Changing and Unchanging Domination in Fuzzy Graph

M. Nithya kalyani[#1], K.Varunika[*2]
P.G Head and Assistant Professor[#], M.Phil Scholar,*
Department Of Mathematics#,*
Sakthi College of Arts and Science for Women[#],*
Oddanchatram-624619[#], Tamilnadu, India.*

***Abstract* - In this paper the concepts of Changing and unchanging domination in Fuzzy Graph and for some standard theorems and examples are discussed.**

Keywords - domination in fuzzy graph, changing domination in graph, unchanging domination in graph, changing and unchanging domination in fuzzy graphs.

I. INTRODUCTION

The study of domination set in graphs was begun by [5]V.R. Kulli and Bidarhali Janakiram (2004). The domination in fuzzy graph discussed by the [1]A. NagoorGani, and V.T.Chandrasekaran,(2006). [2] A.Somasundaram, S.Somasundaram, "Domination in fuzzy Graphs I"(1998), Domination alternation sets in graphs discussed by the [3] D. Bauer, F. Harary, J. Nieminen, S.L. Suffer, changing and unchanging domination in graph [4] J.R.Carrington, F.Harary, T.W.Haynes, changing and unchanging domination in fuzzy graphs G(V,ρ, μ) of γ(G).

II. PRELIMINARIES

A ***fuzzy graph*** $G =< \sigma,\mu >$is a set with two function $\sigma: V \to [0,1]$ and $\mu: E \to [0,1]$ such that $\mu(x,y) \le \sigma(x) \wedge \sigma(y)$ for all $x, y \in V$. hereafter we write $\mu(x,y)$or $\mu(xy)$.A fuzzy graph $H =< \tau,\rho >$ is called a ***fuzzy subgraph*** of G if $\tau(v_i) \le \sigma(v_i)$ for all $v_i \in V$and $\rho(v_i,v_j) \le \mu(v_i,v_j)$ for all $v_i,v_j \in V$.

A subset S of V is called a ***dominating set*** in G if for every $v \notin S$, there exists $u \in S$ such that$u$dominates v. The minimum fuzzy cardinality of a dominating set in G is called the ***domination number*** of G and is denoted by $\gamma(G)$orγ.

A dominating set S of a fuzzy graph G is said to be a minimal dominating set if no proper subset of S is a dominating set of G.

The removal of an edge from a graph G can increase by the domination number by at most one and cannot decrease the domination number. $\gamma(G - e) = \gamma(G) + 1$.

The removal of a vertex from a graph G can increase by the domination number by at most one and cannot decrease the domination number. $\gamma(G - v) = \gamma(G) + 1$

Domination color transversal **bondage number** of a graph G denoted by bb_{st} is defined to be the minimum cardinality of collection of sets E '⊆E such that γ_{st}(G) = γ(G – E '). If b_{st} is not defined for K_1 and K_2.

A graph for which the domination number changes when an ***vertex is removal (CVR)*** has V =$V^- \cup V^+$ Observed $V^0$is never empty for a tree, hence, no tree is in CVR.

V^0={ v ∈ V ; $\gamma(G - V) = \gamma(G)$},$V^+$ ={ v ∈ V ; $\gamma(G - V) > \gamma(G)$},$V^-$ ={ v ∈ V ; $\gamma(G - V) < \gamma(G)$}

The domination number is unchanged when an arbitrary **vertex is removed class UVR,** then V = V^0 .

III. CHANGING and UNCHANGING DOMINATION in FUZZY GRAPH

Definition

A Fuzzy graph for which the domination number changes when an arbitrary vertex is removed (CVR)has $V = V^{-} \cup V^{+}$.

$\gamma_f(G - v) \neq \gamma_f(G)$for all $v \in V$

A Fuzzy graph for which the domination number is unchanged when an arbitrary vertex is removed, (UVR) has $V = V^0$

$\gamma_f(G - v) = \gamma_f(G)$for all $v \in V$

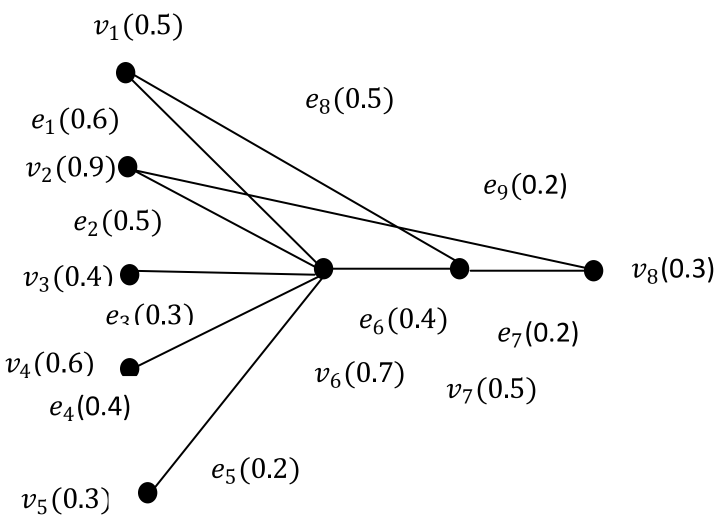

Figure1

Changing and unchanging in fuzzy graph

Theorem

For any fuzzy tree T with $n \geq 2$ there exists a vertex $v \in V$ such that $\gamma(T - V) = \gamma(T)$

Proof

Given that T^f is a fuzzy tree with $n \geq 2$.

Since T^f is a fuzzy tree if G is a acyclic with $n \geq 2$.Let $\gamma(T)$ be a minimum dominating set in a fuzzy tree T^f.

For any vertex $\in V$. Now we remove the vertex v in fuzzy tree T^f.

Therefore $\gamma(T^f - V)$ is a minimum dominating set of $G(T^f - V)$.

Hence G is a acyclic with $n \geq 2$, then $\gamma(T^f) = \gamma(T^f - V)$.

Theorem

For any graph G, $\gamma_f(G) \leq n - \Delta_f(G) - r_f(G) + 1$

Proof

For any fuzzy graph G with $\gamma(G) \geq 2$. Let $\mu = n - \max\{N[S]\}$ for all $S \subset V$ having $|S| = \gamma(G) - 1$. Let $r_f(G)$ to be smallest number of edges which must be added to G to decrease the domination number.

Hence $\gamma_f(G) \leq n - \Delta_f(G) - r_f(G) + 1$.

Theorem

If a graph $G \in CVR$ has order $n = (\Delta(G) + 1)(i(G) - 1) + 1$. Then G is regular fuzzy graph.

Proof

Suppose $G \in CVR$with $n = (\Delta(G) + 1)(i(G) - 1) + 1$

By theorem

Suppose $i(G) \neq i(G - V)$ for all $v \in V$ then $V^0 = \emptyset$ and so $v \in V^+$ or $v \in V^-$.If$v \in V^+$, then for each $v \in V^+ P_{fn}[v, S]$ contains two nonadjacent vertices. These vertices are in $v - S$ and so again if $P_{fn}[v, S] = \{v\}$ then $S - \{v\}$ is an independent fuzzy dominating set of $G - V$ which is a contradiction of $v \in V^+$

Hence V has a private neighbor in $V - S$.

Suppose $< P_{fn}[v, S] >$ is complete. Then $S - \{v\} \cup \{u\}$ for any $u \in P_{fn}[v, S]$ is an$i(G)$ $-$set of G not containing v again a contradiction.

Hence, $< P_{fn}[v, S] >$ contains at least two nonadjacent vertices they are not in V^+ . Since $G \in CVR$ we have $V = V^-$. Let S_u denote an $i(G) -$ set of $G - u$. So that $|S_u| = i(G) - 1$.

In order to dominate the $(\Delta(G) + 1)(i(G) - 1)$ vertices of $G - u$ each element of $S_u$must dominate exactly $(\Delta(G) + 1)$ vertices and has degree $\Delta(G)$.Thus no two vertices in S_u have a common neighbor.

To prove u is regular. It is enough to prove that for any arbitrary vertex, $x \in S_u$ for some u.

Let $r \in S_x$ we prove that $\in S_{rf}$. Suppose $x \notin S_{rf}$

Since $G \in CVR, S_{rF} \cap N[rf] = \emptyset$ each vertex $S_x - \{r\}$ dominates fuzzy unique vertex of $S_{r.}$ So remaining vertex in S_{rf}which is not x must be dominated by S_x and so must be dominated by rf which is a contradiction as $S_{rf} \cap N(rf) = \emptyset$.

Hence $x \in S_{rf}$ and so G is regular fuzzy graph.

Theorem

Let G be a unchanging fuzzy graph. If P_p is a path on P vertices. Then $P_p \in UEA$ if and only if $P = 3k + 2 (k \geq 1)$.

Proof

Suppose $P_p \in UEA$. Let $P = 3k$.

Let $P_p - (1,2, \ldots \ldots k)$ that $k > 1$. Consider the edge $e = (2,5)$. Then, $i(P_p + e) = k + 1$ where $P_p = k$ and so $P_p \notin UEA$ which is a contradiction.

Hence $P \neq 3k$ $(k > 2)$

Similarly,

If $P = 3k + 1$ with $e = (1,3)$. Then , $i(P_p + e) = k$ where as, $i(P_p) = k + 1$ and so $P_p \notin UEA$ which is a contradiction . Hence $n \neq 3k + 1$ and so either $P = 3$ or $P = 3k + 2$.

Conversely,

Assume $P = 3k + 2 \quad (k \geq 1)$. Then $i(P_p) = k + 1$

We prove that $P_p \in UEA$ by induction on k. When $k = 1, P = 5$ and one can verify that , $i(P_5 + e) = i(P_5) = 2$ for all $e \in E(\bar{G})$. Assume that the result is true for k.

Now we prove the result for $P = 3(k + 1) + 2 = 3k + 5$. Let $P_{3k+5} = (1,2 \dots\dots 3k + 5)$ by induction hypothesis any edges joining two vertices among $(1,2, \dots \dots 3k + 2)$ will not change $i(G)$. An adding $e = (3k + 3, 3k + 5)$ we observe that $i(G + e) = k + 2$. Also it is easy to verify that any edge joining a vertex of $\{1,2, \dots \dots 3k + 2\}$ and a vertex of $\{3k + 3, 3k + 4, 3k + 5\}$ does not change $i(G)$.

Hence $P_p \in UEA$

IV. CONCLUSIONS

The concept of changing and unchanging domination are analyzed. In this paper changing and unchanging domination in fuzzy graph are introduced and the some standard theorems are discussed.

REFERENCES

[1] A.Nagoor Gani, and V.T.Chandrasekaran, "Domination in Fuzzy Graph", Advances in Fuzzy Sets and Systems, 1(1) (2006), 17-26.
[2] A.Somasundaram, S.Somasundaram, "Domination in Fuzzy Graphs –I", Elseiver sciences, 19 (1998) 787-791. Discrete Mathematics 33(1981) 249 - 258.
[3] D.Bauer, F.Harary, J.Nieminen, S.L.Suffel, Domination alteration sets in graphs, Discrete math.47(1983) 153-161.
[4] J.R.Carrington, F.Harary, T.W.Haynes, Changing and unchanging domination, J.Combin.Math.Combin.Comput.9(1991) 57-63.
[5] V. R. Kulli and Bidarhalli Janakiram, "*The dominating graph*," Graph Theory Notes of New York XLVI, 5-8 (2004).