Mean Edge-Antimagic Vertex Labeling of Graphs

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Abstract - *A connected graph G(p,q) is said to be Mean Edge-Antimagic vertex labeling if there exists a bijection f : V→{0,1,2,...q} such that the induced mapping g_f : E→{1,2,...q} defined by*

$$g_f\,(e = uv) = \begin{cases} \frac{f(u)+f(v)}{2} & if \quad f(u)+f(v) \text{ is even} \\ \frac{f(u)+f(v)+1}{2} & if \quad f(u)+f(v) \text{ is odd} \end{cases}$$

. In this article we mainly investigate mean edge-antimagic vertex labeling of paths, spider, $K_{1,n}$ and odd and even mean antimagic labeling of paths, dragon, $K_{1,n}$ and cycle graphs.

Keywords - *Mean Edge –Antimagic vertex Labeling, odd and even Mean Antimagic Labeling, Path, Spider, $K_{1,n}$, Cycles, Dragon graphs.*

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1. Introduction

In this paper we mean a *graph* G by finite, connected, undirected graph G = (V, E) without any loops and multiple edges with │V│ = p vertices and │E│= q edges. Terms not defined here are used in the sense of Harary [1].

The concept of graph labeling was introduced by Rosa [4] in 1967. A *graph labeling* is an assignment of integers to the vertices or edges or both subject to certain conditions. The process of vertex labeling is label the vertices with integers. Under this vertex labeling, the edge weight of an edge e = uv is defined as W (e) = W (uv) = f (u) + f (v).

In 1994, N.Hartsfield and Ringel [2] introduced the concept of *antimagic graph*. Each vertex labeling f of a graph G = (p,q) from{0,1,2,…q} induces a edge labeling g_f where g_f (e) is sum the labels of end vertices of an edge e. Labeling f is called *antimagic* if and only if all the edge labelings are pair wisely distinct.

By an *edge antimagic vertex labeling* we mean a one-to-one mapping V(G) into {0,1,2,…q} such that the set of edge weights of all edges in G is {1,2,…q}.

S.Somasundaram and R.Ponraj [5] and [6] introduced the concept of *Mean labeling*.

We follow the notation and terminology of [7]. Different kinds of antimagic graphs were studied by T.Nicholas, S.Somasundaram and V.Vilfred [3].

In this paper we prove that *Mean Edge-Antimagic vertex labeling* of Paths, Spider and Cycles.

Definition 1.1

Let G (V, E) be a graph of order │V│ ≥ 2 with │V│= p vertices and │E│= q edges. A bijective mapping f : V → {0,1,…q} with the induced mapping g_f : E → {1,2,….q} defined by

$$g_f(e = uv) = \begin{cases} \frac{f(u)+f(v)}{2} & \text{if } f(u)+f(v) \text{ is even} \\ \frac{f(u)+f(v)+1}{2} & \text{if } f(u)+f(v) \text{ is odd} \end{cases}$$

is called *Mean Edge-Antimagic vertex labeling*. A graph that admits a mean edge-antimagic vertex labeling is called a *mean antimagic graph.*

Observation 1.2

Every mean edge-antimagic vertex labeling is also antimagic.

Definition 1.3

A *Caterpillar* is a tree which has path $P_n = a_1, a_2 \ldots\ldots a_n$ of order n and is obtained by attaching X_i (possibly zero) end vertices at the vertex a_i of P_n, i = 1,2….n by end edges. It is denoted as $T = S(X_1, X_2 \ldots\ldots X_n)$. The order of T is $n + \sum Xi$.

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Example 1.4

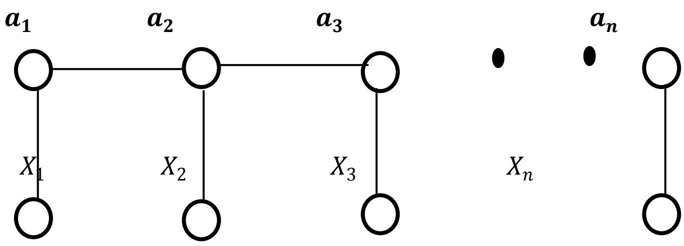


Fig (1.1) - $T = S(X_1, X_2 \ldots\ldots X_n)$

Definition 1.5

A *Spider* SP $(P_{n,2})$ is a Caterpillar $S(X_1, X_2, \ldots X_n)$ where $X_n = 2$ and $X_i = 0$ for i = 1,2,…n-1

Example 1.6

The following graph is the example for the Spider SP $(P_{4,2})$

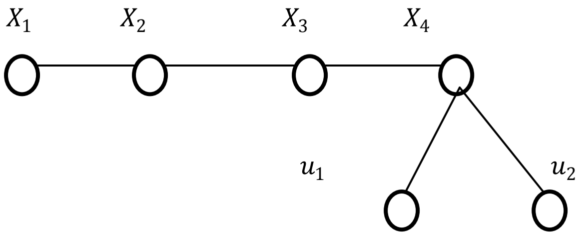
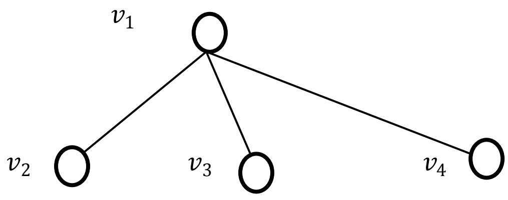


Fig (1.2) - SP $(P_{4,2})$

Definition 1.5

The Complete graph $K_{1,n}$ is called a *Star graph.*

Example 1.6



The above figure represents $K_{1,3}$

Fig (1.3)

2. Main Results

In this section we prove the Mean – Edge Antimagic vertex labeling of some special graphs.

Theorem 2.1

Every path P_n , $n \geq 2$ admits Mean – Edge Antimagic vertex labeling.

Proof:

Let us define a vertex labeling $f : V \to \{0,1,2,\ldots..q\}$ by $f(v_i) = i - 1$, $i = 1,2,\ldots.n$.

Then the induced edge labels are g_f $(e = uv) = f^*$ $(v_iv_{i+1}) = i$, $i = 1,2,\ldots..n$.

The above defined function f provides mean – edge antimagic vertex labeling of path P_n.

Definition 2.2

The starting and end vertices of the *1 – sided infinite path* P_1 is of degree one and all other vertices are of degree two.

Example 2.3

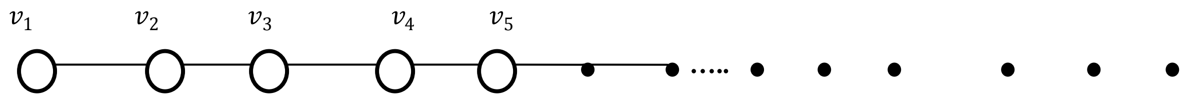


Fig (2.1) – The 1 – sided infinite path P_1

Theorem 2.4

The 1 – sided infinite path P_1 is (1,1) edge antimagic vertex labeling using Partition technique.

Proof:

The vertex labels of P_1 are 0,1,2,…..q.

The induced edge labels are 1,2,…..q.

A partition scheme of numbers is

$0 + 1 = \frac{0+1+1}{2}$, 0 + 1 is odd

$1 + 2 = \frac{1+2+1}{2}$, 1 + 2 is odd

$2 + 3 = \frac{2+3+1}{2}$, 2 + 3 is odd and so on.

The last number of the (a,d) = (1,1) maen – antimagic label is a + (q-1)d = 1 + (q-1)1 = q.

Thus the 1-sided infinite path P_1 is (1,1) mean – antimagic.

Theorem 2.5

The Spider SP ($P_{n,2}$), where $n \geq 2$ is mean – edge antimagic vertex labeling.

Proof:

Here $|V(SP(P_{n,2}))| = n + \sum Xi = n + 2$

$|E(SP(P_{n,2}))| = n + 1$

Define f on $V(SP(P_{n,2}))$ by $f(a_i) = n + 2 - i$, $i = 1,2,\ldots.n$, $f(u_{1)} = 0$, $f(u_2) = 1$.

Therefore the induced mapping $g_f : E \to \{1,2,\ldots.q\}$ defined by

$f^*(a_ia_{i+1}) = n + 2 - i$, $i = 1,2,\ldots..n$,

$f^*(a_nu_1) = 1$

$f^*(a_nu_2) = 2$.

Thus the condition is satisfied.

Definition 2.6

A *regular Spider* SP (k,n) is a tree obtained by identifying one end vertex of k number of paths each of length n.

Observation 2.7

A regular Spiders SP (2, 2), SP (3, 3), SP (3, 2), SP (4,2) are mean – edge antimagic vertex labeling.

Theorem 2.8

The Complete graph $K_{1,n}$ is mean – edge antimagic, if $n \leq 3$.

Proof:

By using trial and error method $K_{1,1}$, $K_{1,2}$ and $K_{1,3}$ is mean – edge antimagic.

3. Odd – Mean edge Antimagic vertex labeling

Here we present the odd mean – edge antimagic vertex labeling and odd mean – edge antimagic vertex labeling of different kinds of graphs.

Definition 3.1

A connected graph G with | V | = p vertices and | E | = q edges is said to be *odd – mean edge antimagic vertex labeling* if there is an bijection $f : V \to \{1,3\ldots.2q + 1\}$ such that the mapping $f^* : E \to \{2,4,\ldots..2q\}$ defined by

$$f^*(v_i v_{i+1}) = \begin{cases} \frac{f(u) + f(v)}{2} & \text{if } f(u) + f(v) \text{ is even} \\ \frac{f(u) + f(v) + 1}{2} & \text{if } f(u) + f(v) \text{ is odd} \end{cases}$$

then the resulting edges are distinct.

Theorem 3.2

The path P_n ($n \geq 2$) admits on odd – mean edge antimagic vertex labeling.

Proof:

The assignment of vertex labels is as follows.

Define $f : V \to \{1,3,\ldots.2q+1\}$ by $f(v_i) = 2i - 1$, $i = 1,2,\ldots n$. Then the induced edge labels are defined as follows.

$$f^*(v_i v_{i+1}) = \begin{cases} \frac{f(u) + f(v)}{2} & \text{if } f(u) + f(v) \text{ is even} \\ \frac{f(u) + f(v) + 1}{2} & \text{if } f(u) + f(v) \text{ is odd} \end{cases}$$

The resulting distinct edge labels are $f^*(v_iv_{i+1}) = 2i$, $i = 1, 2\ldots n$. Thus proved.

Definition 3.3

The graph G = C_n @ P_m is called *dragon* consists of a cycle C_n together with a path P_m one end vertex a_1 of P_m is joined with a node u_n of C_n. That is V(G) = {$V_1 \cup V_2$ } where

V_1 = { $u_1, u_2, \ldots \ldots \ldots u_n$ } of vertices of cycle C_n and V_2 = { $a_1, a_2, \ldots \ldots \ldots a_m$ } of vertices of Path P_m . Therefore V(G) = { $v_1, v_2, \ldots \ldots \ldots v_{n+m}$ }and E(G) = E(C_n) ∪ E(P_m) ∪ {$u_n a_1$} .

Hence C_n @ P_m contains m + n vertices and equal number of edges.

Example 3.4

The below diagram shows the C_5 @ P_4

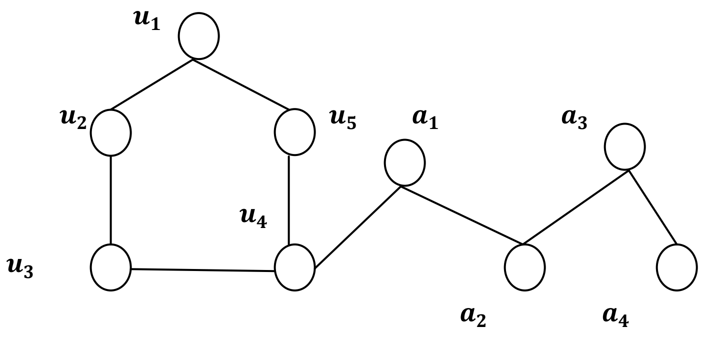


Fig (3.1)

Theorem 3.4

The graph G = C_n @ P_m where n $\geq$ 3 and n is odd admits on odd mean - edge antimagic vertex labeling.

Proof:

We define a vertex labeling of G = C_n @ P_m as follows.

f : V(G) → { 1,3,….2(n+m) + 1 } by f(v_i) = 2i -1 , i = 1,2,…n+m. The induced edge labels are defined by A = f*($v_i v_{i+1}$) = 2i, i = 1,2,…n+m-1, B = f*($v_n v_1$) = n. All these edge labelings are distinct.

Hence the theorem follows.

Theorem 3.5

For n ≥ 1, the Cycle C_{2n+1} is odd – mean edge antimagic vertex labeling.

Proof:

The assignment of vertex labels are f : V → { 1,3,…4n+1 } by f(v_i) = 2i-1, i = 1,2,….2n+1.

Then the induced edge labels are defined by

$$f^*(v_i v_{i+1}) = \begin{cases} \frac{f(u)+f(v)}{2} & \text{if } f(u)+f(v) \text{ is even} \\ \frac{f(u)+f(v)+1}{2} & \text{if } f(u)+f(v) \text{ is odd} \end{cases}$$

and the distinct edge labels are A = f*($v_i v_{i+1}$) = 2i, i = 1,2,…2n.

B = f*($v_i v_1$) = 2n+1.

This completes the proof.

Theorem 3.6

The Complete graph $K_{1,n}$ is odd – mean edge antimagic vertex labeling.

Proof:

Define f : V → {1,3,….2q+1} by $f(v_i) = 2i – 1$, i = 1,2,…n + 1. Then the induced edge labels are defined as follows.

$f^*(v_1 v_i) = i$, i = 2,3,…..n. All these edge labelings are distinct. Hence the condition of antimagic is satisfied.

4. Even – Mean edge Antimagic vertex labeling

Here we present the even mean – edge antimagic vertex labeling and even mean – edge antimagic vertex labeling of different kinds of graphs.

Definition 4.1

A connected graph G with | V | = p vertices and | E | = q edges is said to be *even – mean edge antimagic vertex labeling* if there is an bijection f : V → { 2,4,…2q + 2 } such that the mapping f* : E → {3,5,…..2q+1 } defined by

$$f^*(v_i\, v_{i+1}) = \begin{cases} \frac{f(u)+f(v)}{2} & \text{if } f(u)+f(v) \text{ is even} \\ \frac{f(u)+f(v)+1}{2} & \text{if } f(u)+f(v) \text{ is odd} \end{cases}$$

then the resulting edges are distinct.

Theorem 4.2

The path P_n ($n \geq 2$) admits on even – mean edge antimagic vertex labeling.

Proof:

The assignments of vertex labels are as follows.

Define f : V → {2,4,….2q+2} by $f(v_i) = 2i$, i = 1,2,…n. Then the induced edge labels are defined as follows.

$$f^*(v_i\, v_{i+1}) = \begin{cases} \frac{f(u)+f(v)}{2} & \text{if } f(u)+f(v) \text{ is even} \\ \frac{f(u)+f(v)+1}{2} & \text{if } f(u)+f(v) \text{ is odd} \end{cases}$$

The resulting distinct edge labels are $f^*(v_i v_{i+1}) = 2i + 1$, i = 1,2,…n. Thus proved.

Theorem 4.3

For n ≥ 2, the Cycle C_{2n} is even – mean edge antimagic

vertex labeling.

Proof:

Define f : v → {2,4,……4n } by $f(v_1) = 2$, $f(v_i) = 2i + 2$, i = 2,3,….2n-1 and $f(v_n) = 4$.

Then the induced edge labels are defined by

$A = f^*(v_i v_{i+1}) = 4$, i = 1,

$B = f^*(v_i v_{i+1}) = 2i + 3$, i = 2,3,….2n – 2

$C = f^*(v_i v_{i+1}) = i + 3$, i = 2n-1

$D = f^*(v_i v_1) = 3$, i = 2n.

Here f/A, f/B, f/C and f/D is one – one. Hence f/AUBUCUD is one – one.

Hence the theorem follows.

Theorem 4.4

The Complete graph $K_{1,n}$ is even – mean edge antimagic vertex labeling.

Proof:

Define f : V → { 2,4,…2q + 2 } such that $f(v_i) = 2i$ i = 1,2,…1 + n.

The edge labels are defined by $f^*(v_1 v_i) = i$, $i = 2,3,\ldots..n$ and all are distinct.

Hence $K_{1,n}$ is even – mean edge antimagic vertex labeling.

5. Conclusion

In this article we have investigated Mean – Edge Antimagic vertex labeling of Paths, Spider, and odd and even Mean – Edge Antimagic vertex labeling of Paths, Cycles and dragon graphs. Analogous work can be carried out for other families and in the context of different types of graph labeling techniques.

References

[1] Harary.F, Graph Theory, Addision Wesley, Reading mass, 1972.

[2] Hartsfield.N and Ringel.G, Pearls in Graph Theory, Academic Press, Boston- San Diego – New York- London,1990.

[3] Nicholas.T, Somasundaram.S and Vilfred.V, On (a,d)-antimagic Special trees, Unicyclic graphs and Complete bibartite graphs, Ars.Combin.70(2004),207-220.

[4] Rosa.A, On Certain valuations of the vertices of a graph, Theory of graphs, Gordon and N.Y.Breach and Dunod, Paris(1967), 349-355.

[5] Somasundaram.S and Ponraj.R, Mean labeling of graphs, National academy Science letters, 26(2003), 210-213.

[6] Somasundaram.S and Ponraj.R, Some Results on Mean graph, Pure and Applied Mathematics Science, 58(2003), 29-35.

[7] West.D.B, Introduction to Graph Theory, Prentice Hall of India, New Delhi, 2005.