# Fuzzy Critical Path with Various Measures

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**Abstract**

*In operations research, networks play an important role as quite often the problem of determining an optimum solution can be looked upon as the problem of selecting the best sequence of operations out of a finite number of available alternatives that can be represented as a network. Network diagram plays a vital role to determine project completion time. Network Scheduling is a technique used for planning and scheduling large projects in the various fields such as construction, fabrication, purchasing etc. Network analysis is a technique which determines the various sequences of activities concerning a project and the project completion time. The popular method of this technique is widely used as the critical path method.*

*In this paper, we find the fuzzy critical path in a acyclic project network using magnitude measure to identify the fuzzy critical path from type-2 trapezoidal fuzzy numbers. An illustrative example is also included to demonstrate our proposed approach.*

**Keywords***: Fuzzy critical path, type-2 trapezoidal fuzzy numbers, Acyclic project network, magnitude measure, centroid measure*.

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## I.Introduction

The purpose of critical path method(CPM) is to aid in the planning and control of large, complex projects. The approach requires the consideration of : 1). What activities are to be done, 2). The sequence in which they will be performed,3). The resources required, and 4). The time required for each activity. The critical path method technique provides for the network "critical path," which consists of the sequence of project activities that determine the minimum required project time.

This paper analyze the critical path in a general project network with fuzzy activity times. The fuzzy measures were introduced by sugeno[9]. We propose magnitude and centroid measure[10] for fuzzy numbers to a critical path method in a fuzzy project network, where the duration time of each activity in a fuzzy project network is represented by a type-2 trapezoidal fuzzy number.

The structure of this paper is as follows: In section 2, we have some basic concepts. Section 3 contains some properties regarding calculation of the total slack fuzzy time. Section 4 gives the network terminology. Section 5 gives an algorithm to find the critical path combined with type-2 trapezoidal fuzzy number using magnitude and centroid measure method. To illustrate the proposed algorithm the numerical example is solved in section 6.

## II. Basic concepts

**2.1. Type-2 fuzzy number[8,11,12]:**

Let X be a type-2 fuzzy set defined in the universe of discourse R, if the following conditions are satisfied, then X is called a type-2 fuzzy number.

1. $X$ is normal.
2. $X$ is a convex set.
3. The support of $X$ is closed and bounded.

**2.2. Normal type-2 fuzzy number**

A type-2 fuzzy number(T2fs) X is said to be normal if its Foot of Uncertainty (FOU) is normal interval type-2 fuzzy number (IT2FS) and it has a primary membership function.

**2.3 Addition on type-2 fuzzy numbers**

$$\text{Let } \tilde{A} = \bigcup_{forall\ \tilde{\alpha}} \tilde{\alpha}\, FOU\ (\tilde{A}_{\tilde{\alpha}}) = (A^L, A^M, A^N, A^U)$$

$$= ((a_1^L, a_2^L, a_3^L, a_4^L), (a_1^M, a_2^M, a_3^M, a_4^M)(a_1^N, a_2^N, a_3^N, a_4^N)(a_1^U, a_2^U, a_3^U, a_4^U)$$

$$\text{Let } \tilde{B} = \bigcup_{forall\ \tilde{\alpha}} \tilde{\alpha}\, FOU\ (\tilde{B}_{\tilde{\alpha}}) = (B^L, B^M, B^N, B^U)$$

$$= ((b_1^L, b_2^L, b_3^L, b_4^L), (b_1^M, b_2^M, b_3^M, b_4^M)(b_1^N, b_2^N, b_3^N, b_4^N)(b_1^U, b_2^U, b_3^U, b_4^U)$$

be two normal type-2 fuzzy numbers. By using extension principle,

we have let $\tilde{A} + \tilde{B} =$

$$\left[\bigcup_{forall\ \tilde{\alpha}} \tilde{\alpha}\, FOU\ (\tilde{A}_{\tilde{\alpha}})\right] + \left[\bigcup_{forall\ \tilde{\alpha}} \tilde{\alpha}\, FOU\ (\tilde{B}_{\tilde{\alpha}})\right] = (A^L + B^L, A^M + B^M, A^N + B^N, A^U + B^U)$$

$$= \begin{matrix} ((a_1^L + b_1^L, a_2^L + b_2^L, a_3^L + b_3^L, a_4^L + b_4^L), (a_1^M + b_1^M, a_2^M + b_2^M, a_3^M + b_3^M, a_4^M + b_4^M) \\ (a_1^N + b_1^N, a_2^N + b_2^N, a_3^N + b_3^N, a_4^N + b_4^N)(a_1^U + b_1^U, a_2^U + b_2^U, a_3^U + b_3^U, a_4^U + b_4^U)) \end{matrix}$$

**2.5 Magnitude Measure**

Let $\tilde{A}$ = (a,b,c,d:λ) be a trapezoidal fuzzy number such that a<b<c<d. It is converted to triangular fuzzy number $\tilde{A} = (a, b_1 = \frac{b+c}{2}, d)$ such that a<$b_1$<d. The magnitude measure of the triangular fuzzy number $\tilde{A} = (a, b_1, d)$ with parametric form

$$\tilde{A}_\alpha = [A_L(\alpha), A_R(\alpha)] = [(b_1 - a)\alpha + a, d - (d - b_1)\alpha]$$

is given by

$$Mag\ \tilde{A} = \int_0^1 \frac{(A_L(\alpha) + A_R(\alpha) + b_1)}{2} \alpha\, d\alpha, \qquad \alpha \in [0,1]$$

$$= \frac{a + 7b_1 + d}{12}$$

**2.6. Centroid Measure**

Let $\tilde{A}_\alpha = [A_L(\alpha), A_R(\alpha)]$ be a α-cut interval number, then the centroid measure of

$$(\tilde{A}_\alpha) = \frac{\int_{(\mu-\sigma)+\sigma\alpha}^{(\mu+\sigma)-\sigma\alpha} \alpha x dx}{\int_{(\mu-\sigma)+\sigma\alpha}^{(\mu+\sigma)-\sigma\alpha} \alpha\, dx} = \mu = \frac{1}{2}(A_L(\alpha) + A_R(\alpha))$$

**2.7.Notations:**

$t_{ij}$= The activity between node i and j.

$ESF_j$ =The earliest starting fuzzy time of node j.

$LFF_i$ = The latest finishing fuzzy time of node i.

$TSF_{ij}$ = The total slack fuzzy time of $t_{ij}$.
$p_n$= the $n^{th}$ fuzzy path.
P = the set of all fuzzy paths in a project network
$F(p_n)$= The total slack fuzzy time of path $p_n$ in a project network.

## III.Properties[3,4,5,6]:

**Property :3.1** (Forward pass calculation)

To calculate the earliest starting fuzzy time in the project network, set the initial node to zero for starting (ie) $ESF_1 = (0.0,0.0,0.0,0.0)$

$ESF_j = \max_i \{ESF_j + TSF_{ij}\},\ j \neq i,\ j \in N$, i=number of preceding nodes.($ESF_j$ =The earliest starting fuzzy time of node j).

Ranking value is utilized to identify the maximum value. Earliest finishing fuzzy time = Earliest starting fuzzy time (+) Fuzzy activity time.

**Property 3.2.** (Backward pass calculation)

To calculate the latest finishing time in the project network set $LFF_n = ESFn$ .

$LFF_j = \min_j \{LFF_j (-) SET_{ij}\},\ i \neq n, i \in N$, j = number of succeeding nodes. Ranking value is utilized to identify the minimum value. Latest starting fuzzy time= Latest finishing Fuzzy time (-) Fuzzy activity time.

**Property 3.3.**

For the activity $t_{ij}$, i<j

Total fuzzy slack:

$SFT_{ij} = LFF_j (-)(ESF_i (+) SFT_{ij})$ (or) $(LFF_j (-) SFT_{ij})(-) ESF_i$, $1 \leq i \leq j \leq n;\ i, j \in N$,

**Property 3.4.**

$F(p_n) = \sum_{\substack{1 \leq i \leq j \leq n \\ i, j \in p_k}} SFT_{ij}$ , $p_k \in P$, $p_n$ is the possible paths in a fuzzy acyclic project network from source node to the destination node, k=1 to m.

## IV. Network terminology[1]:

Consider a directed acyclic project network G(V,E) consisting of a finite set of nodes V={1,2,…..n} and a set of m directed edges $E \subseteq V \times V$ . Each edge is denoted by an ordered pair (i,j) where $i. j \in V$ and $i. \neq j$ . In this network, we specify two nodes, denoted by s and t, which are the source node and the destination node, respectively. We define a path $p_{ij}$ as a sequence $p_{ij}=[i=i_1,(i_1,i_2),i_2,\ldots.i_{l-1},i_l=j]$ of alternating nodes and edges. The existence of at least one path $p_{si}$ in G(V,E) is assumed for every node $i \in V - \{S\}$ . $\tilde{d}_{ij}$ denotes a trapezoidal type–2 fuzzy number associated with the edge (i,j), corresponding to the length necessary to transverse (i,j) from i to j.

## V. Algorithm

### 5.1 Algorithm for finding critical path[7,8]:

**Step 1:** Estimate the fuzzy activity time with respect to each activity.

**Step 2:** Let $ESF_1 = (0.0,0.0,0.0,0.0)$ and calculate $ESF_j$ , j =2,3,…. n by using property 1.

**Step 3:** Let $LFF_n = ESF_n$ and calculate $LFF_i$ ,i=n-1, n-2,….. 2,1. By using property 2.

**Step 4:** Calculate $SFT_{ij}$ with respect to each activity in a project network by using property 3.

**Step 5 :** Calculate magnitude measure and centroid measure for each activity using definition 2.5.

**Step 6 :** If magnitude measure and centroid measure are zero those activities are called as fuzzy critical activity and the corresponding path is identified as fuzzy critical path.

## VI. Numerical example [2]:

**Example 6.1:**

The problem is to find the critical path and critical path length between source node to destination node in the fuzzy acyclic project network having 6 vertices and 7 edges with type-2 fuzzy number.

**Solution :**

The edge lengths are

$\tilde{P} = ((1.5,1.8,2.0,2.5), (1.2,1.5,2.0,2.2), (1.1,1.3,1.5,2.0), (1.0,1.3,1.5,2.0))$

$\tilde{Q} = ((1.6,1.7,1.8,2.0), (1.5,1.6,2.0,2.5), (1.3,1.5,2.0,2.3), (1.2,1.4,1.5,1.8))$

$\tilde{R} = ((1.8,1.9,2.2,2.6), (1.6,1.9,2.2,2.6), (1.5,1.7,2.1,2.3), (1.2,1.8,2.3,2.6))$

$\tilde{S} = ((1.3,1.5,1.9,2.2), (1.3,1.4,2.3,2.4,), (1.2,1.6,2.3,2.8), (1.0,1.5,1.9,2.3))$

$\tilde{T} = ((1.8,1.9,2.3,2.5), (1.6,1.8,2.3,2.4,), (1.5,1.6,1.9,2.3), (1.3,1.6,1.9,2.5))$

$\tilde{U} = ((1.5,1.7,2.0,2.5), (1.3,1.6,2.2,2.5), (1.2,1.6,2.3,2.6), (1.1,1.5,1.8,2.0))$

$\tilde{V} = ((1.9,2.0,2.3,2.4), (1.8,2.1,2.3,2.5), (1.5,1.7,2.0,2.3), (1.4,1.7,2.0,2.5))$

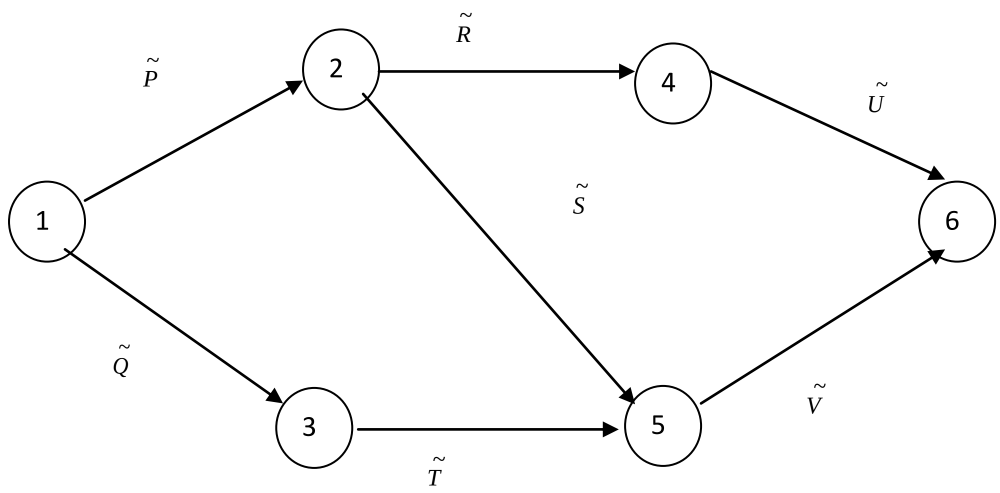


Fig .6.1

**Table:1 Activities, fuzzy durations and total slack fuzzy time for each activity**

| Activity(i-j)i<j | Fuzzy activity time | Defuzzified activity time converted in to triangular fuzzy number | Defuzzified activity time | Total float |
|---|---|---|---|---|
| 1-2 | $((1.5,1.8,2.0,2.5), (1.2,1.5,2.0,2.2), (1.1,1.3,1.5,2.0), (1.0,1.3,1.5,2.0))$ | (1.4,1.2,1.1) | 0.9 | 0.087 |
| 1-3 | $((1.6,1.7,1.8,2.0), (1.5,1.6,2.0,2.5), (1.3,1.5,2.0,2.3), (1.2,1.4,1.5,1.8))$ | (1.3,1.4,1.1) | 0.99 | 0 |

| | | | | |
|---|---|---|---|---|
| 2-4 | ((1.8,1.9,2.2,2.6), (1.6,1.9,2.2,2.6), (1.5,1.7,2.1,2.3), (1.2,1.8,2.3,2.6)) | (1.6,1.52,1.5) | 1.15 | 0.087 |
| 2-5 | ((1.3,1.5,1.9,2.2), (1.3,1.4,2.3,2.4, ), (1.2,1.6,2.3,2.8), (1.0,1.5,1.9,2.3)) | (1.3,1.5,1.3) | 1.1 | 0.107 |
| 3-5 | ((1.8,1.9,2.3,2.5), (1.6,1.8,2.3,2.4, ), (1.5,1.6,1.9,2.3), (1.3,1.6,1.9,2.5)) | (1.6,1.5,1.34) | 1.12 | 0 |
| 4-6 | ((1.5,1.7,2.0,2.5), (1.3,1.6,2.2,2.5), (1.2,1.6,2.3,2.6), (1.1,1.5,1.8,2.0)) | (1.4,1.4,1.2) | 1.1 | 0.087 |
| 5-6 | ((1.9,2.0,2.3,2.4), (1.8,2.1,2.3,2.5), (1.5,1.7,2.0,2.3), (1.4,1.7,2.0,2.5)) | (1.6,1.5,1.4) | 1.13 | 0 |

**Table:2 Activities, fuzzy durations and total slack fuzzy time for each activity**

| Activity(i -j)i<j | Fuzzy activity time | Fuzzy activity time(α-cut interval number) α=0.5 | Total float | Centroid measure |
|---|---|---|---|---|
| 1-2 | ((1.5,1.8,2.0,2.5), (1.2,1.5,2.0,2.2), (1.1,1.3,1.5,2.0), (1.0,1.3,1.5,2.0)) | (1.76,1.76) | (0.91,-0.56) | 0.175 |
| 1-3 | ((1.6,1.7,1.8,2.0), (1.5,1.6,2.0,2.5), (1.3,1.5,2.0,2.3), (1.2,1.4,1.5,1.8)) | (2.2,1.5) | (1.2,-1.2) | 0 |
| 2-4 | ((1.8,1.9,2.2,2.6), (1.6,1.9,2.2,2.6), (1.5,1.7,2.1,2.3), (1.2,1.8,2.3,2.6)) | (2.2,2.03) | (0.87,-0.6) | 0.135 |
| 2-5 | ((1.3,1.5,1.9,2.2), (1.3,1.4,2.3,2.4, ), (1.2,1.6,2.3,2.8), (1.0,1.5,1.9,2.3)) | (1.9,1.9) | (0.8,-0.5) | 0.15 |
| 3-5 | ((1.8,1.9,2.3,2.5), (1.6,1.8,2.3,2.4, ), (1.5,1.6,1.9,2.3), (1.3,1.6,1.9,2.5)) | (2.13,1.93) | (1.2,-1.2) | 0 |
| 4-6 | ((1.5,1.7,2.0,2.5), (1.3,1.6,2.2,2.5), (1.2,1.6,2.3,2.6), (1.1,1.5,1.8,2.0)) | (1.9,1.8) | (0.87,-0.6) | 0.135 |
| 5-6 | ((1.9,2.0,2.3,2.4), (1.8,2.1,2.3,2.5), (1.5,1.7,2.0,2.3), (1.4,1.7,2.0,2.5)) | (2.2,1.9) | (1.2,-1.2) | 0 |

From the table (1,2) we observe that

1. The expected time in terms of trapezoidal fuzzy numbers are defuzzified using definition 2.6 and 2.7 for all activities in the given acyclic project network.
2. By using property 3.4 all possible paths P={(1-2-4-6),(1-2-5-6),(1-3-5-6)} are found.
3. Fuzzy critical path is identified with the help of magnitude and centroid measure.
4. The path (1-3-5-6) is the fuzzy critical path in a given acyclic fuzzy project network by using both the measures.

## VII. Conclusion:

In this work, magnitude measure and centroid measure are used to find the fuzzy critical path in a acyclic project network. we found that the critical path which is obtained by both the measures are same. Hence we conclude that the obtained critical path is unique one.

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