Regulation of Combined Harvesting of a Prey-Predator Fishery with Low Predator Density by Taxation

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Abstract: *In this paper, a model on prey-predator fishery is proposed and analyzed in which the predator density is low compared to the prey density. It is assumed that prey species obeys the logistic law of growth [1] and both the species are allowed to be harvested by the fishermen. In order to control over exploitation the regulatory agencies impose suitable taxes per unit of harvested biomass of landed fish. It is also assumed that the agencies impose a higher tax for harvesting predator fish species compared to the tax for harvesting prey fish species. Different efforts are given by the fishermen to harvest prey and predator species and the efforts are considered as dynamic variables. Different suitable ranges of taxes are determined for existence of all possible steady states. The local and global stability of the steady states are discussed. An optimal harvest policy has been discussed considering taxes as the control variables. All the results are illustrated with the help of a numerical example.*

Keywords: *prey-predator fishery, combined harvesting, regulatory agency, steady states, local and global stability, optimal harvest policy.*

I. INTRODUCTION:

Now-a-days overfishing is a common problem in commercial fisheries. Several fish species like Antractic blue whales, Antractic fin whales, Japanese Sardine, California sardine, etc. are now almost extinct in this century due to overfishing. These incidents had serious economic and social implications also besides causing damage to the marine ecosystem of the concerned regions. To arrest further aggravation of the situation, several countries entered into multilateral agreements which led to the establishment of some international regulatory agencies like International Whaling Commission, Pacific Halibut Commission, Inter-America Tropical Tuna Commission, etc. These agencies are expected to monitor and regulate exploitation of various marine fisheries. In the year 1954, the federal fisheries authorities of Canada asked an economist, H.S. Gordon, to provide an economic analysis of the persistent problem of low income among maritime fishermen of Canada. Gordon [2] developed a static model of the common property (open access) fisheries which not only explained the low income of fishermen, but also provided an economic interpretation of the overfishing problem. This model explained how economic overfishing would be expected to occur in an open access fishery while biological overfishing would take place when the price or cost ratio was considerably high. The fisheries biologist M.B. Schaefer [3] developed a dynamic bioeconomic model for a single species fishery and it is applied to the tuna fisheries of the tropical Pacific. Gordon's model [2] is the equilibrium solution of the Schaefer model [3].

Various methods of counteracting the common property externality in commercial fisheries have been suggested from time to time. These include allocation of fishermen's quotas ([4],[5]), imposition of taxes on landed fish ([6],[7]), license limitation [8], restricting fishing seasons [9],etc. Some of the issues associated with the choice and enforcement of optimal governing instruments in regulating fisheries were discussed Anderson and Lee [10]. The economic implications of enforcing laws for regulating marine fisheries were discussed by Sutinen and Andersen [11]. Among these methods, taxation is superior to the other control policies because of its flexibility described by Clark [6].

A single species fishery model using taxation as a control measure was first discussed by Clark [6]. Chaudhuri and Johnson [12] extended that model using a catch-rate function which was more realistic than that in [6]. Ganguly and Chaudhuri [13] made a capital theoretic study of a single species fishery with taxation as control

policy. Pradhan and Chaudhuri [14] developed a mathematical model for growth and exploitation of a schooling fish species, using a realistic catch-rate function and imposing a tax per unit biomass of landed fish to control harvesting. Pradhan and Chaudhuri [15] also studied a fully dynamic reaction model of fishery consisting two competing fish species with taxation as a control instrument.

Pradhan [16] developed a prey-predator fishery model with low predator density where taxation is the control instrument. In that paper only the predator fish species is allowed for harvesting by the fishermen after imposing suitable tax by the regulatory agencies. But in reality it is very difficult to prevent harvesting of prey fish species because the low cost for harvesting of prey species and the high density of prey population may be attracted the fishermen to fish prey species. Moreover, if the fishing of prey species is totally stopped, then the revenue earned by the Government or the regulatory agencies from fishery will be decreased. Again, the market price of the predator fish species is always high compared to the market price of the prey fish species, so the fishermen will be more attracted to harvest the predator species. This may cause over exploitation of the predator species. So the regulatory agencies should impose a higher tax for harvesting the predator species compared to the tax for harvesting the prey species.

In this paper, both populations are allowed to be harvested after imposing suitable taxes for harvesting prey and predator fish species. Here two different efforts are considered to harvest prey and predator fish and all the efforts are dynamic variables i.e. time dependent variables depending on the net revenue earned by the society. Suitable rages of taxes are determined for existence of the steady states. The local and global stability of the steady states are discussed. An optimal harvest policy has been discussed considering taxes as the control variables. A numerical example is given to illustrate all the results.

II. THE MATHEMATICAL MODEL:

Let at any time t, $x(t)$ and $y(t)$ be the population densities of the prey and predator populations respectively. It is assumed that predator density is very low compared to the prey density and the prey species obeys the logistic law of growth. In such a situation the prey and predator populations obey the following differential equations.

$$\left.\begin{aligned} \frac{dx(t)}{dt} &= rx\left(1-\frac{x}{k}\right)-\alpha xy \\ \frac{dy(t)}{dt} &= \frac{\beta xy}{k}-sy \end{aligned}\right\} \qquad (1)$$

where r, k, α, β, s are all positive constants.

Here $r =$ natural growth rate of the prey population,

$\alpha =$ catchability rate (the rate at which the prey species is removed due to predation effect),

$\beta =$ the reproductive rate of the predator population,

$s =$ per capita death rate of the predator population and

$k =$ environmental carrying capacity for both the prey and predator populations.

Since the predator density is low, so due to intra-specific competition, the crowing effect term like $\frac{rx^2}{k}$ is absent in the growth equation of the predator species but present in the growth equation in the prey species. From the second equation of the system (1) it is clear that in absence of the prey species the predator species dies out exponentially. Pradhan [16] discussed such a model assuming that only the predator species is allowed to be harvested after imposing a suitable tax by the Government or the private agencies.

In this model it is assumed that both the species are allowed to be harvested by the fishermen. In order to control over exploitation of both species, the Government or the private agencies should impose the tax per unit biomass of the landed fish. Since the market price of the predator fish species is always high compared to the market price of the prey fish species, so the fishermen will be more attracted to harvest the predator species. It may be a cause of over exploitation of the predator species.

So the regulatory agencies should impose a higher tax for harvesting the predator species compared to the tax for harvesting the prey species.

Let $E_1(t)$ and $E_2(t)$ be the efforts for harvesting of the prey and predator species respectively. The regulatory agencies impose the taxes τ_1 and τ_2 per unit biomass of the harvested fish of the prey and predator species respectively and $\tau_1 < \tau_2$. If p_1 be the market price per unit biomass of the harvested prey fish species and c_1 be the cost per unit effort for harvesting the prey species then the net economic revenue to the fishermen (perceived

rent) is $\{q_1(p_1-\tau_1)x-c_1\}E_1$, where q_1 is the catchability coefficient of the prey population. Similarly, if p_2 be the market price per unit biomass of the harvested predator fish species and c_2 be the cost per unit effort for harvesting the predator species then the net economic revenue to the fishermen is $\{q_2(p_2-\tau_2)y-c_2\}E_2$ where q_2 is the catchability coefficient of the predator population.

In this model it is considered that $E_i(t)$ $(i=1,2)$ as the dynamic variables i.e. time dependent variables governed by the differential equations

$\frac{dE_1(t)}{dt}=\lambda_1\{q_1(p_1-\tau_1)x-c_1\}E_1$, and

$\frac{dE_2(t)}{dt}=\lambda_2\{q_2(p_2-\tau_2)y-c_2\}E_2$,

where $\lambda_i(i=1,2)$ are the stiffness parameters measuring the effort and the perceived rent for the prey and predator populations respectively.

Therefore, we have the following system of differential equations:

$$\left.\begin{aligned}\frac{dx(t)}{dt}&=rx\left(1-\frac{x}{k}\right)-\alpha xy-q_1E_1x\\ \frac{dy(t)}{dt}&=\frac{\beta xy}{k}-sy-q_2E_2y\\ \frac{dE_1(t)}{dt}&=\lambda_1\{q_1(p_1-\tau_1)x-c_1\}E_1\\ \frac{dE_2(t)}{dt}&=\lambda_2\{q_2(p_2-\tau_2)y-c_2\}E_2,\end{aligned}\right\}\qquad(2)$$

III. STEADY STATE ANALYSIS:

The steady states of the system of equations (2) are given by

$$\frac{dx}{dt}=\frac{dy}{dt}=\frac{dE_1}{dt}=\frac{dE_2}{dt}=0\qquad(3)$$

We have the following steady states $P_i\left(x^{(i)},y^{(i)},E_1^{(i)},E_2^{(i)}\right)$ $(i=0,1,2,3,4)$ of the system:

i) $P_0(0,0,0,0)$ is the trivial steady state of the system of equations (2). In absence of the prey species only the trivial solution is the solution of the system.

ii) $P_1(k,0,0,0)$ is the axial steady state of the system (2). In absence of the predator, the environmental carrying capacity is the steady state of the prey species for the unexploited $(E_1=E_2=0)$ system.

iii) $P_2\left(x^{(2)},y^{(2)},0,0\right)$ is the non-trivial steady state for the unexploited system where

$$x^{(2)}=\frac{ks}{\beta}>0\qquad(4)$$

and $y^{(2)}=\frac{r(\beta-s)}{\alpha\beta}$ (5)

Now, $y^{(2)}>0$ iff $\beta>s$ i.e. the reproductive rate of the predator species is greater than the natural mortality rate of the predator species and this is always true in the ideal living conditions for the fish species.

iv) $P_3\left(x^{(3)},0,E_1^{(3)},0\right)$ is the non-trivial steady state for the exploited prey population in absence of predator.

Here $x^{(3)}=\frac{c_1}{q_1(p_1-\tau_1)}$ (6)

and $E_1^{(3)}=\frac{r}{q_1}\left\{1-\frac{c_1}{kq_1(p_1-\tau_1)}\right\}$ (7)

$E_1^{(3)}>0$ iff $0<\tau_1<p_1-\frac{c_1}{kq_1}$ (8)

Condition (8) is the necessary and sufficient condition for the existence of the non-trivial steady state of the exploited prey population when predation effect is not considered.

v) $P_4\left(x^{(4)},y^{(4)},0,E_2^{(4)}\right)$ is the non-trivial steady state of the selective harvesting prey-predator system when the predator species is harvested and the prey species is not allowed to be harvested. In this case

$$x^{(4)}=k\left\{1-\frac{\alpha c_2}{rq_2(p_2-\tau_2)}\right\}\qquad(9)$$

$$y^{(4)}=\frac{c_2}{q_2(p_2-\tau_2)}\qquad(10)$$

$$E_2^{(4)}=\frac{1}{q_2}\left\{\beta-s-\frac{\alpha\beta c_2}{rq_2(p_2-\tau_2)}\right\}\qquad(11)$$

Now, $x^{(4)}>0$ iff $0<\tau_2<p_2-\frac{\alpha c_2}{rq_2}$ (12)

$y^{(4)} > 0$ iff $0 < \tau_2 < p_2$

$E_2^{(4)} > 0$ iff $0 < \tau_2 < p_2 - \frac{\alpha\beta c_2}{rq_2(\beta - s)}, \beta > s$ (13)

So the non-trivial steady state $P_4\left(x^{(4)}, y^{(4)}, 0, E_2^{(4)}\right)$ exists if and only if

$0 < \tau_2 < min\left\{p_2, p_2 - \frac{\alpha c_2}{rq_2}, p_2 - \frac{\alpha\beta c_2}{rq_2(\beta - s)}\right\} = p_2 - \frac{\alpha\beta c_2}{rq_2(\beta - s)}$ since $\frac{\beta}{\beta - s} > 1$.

This case is discussed in details by Pradhan [16].

vi) $P_5\left(x^{(5)}, y^{(5)}, E_1^{(5)}, E_2^{(5)}\right)$ is the non-trivial interior steady state of the system (2) where

$$x^{(5)} = \frac{c_1}{q_1(p_1 - \tau_1)}. \quad (14)$$

$$y^{(5)} = \frac{c_2}{q_2(p_2 - \tau_2)}. \quad (15)$$

$$E_1^{(5)} = \frac{1}{q_1}\left[r\left\{1 - \frac{c_1}{kq_1(p_1 - \tau_1)}\right\} - \frac{\alpha c_2}{q_2(p_2 - \tau_2)}\right]. \quad (16)$$

$$E_2^{(5)} = \frac{1}{q_2}\left\{\frac{\beta c_1}{kq_1(p_1 - \tau_1)} - s\right\}. \quad (17)$$

Here $x^{(5)} > 0$ since $0 < \tau_1 < p_1$ and $y^{(5)} > 0$ since $0 < \tau_2 < p_2$.

$E_2^{(5)} > 0$ iff $max\left(0, p_1 - \frac{\beta c_1}{kq_1 s}\right) < \tau_1 < p_1$. (18)

$E_1^{(5)} > 0$ iff $\frac{rc_1}{kq_1(p_1 - \tau_1)} + \frac{\alpha c_2}{q_2(p_2 - \tau_2)} < r$. (19)

(18) and (19) are the necessary and sufficient conditions for existence of the non-trivial interior equilibrium point $P_5\left(x^{(5)}, y^{(5)}, E_1^{(5)}, E_2^{(5)}\right)$ of the system of equations (2).

$E_2^{(5)} > 0$ implies $\frac{c_1}{kq_1(p_1 - \tau_1)} > \frac{s}{\beta}$, by (17).

If $E_1^{(5)} > 0$, then $\frac{\alpha c_2}{q_2(p_2 - \tau_2)} < r - \frac{rs}{\beta} \Rightarrow \tau_2 < p_2 - \frac{\alpha\beta c_2}{q_2 r(\beta - s)}$. (20)

Therefore, (18) and (19) are the necessary conditions for existence of the non-trivial interior equilibrium point $P_5\left(x^{(5)}, y^{(5)}, E_1^{(5)}, E_2^{(5)}\right)$ of the system of equations (2).

Again $\frac{c_1}{kq_1(p_1 - \tau_1)} < \frac{1}{2}$ and $\frac{\alpha c_2}{q_2(p_2 - \tau_2)} < \frac{r}{2}$ imply $\frac{rc_1}{kq_1(p_1 - \tau_1)} + \frac{\alpha c_2}{q_2(p_2 - \tau_2)} < r$.

Now, $\frac{c_1}{kq_1(p_1 - \tau_1)} < \frac{1}{2} \Rightarrow \tau_1 < p_1 - \frac{2c_1}{kq_1}$ and $\frac{\alpha c_2}{q_2(p_2 - \tau_2)} < \frac{r}{2} \Rightarrow \tau_2 < p_2 - \frac{2\alpha c_2}{rq_2}$.

So $max\left(0, p_1 - \frac{\beta c_1}{kq_1 s}\right) < \tau_1 < p_1 - \frac{2c_1}{kq_1}$ (21)

and $0 < \tau_2 < p_2 - \frac{2\alpha c_2}{rq_2}$ (22)

are the sufficient conditions for existence of the non-trivial interior equilibrium point $P_5\left(x^{(5)}, y^{(5)}, E_1^{(5)}, E_2^{(5)}\right)$ of the system of equations (2).

Let $\frac{1}{p_1 - \tau_1} = T_1$ and $\frac{1}{p_2 - \tau_2} = T_2$.

Therefore, $E_2^{(5)} > 0$ iff $T_1 > \frac{kq_1 s}{\beta c_1}$ and $E_1^{(5)} > 0$ iff $\frac{rc_1}{kq_1}T_1 + \frac{\alpha c_2}{q_2}T_2 < r$.

Thus we have the following system of linear inequalities:

$$\left.\begin{aligned} \frac{rc_1}{kq_1}T_1 + \frac{\alpha c_2}{q_2}T_2 &< r \\ T_1 &> \frac{kq_1 s}{\beta c_1} \\ T_2 &> 0 \end{aligned}\right\} \quad (23)$$

The region bounded by the system of inequalities (23) is the feasible region or the solution space of the system of inequalities (23). Since this region is bounded, so the solutions (T_1, T_2) of the system of inequalities (23) are also bounded. Due to boundedness of T_1 and T_2, τ_1 and τ_2 are also bounded. Hence for existence of the non-trivial interior equilibrium point $P_5\left(x^{(5)}, y^{(5)}, E_1^{(5)}, E_2^{(5)}\right)$ of the dynamical system (2) there exist the bounded solutions (τ_1, τ_2) satisfying the inequalities (18) and (19).

IV. LOCAL STABILITY ANALYSIS OF STEADY STATES:

The variational matrix of the unexploited $(E_1 = E_2 = 0)$ system corresponding to the system of equations (2) is

$$V(x,y)=\begin{pmatrix} r-\frac{2rx}{k}-\alpha x & -\alpha x \\ \frac{\beta y}{k} & \frac{\beta x}{k}-s \end{pmatrix}. \qquad (24)$$

Therefore, $V(0,0)=\begin{pmatrix} r & 0 \\ 0 & -s \end{pmatrix}$.

The eigen values of $V(0,0)$ are $r\ (>0)$ and $-s\ (<0)$. So the trivial steady state $P_0(0,0,0,0)$ is an unstable steady state of the system of equations (2).

$V(k,0)=\begin{pmatrix} -r & 0 \\ 0 & \beta-s \end{pmatrix}$ by (24).

Eigen values of $V(k,0)$ are $-r\ (<0)$ and $\beta-s$. So the axial or boundary steady state $P_1(k,0,0,0)$ of the system (2) is stable or unstable according as $\beta<s$ or $\beta>s$.

If the reproductive rate of the predator is less than its natural mortality rate, then the axial equilibrium point is asymptotically stable. If the reproductive rate of the predator is greater than its natural mortality rate, then the axial equilibrium point is unstable. Biological interpretation of this result is that when the reproductive rate of the predator species is less than its mortality rate then the predator species exponentially dies out and after some time there will be no predation effect. Since the system is unexploited and there is no predation effect, so the steady state of the prey species is equal to the environmental carrying capacity.

$V\left(x^{(2)},y^{(2)}\right)=\begin{pmatrix} -\frac{rx^{(2)}}{k} & -\alpha x^{(2)} \\ \frac{\beta y^{(2)}}{k} & 0 \end{pmatrix}$ by (24).

Characteristic equation of the matrix $V\left(x^{(2)},y^{(2)}\right)$ is $\begin{vmatrix} -\frac{rx^{(2)}}{k}-\lambda & -\alpha x^{(2)} \\ \frac{\beta y^{(2)}}{k} & -\lambda \end{vmatrix}=0.$

or, $\lambda^2+\frac{rx^{(2)}}{k}\lambda+\frac{\alpha\beta\, x^{(2)}y^{(2)}}{k}=0.$

Sum of the eigen values is $-\frac{rx^{(2)}}{k}<0$ and product of the eigen values is $\frac{\alpha\beta\, x^{(2)}y^{(2)}}{k}>0.$

So the eigen values of $V\left(x^{(2)},y^{(2)}\right)$ are both negative or complex conjugate with negative real parts. Therefore, the non-trivial steady state $P_2\left(x^{(2)},y^{(2)},0,0\right)$ of the unexploited $(E_1=E_2=0)$ system is either a stable node or stable focus.

The variational matrix of the system for the exploited prey population in absence of predator is

$V(x,E_1)=\begin{pmatrix} r-\frac{2rx}{k}-\alpha x & -q_1x \\ \lambda_1q_1(p_1-\tau_1)E_1 & \lambda_1\{q_1(p_1-\tau_1)x-c_1\} \end{pmatrix}$.Therefore,

$$V\left(x^{(3)},E_1^{(3)}\right)=\begin{pmatrix} -\frac{rx^{(3)}}{k} & -q_1x^{(3)} \\ \lambda_1q_1(p_1-\tau_1)E_1^{(3)} & 0 \end{pmatrix}.$$

Sum of the eigen values of $V\left(x^{(3)},E_1^{(3)}\right)$ is $traceV\left(x^{(3)},E_1^{(3)}\right)=-\frac{rx^{(3)}}{k}<0$ and the product of the eigen values is $detV\left(x^{(3)},E_1^{(3)}\right)=\lambda_1q_1^2(p_1-\tau_1)x^{(3)}E_1^{(3)}>0$. So the eigen values of $V\left(x^{(3)},E_1^{(3)}\right)$ are both negative or complex conjugate with negative real parts. Therefore, the non-trivial steady state $P_3\left(x^{(3)},0,E_1^{(3)},0\right)$ for the exploited prey species in absence of predator species is always a stable node or stable focus if it exists i.e. if the regulatory agencies impose a tax τ_1 such that $0<\tau_1<p_1-\frac{c_1}{kq_1}$, by (8). Biologically it is true that if the regulatory agencies control the over exploitation by imposing a suitable tax, then in absence of predator the steady state level of the prey species and the effort level will be stable.

The variational matrix of the unexploited prey $(E_1=0)$ and exploited predator $(E_2\neq 0)$ system is

$$V(x,y,E_2)=\begin{pmatrix} r-\frac{2rx}{k}-\alpha y & -\alpha x & 0 \\ \frac{\beta y}{k} & \frac{\beta x}{k}-s-q_2E_2 & -q_2y \\ 0 & \lambda_2q_2(p_2-\tau_2)E_2 & \lambda_2\{q_2(p_2-\tau_2)y-c_2\} \end{pmatrix}.$$

Therefore, $V\left(x^{(4)}, y^{(4)}, E_2^{(4)}\right) = \begin{pmatrix} -\frac{rx^{(4)}}{k} & -\alpha x^{(4)} & 0 \\ \frac{\beta y^{(4)}}{k} & 0 & -q_2 y^{(4)} \\ 0 & \lambda_2 q_2 (p_2 - \tau_2) E_2^{(4)} & 0 \end{pmatrix}$.

The characteristic equation of $V\left(x^{(4)}, y^{(4)}, E_2^{(4)}\right)$ is

$$\begin{vmatrix} -\frac{rx^{(4)}}{k} - \lambda & -\alpha x^{(4)} & 0 \\ \frac{\beta y^{(4)}}{k} & -\lambda & -q_2 y^{(4)} \\ 0 & \lambda_2 q_2 (p_2 - \tau_2) E_2^{(4)} & -\lambda \end{vmatrix} = 0.$$

or, $\lambda^3 + \frac{rx^{(4)}}{k}\lambda^2 + \left\{\lambda_2 q_2^2 (p_2 - \tau_2) y^{(4)} E_2^{(4)} + \frac{\alpha\beta x^{(4)} y^{(4)}}{k}\right\}\lambda + \frac{1}{k}\lambda_2 r q_2^2 (p_2 - \tau_2) x^{(4)} y^{(4)} E_2^{(4)} = 0$

or, $\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0$, where $a_1 = \frac{rx^{(4)}}{k} > 0$,

$a_2 = \lambda_2 q_2^2 (p_2 - \tau_2) y^{(4)} E_2^{(4)} + \frac{\alpha\beta x^{(4)} y^{(4)}}{k} > 0$ and $a_3 = \frac{1}{k}\lambda_2 r q_2^2 (p_2 - \tau_2) x^{(4)} y^{(4)} E_2^{(4)} > 0$.

Now, $a_1 a_2 - a_3 = \frac{r}{k^2}\alpha\beta\left(x^{(4)}\right)^2 y^{(4)} > 0$.

Therefore, by Routh-Hurwitz criterion [17] the non-trivial steady state $P_4\left(x^{(4)}, y^{(4)}, 0, E_2^{(4)}\right)$ of the unexploited prey and exploited predator system is locally asymptotically stable, if it exists.

Now we discuss the stability of the non-trivial interior equilibrium $P_5\left(x^{(5)}, y^{(5)}, E_1^{(5)}, E_2^{(5)}\right)$ of the exploited $(E_1 \neq 0, E_2 \neq 0)$ system (2). The variational matrix of the system of equations (2) is

$V(x, y, E_1, E_2) = \left(J_{ij}\right)_{4\times4}, (i, j = 1,2,3,4)$ where

$J_{11} = \frac{\partial}{\partial x}\left(\frac{dx}{dt}\right) = r - \frac{2rx}{k} - \alpha y - q_1 E_1$, $J_{12} = \frac{\partial}{\partial y}\left(\frac{dx}{dt}\right) = -\alpha x$, $J_{13} = \frac{\partial}{\partial E_1}\left(\frac{dx}{dt}\right) = -q_1 x$, $J_{14} = \frac{\partial}{\partial E_2}\left(\frac{dx}{dt}\right) = 0$,

$J_{21} = \frac{\partial}{\partial x}\left(\frac{dy}{dt}\right) = \frac{\beta y}{k}$, $J_{22} = \frac{\partial}{\partial y}\left(\frac{dy}{dt}\right) = \frac{\beta x}{k} - s - q_2 E_2$, $J_{23} = \frac{\partial}{\partial E_1}\left(\frac{dy}{dt}\right) = 0$,

$J_{24} = \frac{\partial}{\partial E_2}\left(\frac{dy}{dt}\right) = -q_2 y$, $J_{31} = \frac{\partial}{\partial x}\left(\frac{dE_1}{dt}\right) = \lambda_1 q_1 (p_1 - \tau_1) E_1$, $J_{32} = \frac{\partial}{\partial y}\left(\frac{dE_1}{dt}\right) = 0$,

$J_{33} = \frac{\partial}{\partial E_1}\left(\frac{dE_1}{dt}\right) = \lambda_1\{q_1 (p_1 - \tau_1) x - c_1\}$, $J_{34} = \frac{\partial}{\partial E_2}\left(\frac{dE_1}{dt}\right) = 0$, $J_{41} = \frac{\partial}{\partial x}\left(\frac{dE_2}{dt}\right) = 0$,

$J_{42} = \frac{\partial}{\partial y}\left(\frac{dE_2}{dt}\right) = \lambda_2 q_2 (p_2 - \tau_2) E_2$, $J_{43} = \frac{\partial}{\partial E_1}\left(\frac{dE_2}{dt}\right) = 0$,

$J_{44} = \frac{\partial}{\partial E_2}\left(\frac{dE_2}{dt}\right) = \lambda_2\{q_2 (p_2 - \tau_2) y - c_2\}$.

Therefore, $V\left(x^{(5)}, y^{(5)}, E_1^{(5)}, E_2^{(5)}\right) = \left(J_{ij}\left(x^{(5)}, y^{(5)}, E_1^{(5)}, E_2^{(5)}\right)\right)_{4\times4}$ such that

$J_{11}\left(x^{(5)}, y^{(5)}, E_1^{(5)}, E_2^{(5)}\right) = -\frac{rx^{(5)}}{k}$, $J_{12}\left(x^{(5)}, y^{(5)}, E_1^{(5)}, E_2^{(5)}\right) = -\alpha x^{(5)}$,

$J_{13}\left(x^{(5)}, y^{(5)}, E_1^{(5)}, E_2^{(5)}\right) = -q_1 x^{(5)}$, $J_{14}\left(x^{(5)}, y^{(5)}, E_1^{(5)}, E_2^{(5)}\right) = 0$,

$J_{21}\left(x^{(5)}, y^{(5)}, E_1^{(5)}, E_2^{(5)}\right) = \frac{\beta y^{(5)}}{k}$, $J_{22}\left(x^{(5)}, y^{(5)}, E_1^{(5)}, E_2^{(5)}\right) = 0 = J_{23}\left(x^{(5)}, y^{(5)}, E_1^{(5)}, E_2^{(5)}\right)$,

$J_{24}\left(x^{(5)}, y^{(5)}, E_1^{(5)}, E_2^{(5)}\right) = -q_2 y^{(5)}$, $J_{31}\left(x^{(5)}, y^{(5)}, E_1^{(5)}, E_2^{(5)}\right) = \lambda_1 q_1 (p_1 - \tau_1) E_1^{(5)}$,

$J_{32}\left(x^{(5)}, y^{(5)}, E_1^{(5)}, E_2^{(5)}\right) = 0 = J_{33}\left(x^{(5)}, y^{(5)}, E_1^{(5)}, E_2^{(5)}\right) = J_{34}\left(x^{(5)}, y^{(5)}, E_1^{(5)}, E_2^{(5)}\right)$,

$J_{41}\left(x^{(5)}, y^{(5)}, E_1^{(5)}, E_2^{(5)}\right) = 0$, $J_{42}\left(x^{(5)}, y^{(5)}, E_1^{(5)}, E_2^{(5)}\right) = \lambda_2 q_2 (p_2 - \tau_2) E_2^{(5)}$,

$J_{43}\left(x^{(5)}, y^{(5)}, E_1^{(5)}, E_2^{(5)}\right) = 0 = J_{44}\left(x^{(5)}, y^{(5)}, E_1^{(5)}, E_2^{(5)}\right)$.

The characteristic equation of the matrix $V\left(x^{(5)}, y^{(5)}, E_1^{(5)}, E_2^{(5)}\right)$ is

$det\left(V\left(x^{(5)}, y^{(5)}, E_1^{(5)}, E_2^{(5)}\right) - \lambda I_4\right) = 0$.

$\Rightarrow \lambda^4 + \frac{rx^{(5)}}{k}\lambda^3 + \left\{\frac{\alpha\beta x^{(5)} y^{(5)}}{k} + q_1^2 \lambda_1 (p_1 - \tau_1) x^{(5)} E_1^{(5)} + q_2^2 \lambda_2 (p_2 - \tau_2) y^{(5)} E_2^{(5)}\right\}\lambda^2 +$

$\frac{rx^{(5)}}{k} q_2^2 \lambda_2 (p_2 - \tau_2) y^{(5)} E_2^{(5)}\lambda + q_1^2 q_2^2 \lambda_1 \lambda_2 (p_1 - \tau_1)(p_2 - \tau_2) x^{(5)} y^{(5)} E_1^{(5)} E_2^{(5)} = 0$.

$\Rightarrow \lambda^4 + m_3\lambda^3 + m_2\lambda^2 + m_1\lambda + m_0 = 0$, where

$m_3 = \frac{rx^{(5)}}{k} > 0$,

$m_2 = \frac{\alpha\beta x^{(5)}y^{(5)}}{k} + q_1^2\lambda_1(p_1-\tau_1)x^{(5)}E_1^{(5)} + q_2^2\lambda_2(p_2-\tau_2)y^{(5)}E_2^{(5)} > 0,$

$m_1 = \frac{rx^{(5)}}{k}q_2^2\lambda_2(p_2-\tau_2)y^{(5)}E_2^{(5)} > 0,$

$m_0 = q_1^2q_2^2\lambda_1\lambda_2(p_1-\tau_1)(p_2-\tau_2)x^{(5)}y^{(5)}E_1^{(5)}E_2^{(5)} > 0.$

Now, $m_3m_2 - m_1 = \frac{\alpha\beta r\left(x^{(5)}\right)^2y^{(5)}}{k} + \frac{rx^{(5)}}{k}q_1^2\lambda_1(p_1-\tau_1)x^{(5)}E_1^{(5)} > 0.$

$m_3m_2m_1 - m_1^2 - m_3^2m_0 = \frac{\alpha\beta r^2q_2^2\lambda_2(p_2-\tau_2)\left(x^{(5)}\right)^3\left(y^{(5)}\right)^2E_2^{(5)}}{k^3} > 0.$

Therefore, by Routh-Hurwitz criterion [17] the non-trivial interior steady state $P_5\left(x^{(5)},y^{(5)},E_1^{(5)},E_2^{(5)}\right)$ of the exploited system (2) is always locally asymptotically stable, if it exists.

V. GLOBAL STABILITY ANALYSIS OF THE STEADY STATES:

We now prove whether the non-trivial interior steady state $P_5\left(x^{(5)},y^{(5)},E_1^{(5)},E_2^{(5)}\right)$ of the system of equations (2) is globally asymptotically stable or not. For the fixed environmental carrying capacity for the populations, the prey and predator densities are bounded. Since the regulatory agencies control the over exploitation of fish populations by imposing suitable taxes, the effort levels E_1 and E_2 are also bounded. Thus the solutions of the dynamical system (2) are uniformly bounded in the finite region
$R_4^+ = \{(x,y,E_1,E_2): x,y,E_1,E_2 \in R, x>0, y>0, E_1>0, E_2>0\}.$

Let us consider the following Lypunov function [18]:

$$L(x,y,E_1,E_2) = x - x^{(5)} - x^{(5)}ln\left(\frac{x}{x^{(5)}}\right) + L_1\left\{y - y^{(5)} - y^{(5)}ln\left(\frac{y}{y^{(5)}}\right)\right\}$$
$$+L_2\left\{E_1 - E_1^{(5)} - E_1^{(5)}ln\left(\frac{E_1}{E_1^{(5)}}\right)\right\} + L_3\left\{E_2 - E_2^{(5)} - E_2^{(5)}ln\left(\frac{E_2}{E_2^{(5)}}\right)\right\}$$

where L_1, L_2, L_3 are positive constants to be determined in the subsequent steps.
Here $L\left(x^{(5)},y^{(5)},E_1^{(5)},E_2^{(5)}\right) = 0$ and
$\lim_{(x,y,E_1,E_2)\to(0,0,0,0)} L(x,y,E_1,E_2) = \lim_{(x,y,E_1,E_2)\to(\infty,\infty,\infty,\infty)} L(x,y,E_1,E_2) = \infty.$

The time derivative of $L(x,y,E_1,E_2)$ along the solution of (2) is

$$\frac{d}{dt}\{L(x,y,E_1,E_2)\} = \frac{x-x^{(5)}}{x}\frac{dx}{dt} + L_1\frac{y-y^{(5)}}{y}\frac{dy}{dt} + L_2\frac{E_1-E_1^{(5)}}{E_1}\frac{dE_1}{dt} + L_3\frac{E_2-E_2^{(5)}}{E_2}\frac{dE_2}{dt}$$
$$= \left(x-x^{(5)}\right)\left\{r\left(1-\frac{x}{k}\right) - \alpha y - q_1E_1\right\} + L_1\left(y-y^{(5)}\right)\left\{\frac{\beta x}{k} - s - q_2E_2\right\}$$
$$+L_2\lambda_1\left(E_1-E_1^{(5)}\right)\{q_1(p_1-\tau_1)x - c_1\} + L_3\lambda_2\left(E_2-E_2^{(5)}\right)\{q_2(p_2-\tau_2)y - c_2\}$$
$$= \left(x-x^{(5)}\right)\left[\left\{r\left(1-\frac{x}{k}\right) - \alpha y - q_1E_1\right\} - \left\{r\left(1-\frac{x^{(5)}}{k}\right) - \alpha y^{(5)} - q_1E_1^{(5)}\right\}\right]$$
$$+L_1\left(y-y^{(5)}\right)\left[\left\{\frac{\beta x}{k} - s - q_2E_2\right\} - \left\{\frac{\beta x^{(5)}}{k} - s - q_2E_2^{(5)}\right\}\right]$$
$$+L_2\lambda_1\left(E_1-E_1^{(5)}\right)\left[\{q_1(p_1-\tau_1)x - c_1\} - \{q_1(p_1-\tau_1)x^{(5)} - c_1\}\right]$$
$$+L_3\lambda_2\left(E_2-E_2^{(5)}\right)\left[\{q_2(p_2-\tau_2)y - c_2\} - \{q_2(p_2-\tau_2)y^{(5)} - c_2\}\right]$$
$$= \left(x-x^{(5)}\right)\left\{-\frac{r}{k}\left(x-x^{(5)}\right) - \alpha\left(y-y^{(5)}\right) - q_1\left(E_1-E_1^{(5)}\right)\right\}$$
$$+L_1\left(y-y^{(5)}\right)\left\{\frac{\beta}{k}\left(x-x^{(5)}\right) - q_2\left(E_2-E_2^{(5)}\right)\right\}$$
$$+L_2\lambda_1q_1(p_1-\tau_1)\left(x-x^{(5)}\right)\left(E_1-E_1^{(5)}\right) + L_3\lambda_2q_2(p_2-\tau_2)\left(y-y^{(5)}\right)\left(E_2-E_2^{(5)}\right)$$
$$= -\frac{r}{k}\left(x-x^{(5)}\right)^2 < 0 \text{ for } L_1 = \frac{\alpha k}{\beta} > 0, L_2 = \frac{1}{\lambda_1(p_1-\tau_1)} > 0 \text{ and } L_3 = \frac{\alpha k}{\lambda_2\beta(p_2-\tau_2)} > 0.$$

Therefore, $\frac{d}{dt}\{L(x,y,E_1,E_2)\} < 0\ \forall(x,y,E_1,E_2) \in R_4^+$ and $\frac{d}{dt}\{L(x,y,E_1,E_2)\} = 0$ at $P_5\left(x^{(5)},y^{(5)},E_1^{(5)},E_2^{(5)}\right)$. This shows that $\frac{d}{dt}\{L(x,y,E_1,E_2)\}$ is negative definite in the region R_4^+ and hence by Lassel's invariance principle [19] the non-trivial interior steady state $P_5\left(x^{(5)},y^{(5)},E_1^{(5)},E_2^{(5)}\right)$ of the exploited system of equations

(2) is globally asymptotically stable in the region $R_4^+ = \{(x, y, E_1, E_2): x, y, E_1, E_2 \in R, x > 0, y > 0, E_1 > 0, E_2 > 0\}$.

VI. OPTIMAL HARVEST POLICY:

In this section an optimal harvest policy is determined to maximize the total discounted net revenue from the harvesting biomass using taxes as control parameters.

The objective of the regulatory agencies is to maximize $J = \int_0^\infty \Pi(x, y, E_1, E_2, t)e^{-\delta t}\, dt$ where δ denotes the instantaneous annual rate of discount and $\Pi(x, y, E_1, E_2, t)$ is the net revenue of the society.

Therefore, $\Pi(x, y, E_1, E_2, t) =$ net revenue of the fishermen + net revenue of the regulatory agencies.

$$= (p_1 - \tau_1)q_1E_1x - c_1E_1 + (p_2 - \tau_2)q_2E_2y - c_2E_2 + \tau_1q_1E_1x + \tau_2q_2E_2y$$
$$= (p_1q_1x - c_1)E_1 + (p_2q_2y - c_2)E_2.$$

The objective of the regulatory agencies is to determine the optimal values of the taxes τ_1 and τ_2 in order maximize J subject to the state equations (3) i.e. $\frac{dx}{dt} = \frac{dy}{dt} = \frac{dE_1}{dt} = \frac{dE_2}{dt} = 0$ and the constraints $0 < \tau_1 < \tau_{1(max)}$ and $0 < \tau_2 < \tau_{2(max)}$.

The Pontryagin Maximum Principle [20] is applied to obtained the optimal equilibrium solution of the system of equations (2).

The Hamiltonian of this control problem is

$$H = e^{-\delta t}\{(p_1q_1x - c_1)E_1 + (p_2q_2y - c_2)E_2\} + \mu_1(t)\left\{rx\left(1 - \frac{x}{k}\right) - \alpha xy - q_1E_1x\right\}$$
$$+ \mu_2(t)\left\{\frac{\beta xy}{k} - sy - q_2E_2y\right\}$$
$$+\mu_3(t)\lambda_1\{q_1(p_1 - \tau_1)x - c_1\}E_1 + \mu_4(t)\lambda_2\{q_2(p_2 - \tau_2)y - c_2\}E_2. \qquad (25)$$

where $\mu_i(t)(i = 1,2,3,4)$ are adjoints variables.

Since H is the linear function of τ_1 and τ_2, the conditions that the Hamiltonian H be maximum for τ_1 and τ_2 satisfying the conditions (3) are

$$\frac{\partial H}{\partial \tau_1} = \frac{\partial H}{\partial \tau_2} = 0 \Rightarrow \mu_3(t) = \mu_4(t) = 0. \qquad (26)$$

The adjoint equations are $\frac{d\mu_1}{dt} = -\frac{\partial H}{\partial x}, \frac{d\mu_2}{dt} = -\frac{\partial H}{\partial y}, \frac{d\mu_3}{dt} = -\frac{\partial H}{\partial E_1}, \frac{d\mu_4}{dt} = -\frac{\partial H}{\partial E_2}$ (27)

Therefore, $\frac{d\mu_1}{dt} = -\left\{e^{-\delta t}p_1q_1E_1 + \mu_1(t)\left(r - \frac{2rx}{k} - \alpha y - q_1E_1\right) + \mu_2(t)\frac{\beta y}{k}\right\}$. (28)

$$\frac{d\mu_2}{dt} = -\left\{e^{-\delta t}p_2q_2E_2 - \mu_1(t)\alpha x + \mu_2(t)\left(\frac{\beta x}{k} - s - q_2E_2\right)\right\}. \qquad (29)$$

$$\frac{d\mu_3}{dt} = -e^{-\delta t}(p_1q_1x - c_1) + \mu_1(t)q_1x \Rightarrow \mu_1(t) = e^{-\delta t}\left(p_1 - \frac{c_1}{q_1x}\right) \text{ by (26).} \qquad (30)$$

$$\frac{d\mu_4}{dt} = -e^{-\delta t}(p_2q_2y - c_2) + \mu_2(t)q_2y \Rightarrow \mu_2(t) = e^{-\delta t}\left(p_2 - \frac{c_2}{q_2y}\right) \text{ by (26)} \qquad (31)$$

From (28), (30) and (31), we have

$$-\delta e^{-\delta t}\left(p_1 - \frac{c_1}{q_1x}\right) = -\left\{e^{-\delta t}p_1q_1E_1 + e^{-\delta t}\left(p_1 - \frac{c_1}{q_1x}\right)\left(r - \frac{2rx}{k} - \alpha y - q_1E_1\right) + e^{-\delta t}\left(p_2 - \frac{c_2}{q_2y}\right)\frac{\beta y}{k}\right\}$$

using the state equation $\frac{dx}{dt} = 0$.

$$\Rightarrow \delta(p_1q_1x - c_1) = p_1q_1rx - \frac{2rp_1q_1x^2}{k} - p_1q_1\alpha xy + \frac{c_1rx}{k} + \frac{p_2q_1\beta xy}{k} - \frac{c_2\beta q_1x}{q_2k}$$
$$\Rightarrow Ax^2 + Bx + Cxy - \delta c_1 = 0 \qquad (32)$$

where $A = \frac{2rp_1q_1}{k}, B = \left(\delta p_1q_1 - p_1q_1r - \frac{c_1r}{k} + \frac{c_2\beta q_1}{q_2}\right)$ and $C = p_1q_1\alpha - \frac{p_1q_1\beta}{k}$.

From (29), (30) and (31), we have

$$-\delta e^{-\delta t}\left(p_2 - \frac{c_2}{q_2y}\right) = -\{e^{-\delta t}p_2q_2E_2 - e^{-\delta t}\left(p_1 - \frac{c_1}{q_1x}\right)\alpha x + e^{-\delta t}\left(p_2 - \frac{c_2}{q_2y}\right)\left(\frac{\beta x}{k} - s - q_2E_2\right)\}$$

using the state equation $\frac{dy}{dt} = 0$.

$$\Rightarrow \delta p_2q_2y - \delta c_2 = -p_1q_2\alpha xy + \frac{c_1q_2\alpha y}{q_1} + \frac{p_2q_2\beta xy}{k} - p_2q_2sy$$
$$\Rightarrow y = \frac{\delta c_2}{D + Ex} \qquad (33)$$

where $D = \delta p_2q_2 + p_2q_2s - \frac{c_1q_2\alpha}{q_1}$ and $E = p_1q_2\alpha - \frac{p_2q_2\beta}{k}$.

From (32) and (33), we have

$$Ax^2 + Bx + Cx\left(\frac{\delta c_2}{D+Ex}\right) - \delta c_1 = 0$$

$$\Rightarrow x^3 + \left(\frac{D}{E} + \frac{B}{A}\right)x^2 + \left(\frac{BD}{AE} + \frac{\delta c_2 C}{AE} - \frac{\delta c_1}{A}\right)x - \frac{\delta c_1 D}{AE} = 0 \quad (34)$$

The cubic polynomial equation (34) with real coefficients has at least one real root and the product of the root is $\frac{\delta c_1 D}{AE}$.

Equation (34) has at least one positive real root if D and E have same sign since $A > 0$.

Let x_δ be one of the positive real root of (34).

Therefore, $y_\delta = \frac{\delta c_2}{D+Ex_\delta}$ by (33). (35)

$$E_{1\delta} = \frac{1}{q_1}\left(r - \frac{rx_\delta}{k} - \alpha y_\delta\right) \quad (36)$$

$$E_{2\delta} = \frac{1}{q_2}\left(\frac{\beta x_\delta}{k} - s\right) \quad (37)$$

$$\tau_{1\delta} = p_1 - \frac{c_1}{q_1 x_\delta} \quad (38)$$

$$\tau_{2\delta} = p_2 - \frac{c_2}{q_2 y_\delta} \quad (39)$$

Therefore, $(x_\delta, y_\delta, E_{1\delta}, E_{2\delta})$ be the optimal equilibrium solution of the system of equations (2) and the corresponding optimal taxes are $\tau_{1\delta}$ and $\tau_{2\delta}$ given by (38) and (39) respectively.

VII. NUMERICAL EXAMPLE:

Let us consider the hypothetical parameter values as follows:
$r = 10, k = 100, \alpha = 10\%, \beta = 5, s = 30\%, q_1 = 2\%, q_2 = 1\%, \lambda_1 = \lambda_2 = 1, p_1 = 20, p_2 = 50, c_1 = 15,$
$c_2 = 20, \delta = 10$ in appropriate units.

Here the reproductive rate of the predator species $(\beta = 5)$ is greater than its natural mortality rate $(s = 0.3)$. So the non-trivial steady state $P_2\left(x^{(2)}, y^{(2)}, 0,0\right)$ of the unexploited system exists where $x^{(2)} = 6$ and $y^{(2)} = 94$ by (4) and (5) respectively.

Now, $p_1 - \frac{c_1}{kq_1} = 12.5$.

Therefore, the necessary and sufficient condition for existence of the non-trivial steady state $P_3\left(x^{(3)}, 0, E_1^{(3)}, 0\right)$ of the exploited prey population when the predation effect is not considered is that $0 < \tau_1 < 12.5$.

Again, $p_2 - \frac{\alpha\beta c_2}{rq_2(\beta - s)} = 28.723$. If the regulatory agency imposes the tax τ_2 such that $0 < \tau_2 < 28.723$, then the non-trivial steady state $P_4\left(x^{(4)}, y^{(4)}, 0, E_2^{(4)}\right)$ of the unexploited prey and exploited predator system exists.

Now, $max\left(0, p_1 - \frac{\beta c_1}{kq_1 s}\right) < \tau_1 < p_1 \Rightarrow max(0, -105) < \tau_1 < 20 \Rightarrow 0 < \tau_1 < 20$ by (18) and

$\frac{rc_1}{kq_1(p_1 - \tau_1)} + \frac{\alpha c_2}{q_2(p_2 - c_2)} < r \Rightarrow \frac{75}{20 - \tau_1} + \frac{200}{50 - \tau_2} < 10$ by (19).

Therefore, the necessary and sufficient conditions for existence of the non-trivial interior equilibrium point $P_5\left(x^{(5)}, y^{(5)}, E_1^{(5)}, E_2^{(5)}\right)$ of the system of equations (2) are $0 < \tau_1 < 20$ and $\frac{75}{20 - \tau_1} + \frac{200}{50 - \tau_2} < 10$.

From (21) and (22), we have $0 < \tau_1 < 5$ and $0 < \tau_2 < 10$, these are the sufficient conditions for existence of the non-trivial interior equilibrium point $P_5\left(x^{(5)}, y^{(5)}, E_1^{(5)}, E_2^{(5)}\right)$ of the system of equations (2).

If the regulatory agencies impose the taxes τ_1 and τ_2 such that $0 < \tau_1 < 5$ and $0 < \tau_2 < 10$, then all the steady states of the system of equations (2) exist.

Let the regulatory agency imposes the taxes $\tau_1 = 4$ and $\tau_2 = 8$ units. Then

(i) $P_1(100,0,0,0)$ is the axial equilibrium of the system (2).

(ii) $P_2(6,94,0,0)$ is the non-trivial steady state for the unexploited system. This result shows that if both the species are not harvested then the prey steady state will be very low due to predation. So harvesting of predator species is necessary for existence of both species.

(iii) $P_3(46.875, 0, 265.625, 0)$ is the non-trivial steady state for the exploited prey population in absence of predator.

(iv) $P_4(52.381,47.619,0,231.905)$ is the steady state of the exploited predator and unexploited prey system.

(v) $P_5(46.875,47.619,27.525,205.375)$is the non-trivial interior equilibrium point of the exploited prey-predator system (2).

All the above steady states are locally and also globally asymptotically stable.

For the above parameter values the equation (34) becomes

$x^3 - 873.75x^2 - 2375x + 1650000 = 0.$

43.164 is one of the positive real roots of the above equation and let $x_\delta = 43.164$.

Therefore, $y_\delta = 47.80$ by (35), $E_{1\delta} = 45.18$ by (36) and $E_{2\delta} = 185.82$ by (37).

So $(43.164,47.80,45.18,185.82)$ is the optimal equilibrium solution of the system and the optimal taxes are $\tau_{1\delta} = 2.624$ by (38) and $\tau_{2\delta} = 8.159$ by (39).

Comparing the non-trivial interior steady state with the optimal steady state for the exploited prey-predator fishery we see that the prey steady state decreases and the predator steady state slightly increases and the corresponding steady state levels of efforts are increases and decreases respectively in the optimal level. The optimal tax for harvesting prey species is less and the optimal tax for harvesting predator species is high compared to the taxes imposed by the regulatory agencies for existence of the biological equilibrium point of the dynamical system. The optimal values of taxes depend on δ, the instantaneous annual rate of discount, when the other parameters remain fixed. Thus the regulatory agencies choose the taxes in order to reach the optimal revenue for the society considering all parameters.

VIII. CONCLUSION:

This model is very important for the fishery having a prey-predator community in which prey density is high and the predator density is low. Since the predator density is low so it is very sensitive to harvest the predator species. The regulatory agencies should always monitor on harvesting of predator species very carefully. Krill-whale fishery is an example of such model. The important feature of this model is that, in spite of low predator density and low market price of the prey fish species both species are harvested and if the regulatory agencies impose suitable taxes for which the steady states exist then all the non-trivial steady states are locally and globally asymptotically sable.

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