Cost Minimization of Fuzzy Assignment Problem using Two Types of Symmetric Intuitionistic Fuzzy Numbers

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Abstract: *This paper illustrates the method of obtaining the minimum cost of the Fuzzy Assignment Problem using a Symmetric Intuitionistic Fuzzy Number. The comparative study of different average rankings of Symmetric Triangular Intuitionistic Fuzzy Number (STIFN) and Symmetric Trapezoidal Intuitionistic Fuzzy Number (STrIFN) is discussed. The cost values of the given Fuzzy Assignment Problem are taken as STIFN and STrIFN. The STIFN and STrIFN are converted into crisp values using proposed rankings. The Fuzzy Assignment Problem is solved by usual Hungarian Method. A numerical example is given to illustrate the proposed approach.*

Keywords: *Triangular Fuzzy Number, Triangular Intuitionistic Fuzzy Number, Symmetric Triangular Intuitionistic Fuzzy Number(STIFN), Symmetric Trapezoidal Intuitionistic Fuzzy Number (STrIFN), Fuzzy Assignment Problem*

I. INTRODUCTION

Assignment problem is a special type of Transportation Problem. The objective of the assignment problem is to assign 'n' number of origins to 'n' number of destinations at a minimum cost (or maximum profit) under the assumption that each machine can perform each job but with varying degree of effectiveness. Therefore the assignment problem is a special case of a number of tasks (Jobs or Origins or Sources) assigned to an equal number of facilities (Machines or persons or Destinations) at a minimum cost (or maximum profit). In real life situations the parameters of assignment problems are imprecise numbers instead of fixed real numbers. In assignment problems the parameters like time and cost might vary due to different reasons.

The concept of fuzzy set was introduced by Zadeh [15] in 1965 and it dealt with imprecision, vagueness in real life situations. In 1970, Bellman & Zadeh [4] proposed the concept of decision making problems involving uncertainty and imprecision. K.T. Attanasov [1], [2], [3] introduced the Intutitionistic fuzzy sets and their applications to deal with vagueness. W.L. Gau and D.J. Buehrer [6] reviewed the concept of vagueness sets. Bustine and Burillo [5] discussed that intuitionistic fuzzy sets are vague sets. M. H. Shu, C. H. Cheng and J.R. Chang [13] gave the definition and operational laws of triangular intuitionistic fuzzy number and proposed an algorithm of the intuitionistic fuzzy set fault-tree analysis. R. Parvathi, C. Malathi [8] introduced the Symmetric Trapezoidal Intuitionistic Fuzzy Numbers and proposed its Arithmetic Operations. J.Q. Wang [14] analyzed the overview of fuzzy multi-criteria decision-making approach. K. Prabakaran, K. Ganesan [9] solved the intuitionistic fuzzy assignment problem using the fuzzy Hungarian method. Sagaya Roseline, Henry Amirtharaj [10] finds the solution for the intuitionistic fuzzy assignment problem with the ranking of intuitionistic fuzzy numbers. P.Senthil Kumar, R. Jahir Hussain [11] examine a method for solving balanced Intuitionistic fuzzy assignment problem. Shiny Jose, Sunny Kuriakose [12] review an algorithm for solving an assignment model in intuitionistic fuzzy context. S. Krishna Prabha, S. Vimala [7] discover the optimal solution for the intuitionistic fuzzy assignment problem via three methods-IFRMM, IFOAM, IFAM.

In this paper a Fuzzy Assignment problem is considered. Two types of Symmetric Intutiionistic Fuzzy Numbers are considered. The assignment cost of the Fuzzy Assignment Problem is taken as STIFN and STrIFN. The STIFN and STrIFN are converted into crisp values using the proposed ranking. The problem is then solved by the usual Hungarian method to get the optimum assignment schedule and the minimum cost (or maximum profit).

The rest of this paper is organized as follows. In Section 2, some basic definitions, Arithmetic Operations of STrIFN and the proposed ranking of STIFN and STrIFN are given. Section 3, presents an introduction of the Fuzzy Assignment Problem. In Section 4, procedure, numerical example for the proposed method and the comparative study are provided followed by conclusion in section-5.

II. DEFINITIONS

Definition: 2.1. Fuzzy Set

Let A be a classical set, $\mu_A(x)$ be a function from A to [0,1]. A fuzzy set A with the membership function $\mu_A(x)$ is defined as $A = \{x, \mu_A(x); x \in A \text{ and } \mu_A(x) \in [0,1]\}$.

Definition: 2.2. Intuitionistic Fuzzy Set

Let X be a given set. An Intuitionistic Fuzzy Set A in X is given by, where $A=\{(x, \mu_A(x), \vartheta_A(x)) \mid x \in X\}$ $X \rightarrow [0, 1]$, where $\mu_A(x)$ is the degree of membership of the element x in A and, $\vartheta_A(x)$ is the degree of non membership of x in A and $0 \leq \mu_A(x) + \vartheta_A(x) \leq 1$.

Definition: 2.3. Triangular Fuzzy Number

A fuzzy number $A = (a_1, a_2, a_3)$ is defined to be a triangular fuzzy number if its membership functions $\mu_A : R \rightarrow [0,1]$ is equal to

$$\mu_A(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1} & if\ x \in [a_1, a_2] \\ 1 & x = a_2 \\ \dfrac{a_3 - x}{a_3 - a_2} & if\ x \in [a_2, a_3] \\ 0 & otherwise \end{cases}$$

It is denoted by $A = (a_1, a_2, a_3)$ where a_1 is Core (A), a_2 is left width and a_3 is right width.

Definition: 2.4. Symmetric Triangular Fuzzy Number

If $a_2 = a_3$, then the Triangular Fuzzy Number $A = (a_1, a_2, a_3)$ is called Symmetric Triangular Fuzzy Number. It is denoted by $A = (a_1, a_2)$ where a_1 is Core (A), a_2 is left width and right width of Core(A).

Definition: 2.5. Triangular Intuitionistic Fuzzy Number (TIFN)

A Triangular Intuitionistic Fuzzy Number $A = (a_1, a_2, a_3 ; a_1', a_2, a_3')$ is an Intuitionistic Fuzzy set in R with the following membership function $\mu_A(x)$ and non-membership function $\vartheta_A(x)$ is defined by

$$\mu_A(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1} & if\ a_1 \leq x < a_2 \\ 1 & x = a_2 \\ \dfrac{a_3 - x}{a_3 - a_2} & if\ a_2 < x \leq a_3 \\ 0 & otherwise \end{cases}$$

$$\vartheta_A(x) = \begin{cases} \dfrac{a_2 - x}{a_2 - a_1'} & if\ a_1' \leq x < a_2 \\ 1 & x = a_2 \\ \dfrac{x - a_3}{a_3' - a_2} & if\ a_2 < x \leq a_3' \\ 0 & otherwise \end{cases}$$

Respectively, Where $a_1' \leq a_1 < a_2 < a_3 \leq a_3'$. A Triangular Intuitionistic Fuzzy Number

$A = (a_1, a_2, a_3 ; a_1', a_2, a_3')$ also denoted by $A = (a_2, \alpha, \alpha' ; a_2, \beta, \beta')$

where $\alpha = a_2 - a_1$, $\alpha' = a_3 - a_2$, $\beta = a_2 - a_1'$, $\beta' = a_3' - a_2$.

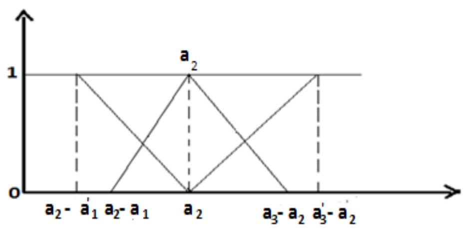

Fig. 1 Diagrammatic representation of Triangular Intuitionistic Fuzzy Number

Definition: 2.6. Symmetric Triangular Intuitionistic Fuzzy Number (STIFN)

A Triangular Intuitionistic Fuzzy Number $A = (a_1, \alpha, \alpha' ; a_1, \beta, \beta')$ whose left spread α is equal to right spread α' of a memembership function as well as left spread β is equal to right spread β' of a non membership function is called STIFN and is denoted by $A = (a_1, \alpha, \alpha ; a_1, \beta, \beta)$

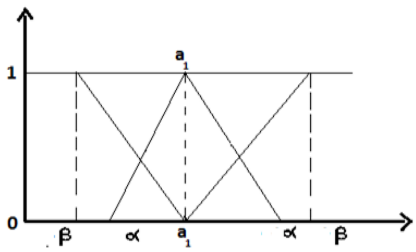

Fig. 2 Diagrammatic representation of STIFN

Definition: 2.7. Trapezoidal Intuitionistic Fuzzy Number

A Trapezoidal Intuitionistic Fuzzy Number A is an Intuitionistic Fuzzy set in R with membership function and non membership function as follows:

$$\mu_A(x) = \begin{cases} \dfrac{x-(a_1-h)}{h}, & x \in [a_1 - h\, , a_1] \\ 1, & x = [a_1, a_2] \\ \dfrac{(a_2+k)-x}{k}, & x \in [a_1, a_2 + k] \\ 0 & otherwise \end{cases}$$

$$\vartheta_A(x) = \begin{cases} \dfrac{(a_1-x)}{h'}, & x \in [a_1 - h'\, , a_1] \\ 0, & x = [a_1, a_2] \\ \dfrac{x-a_2}{k'}, & x \in [a_2, a_2 + k'] \\ 1 & otherwise \end{cases}$$

Where $a_1 \leq a_2,\ h, k \geq 0$ such that $h \leq h'$ and $k \leq k'$.

A Trapezoidal Intuitionistic Fuzzy Number is denoted by $A = (a_1, a_2, h, k\ ;\ a_1, a_2, h', k')$.

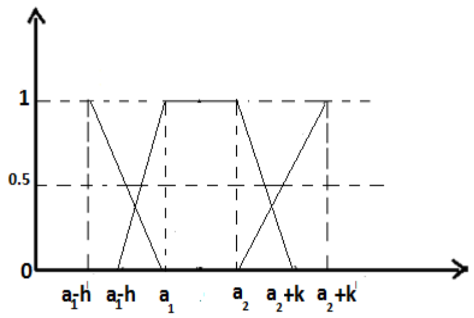

Fig. 3 Diagrammatic representation of Trapezoidal Intuitionistic Fuzzy Number

Definition: 2.8. Symmetric Trapezoidal Intuitionistic Fuzzy Number (STrIFN)

A Trapezoidal Intuitionistic Fuzzy Number is said to be **STrIFN** if h=k (say α) and h' =k' (say β) . Hence the definition of **STrIFN** is as follows.

An Intuitionistic Fuzzy set A in R is said to be a **STrIFN** if there exist real numbers a_1, a_2, α, β where $a_1 \leq a_2\ , \alpha \leq \beta$ and $\alpha, \beta \geq 0$ such that the membership and non membership functions as follows.

$$\mu_A(x) = \begin{cases} \dfrac{x-(a_1-\alpha)}{\alpha}, & x \in [a_1 - \alpha\, , a_1] \\ 1, & x = [a_1, a_2] \\ \dfrac{(a_2+\alpha)-x}{\alpha}, & x \in [a_1, a_2 + \alpha] \\ 0 & otherwise \end{cases}$$

$$\vartheta_A(x)=\begin{cases}\frac{(a_1-x)}{\beta}, & x\in[a_1-\beta, a_1]\\ 0, & x=[a_1,a_2]\\ \frac{x-a_2}{\beta}, & x\in[a_2,a_2+\beta]\\ 1 & otherwise\end{cases}$$

A **STrIFN** is denoted by $A=(a_1,a_2\alpha,\alpha\ ;\ a_1,a_2,\beta,\beta)$

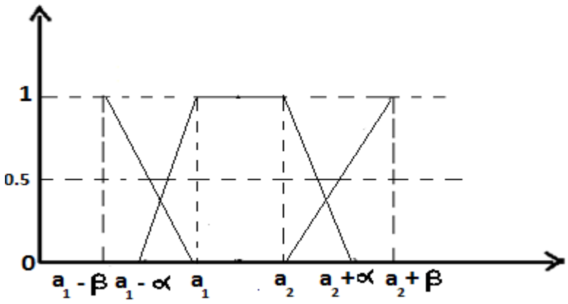

Fig. 4 Diagrammatic representation of **STrIFN**

Arithmetic Operations of Symmetric Trapezoidal Intuitionistic Fuzzy Number (STrIFN)

Let $A=(a_1,a_2,\alpha,\alpha\ ;\ a_1,a_2,\beta,\beta)$ and $B=(b_1,b_2,\alpha',\alpha'\ ;\ b_1,b_2,\beta',\beta')$ be two **STrIFN,** the arithmetic operations on A and B is given below:

1.Addition: $(a_1,a_2,\alpha,\alpha\ ;\ a_1,a_2,\beta,\beta)+(b_1,b_2,\alpha',\alpha'\ ;\ b_1,b_2,\beta',\beta')$

$=(a_1+b_1,a_2+b_2,\alpha+\alpha',\alpha+\alpha';\ a_1+b_1,a_2+b_2,\beta+\beta',\beta+\beta')$

2.Subtraction: $(a_1,a_2,\alpha,\alpha\ ;\ a_1,a_2,\beta,\beta)-(b_1,b_2,\alpha',\alpha'\ ;\ b_1,b_2,\beta',\beta')$

$=(a_1-b_2,a_2-b_1,\alpha+\alpha',\alpha+\alpha';\ a_1-b_1,a_2-b_1,\beta+\beta',\beta+\beta')$ ***3. Scalar Multiplication:***

$$k(a_1,a_2\alpha,\alpha;\ a_1,a_2,\beta,\beta)=\begin{cases}(ka_1,ka_2,k\alpha,k\alpha;ka_1,ka_2,k\beta,k\beta) & if\ k>0\\ (ka_2,ka_1-k\alpha,-k\alpha;ka_2,ka_1,-k\beta,-k\beta) & if\ k<0\end{cases}$$

Proposed Ranking for Symmetric Triangular Intuitionistic Fuzzy Number (STIFN)

If $A=(a_1,\alpha,\alpha\ ;\ a_1,\beta,\beta)$ is a **STIFN** then the average ranking is defined by

$$R(A)=\frac{(a_1+4\alpha+\alpha)+(a_1+4\beta+\beta)}{12}$$

Proposed Ranking for Symmetric Trapezoidal Intuitionistic Fuzzy Number (STrIFN)

If $A=(a_1,a_2\alpha,\alpha\ ;\ a_1,a_2\beta,\beta)$ is a **STrIFN,** then the average ranking is defined by

$$R(A)=\frac{(a_1+a_2+\alpha+\alpha)+(a_1+a_2+\beta+\beta)}{8}$$

III. FUZZY ASSIGNMENT PROBLEM

Consider the situation of assigning *n* machines to *n* jobs and each machine is capable of doing each job at different costs. Let C_{ij}^* be the fuzzy cost of assigning the i^{th} machine to the j^{th} job. Let x_{ij} be the decision variable denoting the assignment of the i^{th} machine to the j^{th} job. The objective is to minimize the total cost. The mathematical model of the Fuzzy Assignment Problem is given by

Minimize $z^*=\sum_{i=1}^{n}\sum_{j=1}^{n}C_{ij}^*\,x_{ij}$

Subject to:

$$\sum_{i=1}^{n}x_{ij}=1, for\ j=1,2\ldots\ldots n$$

$$\sum_{j=1}^{n}x_{ij}=1, for\ i=1,2\ldots\ldots n$$

$x_{ij}\in[0,1]$ Where

$$x_{ij}=\begin{cases}1, & if\ the\ i^{th} machine\ is\ assigned\ to\ j^{th}\ job\\ 0, & if\ the\ i^{th} machine\ is\ not\ assigned\ to\ j^{th}\ job\end{cases}$$

IV. PROCEDURE

1. Consider the Fuzzy assignment problem with 'n' number of rows and 'n' number of columns.
2. First convert the cost values of the fuzzy assignment problem into crisp values by using the proposed ranking .
3. Check the condition that the fuzzy assignment problem is balanced.
   (i) If balanced go to step 5 (Number of rows = Number of Columns)
   (ii) If not balanced go to step 4 (Number of rows ≠ Number of Columns)
4. If the given Fuzzy Assignment problem is not balanced, then add a dummy row (or) dummy Column with cost value as zero to make the fuzzy assignment problem a balanced one.

5. Obtain the optimum assignment schedule by Hungarian method.

1. Example for STIFN

Consider a Fuzzy Assignment Problem with rows representing 3 machines M_1, M_2, M_3 and columns representing the 3 Jobs J_1, J_2, J_3. In this Fuzzy Assignment Problem the cost matrix C^*_{ij} elements are Symmetric Triangular Intuitionistic Fuzzy Numbers. The objective is to find the minimum cost.

$$\begin{pmatrix} (3,1,1\,;\,3,2,2) & (7,4,4\,;7,5,5) & (6,2,2\,\,;6,3,3) \\ (5,2,2\,;5,4,4) & (6,1,1\,;6,3,3) & (4,1,1\,;4,2,2) \\ (8,3,3\,;8,4,4) & (9,3,3\,;9,4,4) & (5,2,2\,;5,3,3) \end{pmatrix}$$

Using the proposed average ranking, the crisp values of the above problem is

$$\begin{pmatrix} 1.75 & 4.92 & 2.83 \\ 3.33 & 2.67 & 1.92 \\ 4.25 & 4.42 & 2.92 \end{pmatrix}$$

Number of rows = Number of columns.
Therefore the fuzzy assignment problem is balanced.
Now, the assignment problem is solved by Hungarian method. The assignment schedule of the fuzzy assignment problem is given by

$$\begin{pmatrix} (0) & 2.42 & 1.08 \\ 1.41 & (0) & 0 \\ 1.33 & 0.75 & (0) \end{pmatrix}$$

The assignment schedule is $M_1 \rightarrow J_1$, $M_2 \rightarrow J_2$, $M_3 \rightarrow J_3$

The assignment cost = 1.75 + 2.67 + 2.92 = 7.34.

2. Example for STrIFN

Consider a Fuzzy Assignment Problem with rows representing 4 machines M_1,M_2,M_3, M_4 and columns representing the 4 Jobs J_1,J_2,J_3, J_4. In this Fuzzy Assignment Problem the cost matrix C^*_{ij} elements are Symmetric Trapezoidal Intuitionistic Fuzzy Numbers. The objective is to find the minimum cost.

$$\begin{pmatrix} (5,7,1,1\,;5,7,3,3) & (5,6,2,2\,;5,6,4,4) & (11,13,8,8\,;11,13,10,10) & (6,8,2,2\,;6,8,4,4) \\ (13,15,6,6\,;13,15,8,8) & (6,8,3,3\,;6,8,5,5) & (14,15,8,8\,;\,14,15,10,10) & (5,6,2,2\,;5,6,4,4) \\ (10,11,6,6\,;10,11,8,8) & (6,8,2,2\,;6,8,4,4) & (4,5,1,1\,;4,5,3,3) & (6,8,1,1\,;6,8,3,3) \\ (11,13,8,8\,;11,13,10,10) & (4,5,1,1\,\,;4,5,3,3) & (13,15,6,6\,;13,15,8,8) & (5,7,2,2\,;5,7,4,4) \end{pmatrix}$$

Using proposed average ranking, the crisp values of the above problem is

$$\begin{pmatrix} 4 & 4.25 & 10.5 & 5 \\ 10.5 & 5.5 & 11.75 & 4.25 \\ 8.75 & 5 & 3.25 & 4.5 \\ 10.5 & 3.25 & 10.5 & 4.5 \end{pmatrix}$$

Number of rows = Number of columns.
Therefore the fuzzy assignment problem is balanced.
Now, the assignment problem is solved by Hungarian method. The assignment schedule of the fuzzy assignment problem is given by

$$\begin{pmatrix} (0) & 0.25 & 6.5 & 1 \\ 6.25 & 1.25 & 7.5 & (0) \\ 5.6 & 1.75 & (0) & 1.25 \\ 7.25 & (0) & 7.25 & 6 \end{pmatrix}$$

∴ Assignment schedule is $M_1 \to J_1$, $M_2 \to J_4$, $M_3 \to J_3, M_4 \to J_2$

∴ ***The assignment cost = 4 + 4.25 +3.25 + 3.25 = 14.75***.

Comparative Study of STIFN using different average ranking:

S.No.	Ranking Functions	Optimal Solution
1	$\frac{(a_1 + 4\alpha + \alpha) + (a_1 + 4\beta + \beta)}{12}$	7.34
2	$\frac{(a_1 + 3\alpha + \alpha) + (a_1 + 3\beta + \beta)}{10}$	7.6
3	$\frac{(a_1 + 2\alpha + \alpha) + (a_1 + 2\beta + \beta)}{8}$	8
4	$\frac{(a_1 + \alpha + \alpha) + (a_1 + \beta + \beta)}{6}$	8.666

Comparative Study of STrIFN using a different average ranking:

S.No.	Ranking Functions	Optimal Solution
1	$\frac{(a_1 + a_2 + \alpha + \alpha) + (a_1 + a_2 + \beta + \beta)}{8}$	14.75
2	$\frac{(a_1 + 2a_2 + 2\alpha + \alpha) + (a_1 + 2a_2 + 2\beta + \beta)}{12}$	15.166
3	$\frac{(a_1 + 3a_2 + 3\alpha + \alpha) + (a_1 + 3a_2 + 3\beta + \beta)}{16}$	15.375
4	$\frac{(a_1 + 4a_2 + 4\alpha + \alpha) + (a_1 + 4a_2 + 4\beta + \beta)}{20}$	15.5

V. CONCLUSION:

This paper proposes an optimal solution of the Fuzzy Assignment Problem whose costs are taken as Symmetric Intuitionistic Fuzzy Numbers. Different ranking functions for STIFN and STrIFN are compared. The comparative study reveals a better ranking for both STIFN and STrIFN. The proposed approach would be appropriate in dealing with assignment problems involving imprecise and vague parameters.

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