C_8 and its heronian mean labeling is shown in Figure 7

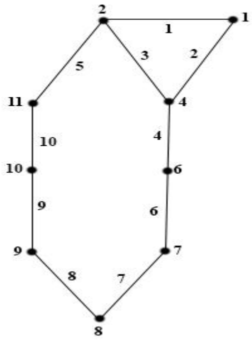

Figure :7

Conclusion:

Here we discuss Heronian Mean Labeling in the Context of duplication of graph elements. The results are demonstrated by means of sufficient illustrations which provide better understanding**.**

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