Some Graph Labelings in Triplication

S.Bala [#1], S.Gurupriya [*2]

[#1] *S.I.V.E.T. College, Gowrivakkam, Chennai – 600 073. India.*

[*2] *Sri Sairam Engineering College, West Tambaram, Chennai – 600 044. India*

Abstract - *In this paper, we prove the existence of 3-total cordial vertex-magic labeling and 3-total cordial edge-magic labeling for the extended triplicate graph of a ladder graph by presenting algorithms.*

Keywords : Graph Labelings, Triplicate graph of path, Ladder graph

I. INTRODUCTION

Graph theory has various applications in the field of computer programming and networking, marketing and communications, business administration and so on. Some major research topics in graph theory are Graph coloring, Spanning trees, Planar graphs, Networks and Graph labeling. Graph labeling has been observed and identified for its usage towards communication networks. That is, the concept of graph labeling can be applied to network security, network addressing, channel assignment process and social networks [4].

I.1 Graph Labeling :

In 1967, Rosa introduced the concept of graph labeling [9]. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain condition(s). If the domain of mapping is the set of vertices (edges) then the labeling is called a vertex (an edge) labeling.

I.2 Cordial Labeling:

In 1987, Cahit introduced the notion of cordial labeling [3].

A graph G is said to admit a cordial labeling if there exists a function $f : V \to \{0, 1\}$ such that the induced function $f^* : E \to \{0, 1\}$ defined as $f^*(v_iv_j) = | f(v_i) - (f(v_j) |$ or $(f(v_i) + f(v_j))$ (mod 2) satisfies the property that the number of vertices labeled '0' and the number of vertices labeled '1' differ by atmost one and the number of edges labeled '0' and the number of edges labeled '1' differ by atmost one.

A graph G is said to admit a total cordial labeling if there exists a function $f : V \to \{0, 1\}$ such that the induced function $f^* : E \to \{0, 1\}$ defined as $f^*(v_iv_j) = | f(v_i) - (f(v_j) |$ or $(f(v_i) + f(v_j))$ (mod 2) satisfies the property that the number of vertices and edges labeled with '0' and the number of vertices and edges labeled with '1' differ by atmost one.

I.3 Magic Labeling:

Kotzig and Rosa [6] defined a magic valuation of a graph G(V,E) as a bijection *f* from $V \cup E$ to $\{1, 2,\dots, |V \cup E|\}$ such that for all edges xy, $f(x) + f(y) + f(xy)$ is constant (called the magic constant). This notion was rediscovered by Ringel and Llad´0 in 1996 who called this labeling edge-magic [8].

MacDougall, Miller, Slamin, and Wallis [7] introduced the notion of a vertex-magic total labeling in 1999. For a graph G(V,E) an injective mapping *f* from $V \cup E$ to the set $\{1, 2,\dots, |V| + |E|\}$ is a vertex-magic total labeling if there is a constant k, called the magic constant, such that for every vertex v, $f(v) + \Sigma f(vu) = k$ where the sum is over all vertices *u* adjacent to *v*.

I.4 Extended Triplicate Graph of a Path:

In 2011, Bala and Thirusangu introduced the concept of the extended triplicate graph of a path $P_n((ETG(P_n))$ and proved many results on this newly defined concept [1].Let $V = \{ v_1, v_2,\dots,v_{n+1}\}$ and $E = \{ e_1, e_2, \dots , e_n\}$ be the vertex and Edge set of a path P_n. For every $v_i \in V$, construct an ordered triple $\{v_i , v_i' ,v_i''\}$ where $1\leq i \leq n+1$ and for every edge $v_iv_j\in E$, construct four edges v_iv_j', $v_j' v_i''$, v_jv_i' and $v_i' v_j''$ where $j = i +1$, then the graph with this vertex set and edge set is called a Triplicate Graph of a path P_n. It is dentoted by $TG(P_n)$. Clearly the Triplicate graph $TG(P_n)$ is disconnected. Let $V_1 = \{v_1, v_2 \dots,v_{3n+1}\}$ and $E_1 = \{ e_1, e_2,\dots, e_{4n}\}$be the vertex and edge set of $TG(P_n)$.If n is odd, include a new edge(v_{n+1} , v_1) and if n is even, include a new edge (v_n ,v_1) in the edge set of $TG(P_n)$. This graph is called the Extended Triplicate of the path P_n and it is denoted by $ETG(P_n)$.

I.5 Ladder Graph:

In 2014,Thirusangu et.al proved some results on Duplicate Graph of Ladder Graph [10]. A ladder graph Ln is a planar undirected graph with 2n vertices and 3n– 2 edges. It is obtained as the Cartesian product of two path graphs, one of which has only one edge: Ln,1 = Pn × P1, where n is the number of rungs in the ladder.

Motivated by the study, the present work is aimed to provide label for the extended triplicate graph of a ladder graph and prove the existence of 3-total cordial vertex-magic labeling and 3-total cordial edge-magic labeling for the extended triplicate graph of a ladder graph.

II. STRUCTURE OF THE EXTENDED TRIPLICATE GRAPH OF LADDER

In this section we discuss about the structure of the extended triplicate graph of ladder by presenting algorithm [5].

Algorithm II.1

Input *ladder graph L_n*

Procedure triplicate of graph L_n

fori = 1 to n **do**

V ← $\{ v_i \cup v_i' \cup v_i'' \cup u_i \cup u_i' \cup u_i'' \}$

end for

for i = 1 to n-1 **do**

$E_1 \leftarrow (v_i v_{i+1}') \cup (v_i' v_{i+1}'') \cup (u_i u_{i+1}') \cup (u_i' u_{i+1}'')$

end for

for i = 2 to n **do**

$E_2 \leftarrow (v_i v_{i-1}') \cup (u_i u_{i-1}') \cup (u_i' u_{i-1}'') \cup (v_i' v_{i-1}'')$

end for

for i =1 to n do

$E_3 \leftarrow (v_i u_i') \cup (u_i v_i') \cup (u_i' v_i'') \cup (v_i' u_i'')$

end for

$E \leftarrow E_1 \cup E_2 \cup E_3$

end procedure

output : Triplicate graph of ladder L_n

From the above algorithm II.1, the triplicate graph of a ladder denoted by $TG(L_n)$ is a disconnected graph with 6n vertices and 12n - 8 edges. To make it as a connected graph, for convenience, we include an edge $v_n'' u_n''$ to the edge set E as defined in the above algorithm. Thus the graph so obtained is called an extended triplicate graph of ladder L_n with 6n vertices and 12n - 7 edges and is denoted by $ETG(L_n)$.

Illustration II.1:

The structure of extended triplicate graph of ladder $ETG(L_4)$ is given in Fig.1

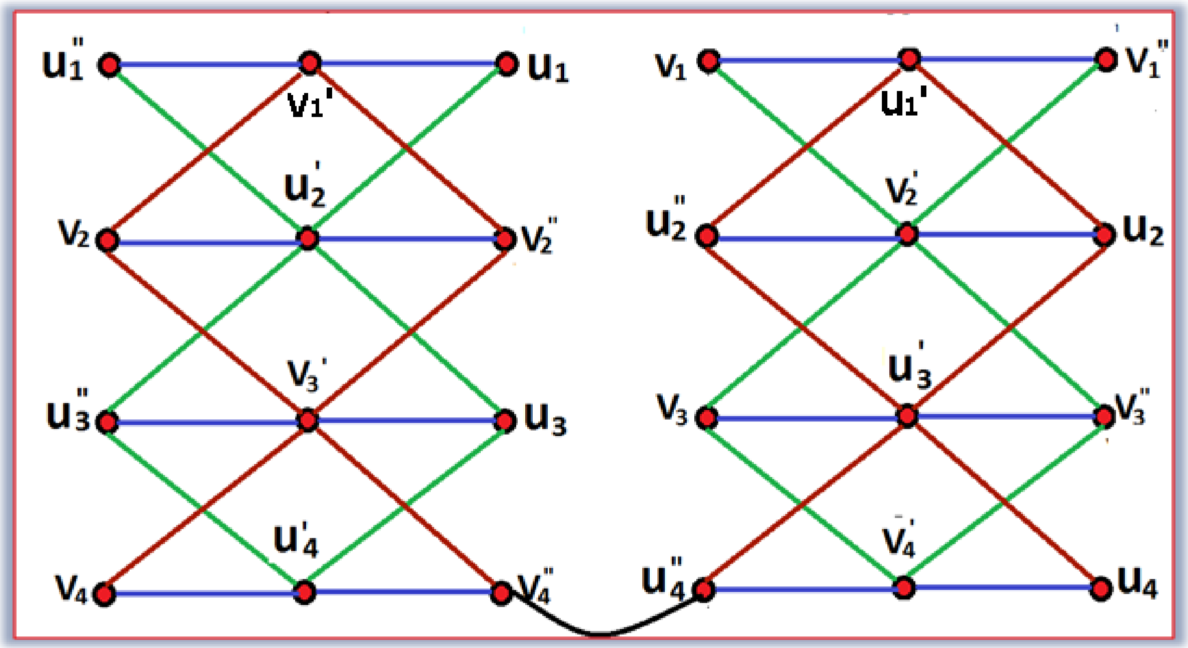

Fig.1

III. 3-TOTAL CORDIAL VERTEX-MAGIC GRAPH

In this section, we present an algorithm to label the vertices and edges of the extended triplicate graph of a Ladder *ETG (L_n)* and prove the existence of the 3-total cordial vertex-magic labeling.

Algorithm III.1

procedure (3-total cordial vertex-magic labeling for ETG(L_n))

for i = 1 to n **do**

  V ← { $v_i \cup v_i' \cup v_i'' \cup u_i \cup u_i' \cup u_i''$}

end for

for i = 1 to n-1 **do**

  $u_i \leftarrow u_i'' \leftarrow v_i \leftarrow v_i'' \leftarrow 0$

endfor

$u_n \leftarrow v_n'' \leftarrow 2$

$v_n \leftarrow u_n'' \leftarrow 1$

for i = 1 to n

$u_i' \leftarrow v_i' \leftarrow 0$

endfor

for i = 1 to n – 1 **do**

  $v_i v_{i+1}' \leftarrow u_i' u_{i+1}'' \leftarrow 1$

  $u_i u_{i+1}' \leftarrow v_i' v_{i+1}'' \leftarrow 2$

end for

for i = 2 to n **do**

  $v_i v_{i-1}' \leftarrow u_i' u_{i-1}'' \leftarrow 1$

  $v_i' v_{i-1}'' \leftarrow u_i u_{i-1}' \leftarrow 2$

end for

for i = 2 to n **do**

  $v_i u_i' \leftarrow v_i' u_i'' \leftarrow 1$

$u_i' v_i'' \leftarrow u_i v_i' \leftarrow 2$

end for

$v_1 u_1' \leftarrow v_1' u_1'' \leftarrow 2$

$u_1' v_1'' \leftarrow u_1 v_1' \leftarrow 1$

if n $\equiv 0 (mod\ 2)$

$v_n'' u_n'' \leftarrow 0$

else

$u_n v_n \leftarrow 0$

end if

end procedure

output labeled vertices and edges of *ETG(L_n)*

Theorem III.1: *ETG (L_n) is a 3-total cordial vertex-magic graph.*

Proof:

We know that, the extended triplicate graph of a ladder has 6n vertices and 12n – 7 edges.

Using algorithm 3.1, define a function f: V∪E → {0,1,2} to label the vertices and edges of *ETG(Ln)*.

Thus the number of 2's, 1's and 0's on the vertices and edges are as follows:

ETG (T_n)	**2**	**1**	**0**
Vertex label	2	2	6n-4
Edge label	6n-4	6n-4	1
Total	6n-2	6n-2	6n-3

From the table, it is clear that the number of vertices and edges labeled together with '2', '1' and '0' differ by atmost one.

In order to prove the extended triplicate graph of a ladder is a 3-total cordial vertex-magic graph , define the induced map f^*: V→ {0,1,2} such that for any $v_i v_j \in E$, $f^*(v_i) = (f(v_i) + \Sigma f(v_i v_j))\ (mod\ 3) = k$, for all $v_i \in V$ and the sum is over all vertices v_i adjacent to v_j.

Thus for all $v_i \in V$, the induced function yields a constant '0'.

Hence ETG (L_n) is a 3-total cordial vertex-magic graph.

Illustration III.1: ETG(L_4) with its 3-total cordial vertex-magic labeling is given below in Fig.2.

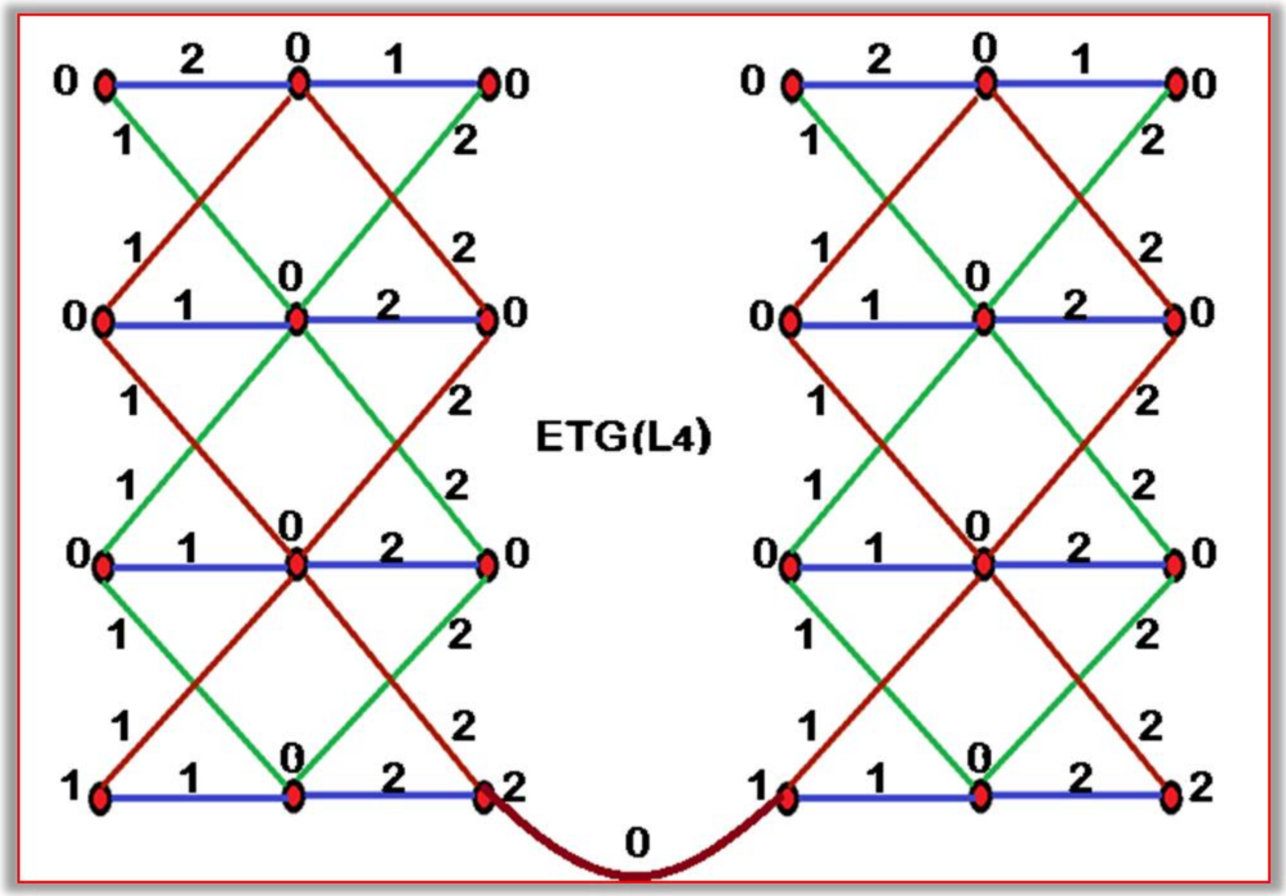

Fig. 2

IV. 3-TOTAL CORDIAL EDGE-MAGIC GRAPH

In this section, we present an algorithm to label the vertices and edges of the extended triplicate graph of ladder L_n and prove the existence of the 3-total cordial edge-magic labeling for the *(ETG(L_n))*.

Algorithm IV.1:

procedure (3-total cordial edge-magic labeling for ETG(L_n))

for i = 1 to n **do**

$V \leftarrow \{ v_i \cup v_i' \cup v_i'' \cup u_i \cup u_i' \cup u_i'' \}$

end for

for i = 1 to n **do**

$u_i'' \leftarrow v_i \leftarrow 1$; $v_i'' \leftarrow u_i \leftarrow 2$

if $(i \equiv 1 (mod\ 2))$

$u_i' \leftarrow 2$; $v_i' \leftarrow 1$

else

$u_i' \leftarrow 1$; $v_i' \leftarrow 2$

end if

end for

for i = 1 to n – 1 **do**

if $(i \equiv 1 (mod\ 2))$

$v_i v_{i+1}' \leftarrow u_i u_{i+1}' \leftarrow v_i' v_{i+1}'' \leftarrow u_i' u_{i+1}'' \leftarrow 0$

else

$v_i v_{i+1}' \leftarrow u_i' u_{i+1}'' \leftarrow 1$; $v_i' v_{i+1}'' \leftarrow u_i u_{i+1}' \leftarrow 2$

end if

end for

for i = 2 to n **do**
    if $(i \equiv 1(mod\ 2))$
        $v_i v'_{i-1} \leftarrow u'_i u''_{i-1} \leftarrow v'_i v''_{i-1} \leftarrow u_i u'_{i-1} \leftarrow 0$
    else
        $v_i v'_{i-1} \leftarrow u'_i u''_{i-1} \leftarrow 1$; $v'_i v''_{i-1} \leftarrow u_i u'_{i-1} \leftarrow 2$
    end if
end for
for i = 1 to n **do**
    if ($i \equiv 1(mod\ 2)$)
        $v_i u'_i \leftarrow u_i v'_i \leftarrow 0$; $v'_i u''_i \leftarrow 1$; $u'_i v''_i \leftarrow 2$
    else
        $v_i u'_i \leftarrow 1$; $v'_i u''_i \leftarrow u'_i v''_i \leftarrow 0$; $u_i v'_i \leftarrow 2$
    end if
end for
if (n $\equiv 0(mod\ 2)$)
        $v''_n u''_n \leftarrow 0$
else
        $u_n v_n \leftarrow 0$
end if
end procedure
output labeled vertices and edges of *ETG(L_n)*.
Theorem IV.1: *ETG(L_n)* is a 3-total cordial edge-magic graph.
Proof:

We know that, the extended triplicate graph of a ladder has 6n vertices and 12n – 7 edges. Using algorithm IV.1, define a function f: V∪E → {0,1,2} to label the vertices and edges. Thus the number of 2's, 1's and 0's on the vertices and edges are as follows:

ETG (T_n)	**2**	**1**	**0**
Vertex label	3n	3n	--
Edge label	3n-2	3n-2	6n-3
Total	6n-2	6n-2	6n-3

From the table, it is clear that the number of vertices and edges labeled together with '2', '1' and '0' differ by atmost one.

In order to prove the extended triplicate graph of a ladder is a 3-total cordial edge-magic graph , define the induced map f^*: E → {0,1,2} such that for any v_iv_j∈ E, $f^*(v_iv_j) = (f(v_i) + f(v_j) + f(v_iv_j))$ (mod 3) = k, a constant.

Thus for all v_iv_j ∈ E, the induced function yields a constant '0'.

Hence ETG (L_n) is a 3-total cordial edge-magic graph.

Illustration IV.1: ETG(L_5) with its 3-total cordial edge-magic labeling is given below in Fig. 3 .

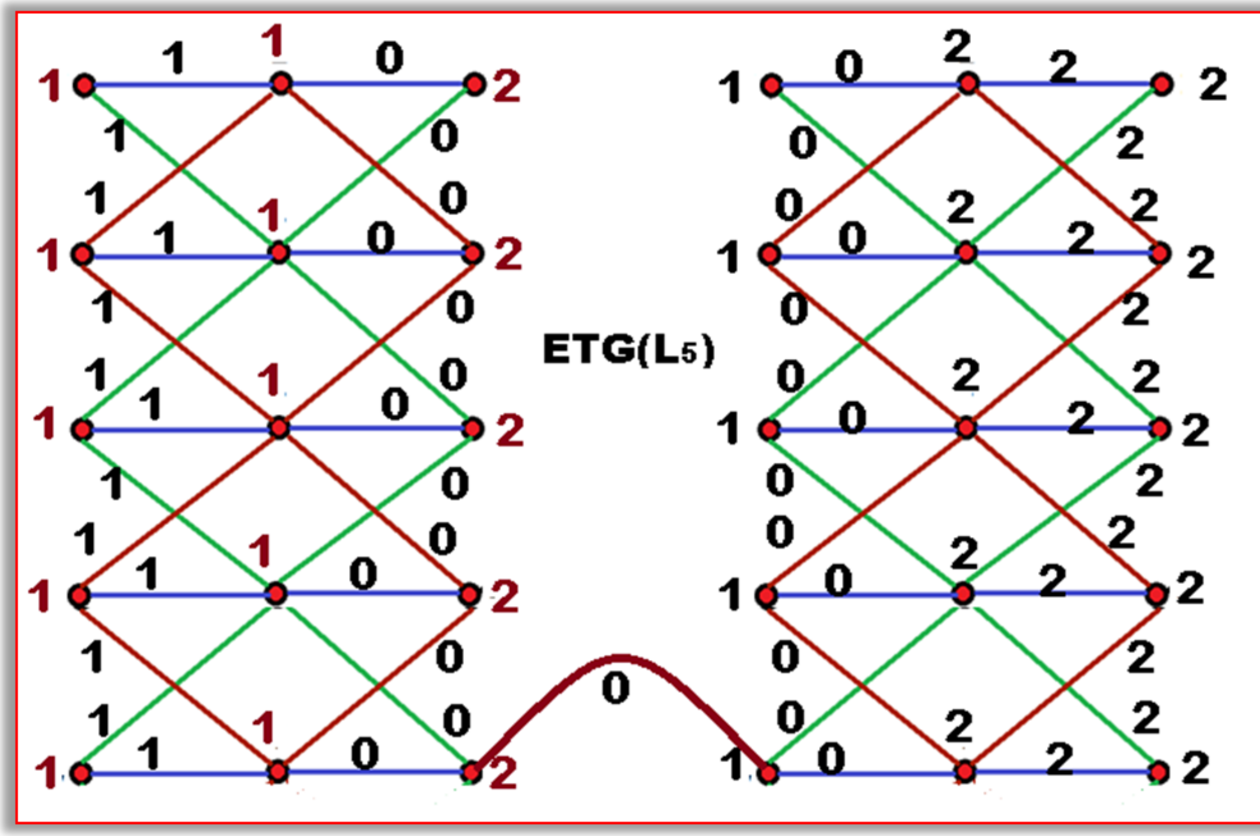

Fig.3

V. CONCLUSIONS

In this paper, we have presented the structure of extended triplicate graph of a ladder and proved the existence of 3-total cordial vertex-magic labeling and 3-total cordial edge-magic labeling for the extended triplicate graph of ladder by presenting algorithms.

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