Global Connected Accurate and Maximal Domination in Graphs

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Abstract - *In this paper the concepts global connected accurate and maximal domination sets are introduced. The numbers $\gamma_{gca}(G)$ and $\gamma_{gcm}(G)$ of a global connected accurate and maximal domination sets are determined and also a Nordhaus - Gaddum type of result is established.*

Keywords - *Global domination set, Connected accurate domination set, Connected Maximal domination set, Global connected accurate domination set, Global connected maximal domination set.*

I. INTRODUCTION

The study of domination set in graphs was begun by V.R. Kulli and Bidarhali Janakiram [2]. The accurate domination in graphs discussed by the V.R. Kulli and M.B. Kattimani [3]. The concept of connected introduced and discuss in the connected accurate domination in graphs by V.R. Kulli and M.B. Kattimani [5] and also the global domination is join to the accurate domination in graph that is global accurate domination in graph is study by V.R. Kulli and M.B. Kattimani [4]. The new concept of global connected accurate domination and the global connected maximal domination sets of G are introduced.

II. PRELIMINARIES

All the graphs considered here are finite, undirected without loops and multiple edges.

Definition: 2.1

A *graph* – usually denoted G (V, E) or G = (V, E) consists of set of vertices V together with a set of edges E. The number of vertices in a graph is usually denoted p while the number of edges is usually denoted q [1].

Definition: 2.2

A set D of vertices in a graph $G = (V, E)$ is *dominating set* of G if every vertex in $V - D$ is adjacent to some vertex in D. The *domination number* $\gamma(G)$ of G is the minimum cardinality of a domination set.

Definition: 2.3

A dominating set D of a graph G is a *connected dominating set* if the induced subgraph $\langle D \rangle$ is connected in G. The *connected domination number* $\gamma_c(G)$ of G is the minimum cardinality of a connected dominating set.

Definition: 2.4

A dominating set D of a graph G is a *global dominating set* if D is also a dominating set of $\underline{G}$. The *global domination number* $\gamma_g(G)$ of G is the minimum cardinality of a global dominating set.

Definition: 2.5

A dominating set D of a graph $G = (V, E)$ is an *Accurate dominating set*, if $V - D$ has no dominating set of cardinality $|D|$. The *Accurate domination number* $\gamma_a(G)$ of G is the minimum cardinality of an accurate dominating set.

Definition: 2.6

An accurate dominating set D of a graph G is a *connected accurate dominating set* if the induced subgraph $\langle D \rangle$ is connected. The *connected accurate domination number* $\gamma_{ca}(G)$ of G is the minimum cardinality of a connected accurate dominating set.

Definition: 2.7

An accurate dominating set D of a graph G is a *global accurate dominating set*, if D is also an accurate dominating set of $\underline{G}$. The *global accurate domination number* $\gamma_{ga}(G)$ of G is the minimum cardinality of a global accurate dominating set.

Definition: 2.8

A maximal dominating set D of G is a *connected maximal dominating set* if the induced subgraph $\langle D \rangle$ is connected. The *connected maximal domination numbers* $\gamma_{cm}(G)$ of G is the minimum cardinality of a connected maximal dominating set.

Definition: 2.9

A set S of vertices in G is independent if no two vertices in S are adjacent. The *independence number* $\alpha_0(G)$ of G is the maximal cardinality of an independent set of vertices.

Let $\lfloor x \rfloor$ denote the greatest integer less than or equal to x.

III. GLOBAL CONNECTED ACCURATE and MAXIMAL DOMINATION in GRAPHS

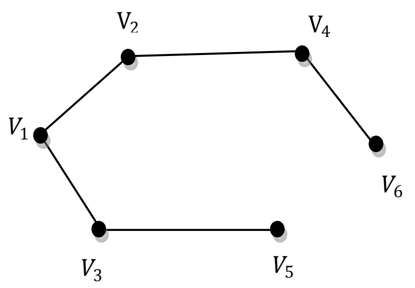
Definition: 3.1

An connected accurate dominating set D of a graph $G(p \geq 5)$ is a *global connected accurate dominating set*, if D is also an connected accurate dominating set of $\underline{G}$. The *global connected accurate domination number* $\gamma_{gca}(G)$ of G is the minimum cardinality of a global connected accurate dominating set.

Example: 3.2

The global connected accurate domination set of given graph (Fig.1) is $D = \{V_1, V_2, V_3, V_4\}$, $V - D = \{V_5, V_6\}$.

The cardinality of a global connected accurate dominating set is 4. The global connected accurate domination number of a graph is $\gamma_{gca}(G) = 4$.



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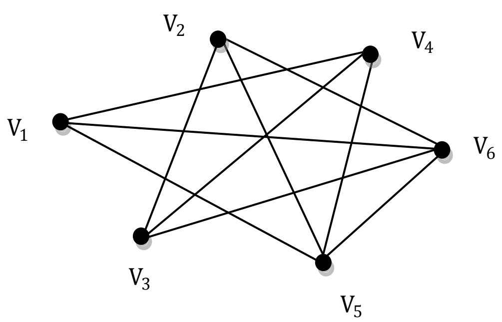

$\underline{G}$

Fig. 1

Theorem: 3.3

Let G be a graph such that neither G nor $\underline{G}$ have an isolated vertex. Then

(i). $\gamma_{gca}(G) = \gamma_{gca}(\underline{G})$ (1)

(ii). $(\frac{\gamma_a(G)+\gamma_a(\underline{G})}{2}) \leq \gamma_{ga}(G) \leq \gamma_{gca}(G) + \gamma_{gca}(\underline{G})$ (2)

Proof:

Let G be a graph without isolated vertex.

By the definition, every global connected accurate domination set is equal to the graph and its complement graph.

Then, The Equation $\gamma_{gca}(G) = \gamma_{gca}(\underline{G})$ is satisfied.

The condition (2) is also hold.

Theorem: 3.4

(i). $\gamma_{gca}(C_p) = \lfloor\frac{p}{2}\rfloor + 1$, if $p \geq 5$[4].

(ii). $\gamma_{gca}(P_p) \leq \lfloor\frac{p}{2}\rfloor + 1$, if $p \geq 6$.

(iii).$\gamma_{gca}(G) \leq \lfloor\frac{p}{2}\rfloor + 2$, if $p \geq 5$.

Theorem: 3.5

Let D be a connected accurate dominating set of G. If there exist two vertices $x \in V - D$ and $y \in D$ such that x is adjacent only to the vertices of D and y is adjacent only to the vertices of $V - D$.Then

$$\gamma_{gca}(G) \leq \gamma_{ga}(G) + 1.$$

Proof:

Let D be a $\gamma_{ga}(G)$ set of G. if these exists a vertex $x \in V - D$ and $y \in D$,

Such that x is adjacent only to the vertices of D.

Then $D \cup \{x\}$ is a global connected accurate dominating set of a graph G.

Thus, $\gamma_{gca}(G) \leq |D \cup \{x\}| \leq |D| + 1 \leq \gamma_{ga}(G) + 1$.

Theorem: 3.6

Let G be a connected graph with $p \geq 5$ vertices and $\underline{G}$ be a connected graph. Then

$\gamma_{gca}(G).\gamma_{gca}(\underline{G}) \leq (p-1)^2$,

$\gamma_{gca}(G) + \gamma_{gca}(\underline{G}) \leq 2(p-1)^2$

Furthermore the bounds are attained if $G = C_p$ [5].

Theorem: 3.7

Let G be a graph such that neither G nor $\underline{G}$ have an isolated vertex. Then

$$\gamma_{gca}(G) \leq p - 1.$$

Proof:

Clearly, G has two adjacent vertices and w and y such that, w is non adjacent to some vertex in $V - \{w\}$.

This implies that $V - \{w\}$ is a global connected accurate dominating set of G. Thus

$$\gamma_{gca}(G) \leq |V - \{w\}| \leq p - 1.$$

Theorem: 3.8

Let G $(p \geq 5)$ be a graph such that neither G nor $\underline{G}$ have an isolated vertex. Then

$$\gamma_{ga}(G) \leq \gamma_{gca}(G). \qquad (3)$$

Proof:

Let G be a graph without isolated vertex.

By the definition, every global accurate dominating set is a global connected accurate dominating set.

Thus (3) holds.

Theorem: 3.9

Let G be a graph without isolated vertices. Then

$$\gamma_{gca}(G) \leq \alpha_0(G) + 1.$$

Proof:

Let S be a maximum independent set of vertices in G.

Then for any vertex $u \in V - S$, $\{V - S\} \cup \{u\}\}$ is a global connected accurate dominating set of G.

Thus,

$$\begin{aligned}\gamma_{gca}(G) &\leq |(V - S) \cup \{u\}| \\ &\leq |V - S| + 1 \\ &\leq \alpha_0(G) + 1\end{aligned}$$

Definition: 3.10

A connected maximal dominating set D of G is a *global connected maximal dominating set* if D is also a connected maximal dominating set of $\underline{G}$. The *global connected maximal domination number* $\gamma_{gcm}(G)$ of G is the minimum cardinality of a global connected maximal dominating set.

Theorem: 3.11

For any connected graph G with $p \geq 5$ vertices.Then $\gamma_{gca}(G) \leq \gamma_{gcm}(G)$. (3)

Proof:

Let G be a connected graph.

Suppose, let $G = C_p$. Then every global connected accurate domination set of a graph G is a global connected maximal dominating set of G.

Hence the condition $\gamma_{gca}(G) \leq \gamma_{gcm}(G)$ is satisfied.

IV. CONCLUSIONS

The concept of domination in graphs is very rich both in the theoretical developments and applications. More than thirty domination parameters have been investigated by different authors, and in this paper we have introduced, the concepts of global connected accurate and maximal domination by using connected accurate and global accurate domination in graphs.

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