An Intuitionistic Uniform Grill Structure Space

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***Abstract** -The purpose of this paper is to introduce the concepts of an intuitionistic uniform B-closed spaces in terms of intuitionistic uniform grills. The concepts of intuitionistic uniform grill, intuitionistic uniform B -converge, intuitionistic uniform B -adhere and intuitionistic uniform section grill are introduced and studied. The concepts of intuitionistic uniform B-closed relative, intuitionis-tic uniform B -linked, intuitionistic uniform B -conjoint, intuitionistic uniform B-regular open symmetric member and intuitionistic uniform B -space are intro-duced. Besides providing some interesting properties and characterizations are introduced and discussed.*

***Keywords**-Intuitionistic symmetric member, intuitionistic t-open symmetric member, intuitionistic B-open symmetric member, intuitionistic uniform B -converge, in-tuitionistic B -adherence, intuitionistic uniform grill, intuitionistic uniform lter, intuitionistic uniform section grill, intuitionistic uniform B-closed space, in-tuitionistic uniform B -linked, intuitionistic uniform B -conjoint, intuitionistic uniform B-regular open symmetric member, intuitionistic uniform B-almost reg-ular, intuitionistic uniform B-regular space and intuitionistic uniform B -space.*

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I. INTRODUCTION

The concept of intuitionistic sets in topological spaces was introduced by Coker in [2]. He studied topology on intuitionistic sets in [3]. In 1937, Andre Weil [6] formulated the concept of uniform space which is a generalization of a metric space. J. Tong [5] introduced the concept of B-set in topological space. Grills in a topological space ($\mathbb{X}$, T) is initiated by G.Choquet [4] in 1947.The purpose of this paper is to intro-duce the concepts of an intuitionistic uniform B-closed spaces in terms of intuitionistic uniform grills. The concepts of intuitionistic uniform grill, intuitionitic uniform B - converge, intuitionitic uniform B -adhere and intuitionistic uniform section grill are introduced and studied. The concepts of intuitionistic uniform B-closed relative, in-tuitionitic uniform B -linked, intuitionitic uniform B -conjoint, intuitionistic uniform B-regular open symmetric member and intuitionistic uniform B -space are introduced. Besides providing some interesting properties and characterizations are introduced and discussed.

II. PRELIMINARIES

DEFINITION 2.1[4] Let $\mathbb{X}$ be a non empty set. An intuitionistic set (IS for short) A is an object having the form A = $\langle x, A^1, A^2 \rangle$ for every $\mathbb{X} \in \mathbb{X}$, where A^1 and A^2 are subsets of $\mathcal{X}$ satisfying $A^1 \cap A^2 = \emptyset$. The set A^1 is called the set of members of A, while A^2 is called the set of nonmembers of A. Every crisp set A on a nonempty set $\mathbb{X}$ is obviously an intuitionistic set having the form $\langle x, A, A^c \rangle$.

DEFINITION 2.2[5] Let $\mathbb{X}$ be a non empty set and let the intuitionistic sets A and B be in the form A = $\langle x, A^1, A^2 \rangle$, B = $\langle x, B^1, B^2 \rangle$, respectively. Furthermore, let $\{A_i : i \in J\}$ be an arbitrary family of intuitionistic sets in $\mathbb{X}$, where $A_i = \langle x, A_i^{\,1}, A_i^{\,2} \rangle$. Then

(i) A ⊆ B if and only if $A^1 \subseteq B^1$ and $A^2 \supseteq B^2$,
(ii) A = B if and only if A ⊆ B and B ⊆ A,
(iii) $\overline{A} = \langle x, A^2, A^1 \rangle$,
(iv) $\cup A_i = \langle x, \cup A_i^{\,1}, \cap A_i^{\,2} \rangle$,
(v) $\cap A_i = \langle x, \cap A_i^{\,1}, \cup A_i^{\,2} \rangle$,
(vi) $\emptyset_\sim = \langle x, \emptyset, \mathbb{X} \rangle$, $\mathbb{X}_\sim = \langle x, \mathbb{X}, \emptyset \rangle$.

DEFINITION 2.3[5] An intuitionistic topology (IT for short) on a nonempty set $\mathbb{X}$ is a family T of intuitionistic sets in $\mathbb{X}$ satisfying the following a$\mathbb{X}$ioms:

(i) $\emptyset_\sim$, $\mathbb{X}_\sim$ ∈ T.
(ii) $G_1 \cap G_2 \in T$ for any $G_1, G_2 \in T$.
(iii) $\cup G_i \in T$ for any arbitrary family $\{G_i : i \in J\} \subset T$.

In this case the pair (𝕏, T) is called an intuitionistic topological space (ITS for short) and any intuitionistic set in T is called an intuitionistic open set (IOS for short) in 𝕏. The complement $\overline{A}$ of an intuitionistic open set *A* is called an intuitionistic closed set (ICS for short) in 𝕏.

DEFINITION 2.4[5] Let (𝕏, T) be an intuitionistic topological space and $A = \langle x, A^1, A^2 \rangle$ be an intuitionistic set in 𝕏. Then the closure and interior of A are defined by

Icl(A) = ∩{ K : K is an intuitionistic closed set in 𝕏 and A ⊆ K }.

Iint(A) = ∪{ G : G is an intuitionistic open set in 𝕏 and G ⊆ A }.

It can be also shown that cl(A) is an intuitionistic closed set and int(A) is an intuitionistic open set in 𝕏, and A is an intuitionistic closed set in 𝕏 iff cl(A) = A; and A is an intuitionistic open set in 𝕏 iff int(A) = A.

DEFINITION 2.6[5] A uniform space 𝕏 with uniformity ξ is a set 𝕏 with a nonempty collection ξ of subsets containing the diagonal $\Delta_{\mathbb{X}}$ in 𝕏 × 𝕏 satisfying the following properties:
(i) If E, F ∈ ξ, then E ∩ F ∈ ξ .
(ii) If F ⊂ E and E ∈ ξ then F ∈ ξ.
(iii) If E ∈ ξ then E^t = {(𝕏, y) : (y, 𝕏) ∈ E } ∈ ξ.
(iv) For any E ∈ ξ there is some F ∈ ξ such that $F^2 \subset E$.

DEFINITION 2.5 Let (𝕏, T) be a topological space.A subset S in 𝕏 is said to be a
(i) t-set[9] if intcl(S) = int(S)
(ii) B-set[9] if S = U∩ A, where U is an open and A is t-set,

3. AN INTUITIONISTIC UNIFORM B-OPEN SYMMETRIC MEMBER IN INTUITIONISTIC UNIFORM STRUCTURE SPACES

DEFINITION 3.1 Let 𝕏 × 𝕏 be a non empty set. An intuitionistic symmetric member (ISM for short) A is an object having the form $A = \langle x, A^1, A^2 \rangle$ where A^1 and A^2 are subsets of 𝕏 × 𝕏 satisfying $A^1 \cap A^2 = \Delta$. The set A^1 is called the set of members of A, while A^2 is called the set of nonmembers of A.

NOTATION 3.1. Let (𝕏 × 𝕏, ξ) be an intuitionistic uniform structure space and it is simply denoted by (𝕏, ξ).

NOTATION 3.2. Let 𝕏 × 𝕏 be a non empty set.
(i) $\Delta_{\sim} = \langle \Delta, \mathbb{X} \times \mathbb{X} \rangle$,
(ii) $\mathbb{X}_{\sim} = \langle \mathbb{X} \times \mathbb{X}, \Delta \rangle$.

DEFINITION 3.2. An intuitionistic uniform structure (IUS for short) on a non empty set 𝕏 is a collection ξ of subsets in 𝕏 which satisfies the following aXioms
(i) $\Delta_{\sim}, \mathbb{X}_{\sim} \in \xi$.
(ii) $E_1 \cap E_2 \in \xi$ for any $E_1, E_2 \in \xi$.
(iii) $\cup E_i \in \xi$ for any arbitrary family $\{E_i : i \in J\} \subseteq \xi$.
(iv) If $E_1 \subset E_2$ and $E_1 \in \xi$ then $E_2 \in \xi$.
(v) If $E_1 \in \xi$ then $E_1^{\,t}$ = {(𝕏, y) : (y, 𝕏) ∈ E_1 } ∈ ξ.
(vi) For any $E_1 \in \xi$ there is some $E_2 \in \xi$ such that $E_2^{\,2} \subset E_1$.
In this case the pair (𝕏, ξ) is called an intuitionistic uniform structure space (IUSS for short) and any intuitionistic symmetric member in ξ is called an intuitionistic open symmetric member (IOSM for short) in 𝕏. The complement $\overline{E}$ of an intuitionistic open symmetric member E is called an intuitionistic closed symmetric member (ICSM for short) in 𝕏.

DEFINITION 3.3. Let (𝕏, ξ) be an intuitionistic uniform structure space and $A = \langle x, A^1, A^2 \rangle$ be an intuitionistic symmetric member in 𝕏. Then the intuitionistic uniform closure (IUcl for short) of A are defined by
IUcl(A) = ∩{K : K is an intuitionistic closed symmetric member in 𝕏 and A ⊆ K}.

DEFINITION 3.4. Let (𝕏, ξ) be an intuitionistic uniform structure space and $A = \langle x, A^1, A^2 \rangle$ be an intuitionistic symmetric member in 𝕏.Then the intuitionistic uniform interior (IUint for short) of A are defined by IUint(A) = ∪{G : G is an intuitionistic open symmetric member in 𝕏 and G⊆ A}.

REMARK 3.1. (i) Finite intersection of intuitionistic open symmetric member is an intuitionistic open symmetric member.

(ii) Finite union of intuitionistic closed symmetric member is an intuitionistic closed symmetric member.

DEFINITION 3.5. Let (𝕏, ξ) be an intuitionistic uniform structure space. An intuitionistic symmetric member A = $\langle x, A^1, A^2 \rangle$ in (𝕏, ξ) is said to be
(i) an intuitionistic t-open symmetric member if IUint(IUcl(A)) = IUint(A).
(ii) an intuitionistic B-open symmetric member if A = U∩V, where U is an intuitionistic open symmetric member and V is an intuitionistic t-open symmetric member. The complement of an intuitionistic t-open (intuitionistic B-open) symmetric member S is called an intuitionistic t-closed (intuitionistic B-closed) symmetric member in 𝕏.

DEFINITION 3.6. An intuitionistic symmetric grill $\mathcal{G}$ on an intuitionistic uniform structure space (𝕏, ξ) is defined to be a collection of intuitionistic symmetric members of 𝕏 such that
(i) A ∈ $\mathcal{G}$ and A ⊆B ⊆ 𝕏 implies B ∈ $\mathcal{G}$ and
(ii) A, B ⊆𝕏 and A ∪ B ∈ $\mathcal{G}$ implies A ∈ $\mathcal{G}$ or B ∈ $\mathcal{G}$.

DEFINITION 3.7. Let (𝕏, ξ) be an intuitionistic uniform structure space and A = $\langle x, A^1, A^2 \rangle$ be an intuitionistic symmetric member in 𝕏. Then the intuitionistic uniform B-closure of A is defined and denoted by

IUBcl(A) = ∩{K : K is an intuitionistic B-closed symmetric member and A ⊆ K}.

NOTATION 3.3. (i) IBO(𝕏) denotes the family of all intuitionistic B-open symmetric members of (𝕏, ξ).
(ii) IBO(𝕏, 𝕏) denotes the family of all intuitionistic B-open symmetric members of (𝕏, ξ) which contain a given pair (𝕏, 𝕏) of 𝕏.

DEFINITION 3.8. Let (𝕏, ξ) be an intuitionistic uniform structure space and A = $\langle x, A^1, A^2 \rangle$ be an intuitionistic symmetric member in (𝕏, ξ). Then the intuitionistic uniform B_θ-closure of A is defined and denoted by

IU B_θ cl(A) = {(𝕏, 𝕏) ∈ 𝕏 : IUBcl(U) ∩ A ≠ ∅, for all U ∈ IBO(𝕏, 𝕏)}

NOTATION 3.4. An intuitionistic grill uniform structure space is an intuitionistic uniform structure space (𝕏, ξ) with an intuitionistic symmetric grill $\mathcal{G}$ on 𝕏 and is denoted by (𝕏, ξ, $\mathcal{G}$)

DEFINITION 3.9. Let (𝕏, ξ, $\mathcal{G}$) be an intuitionistic grill uniform structure space. (𝕏, ξ, $\mathcal{G}$) is said to be $IUB_{\theta\mathcal{G}}$ -adhere at (𝕏, 𝕏) ∈ 𝕏 if for each U ∈ IBO(𝕏; 𝕏) and each G ∈ $\mathcal{G}$, IUBcl(U) ∩ G ≠∅.

DEFINITION 3.10. Let (𝕏, ξ, $\mathcal{G}$) be an intuitionistic grill uniform structure space. (𝕏, ξ, $\mathcal{G}$) is said to be $IUB_{\theta\mathcal{G}}$ -converge at (𝕏, 𝕏) ∈ 𝕏 if for each U ∈ IBO(𝕏, 𝕏), there is some G ∈ $\mathcal{G}$, such that G ⊆ IUBcl(U).

DEFINITION 3.11. Let 𝕏 be a nonempty set and F be a nonempty family of intuitionistic symmetric member of 𝕏. Then the collection F is said to be an intuitionistic symmetric filter on 𝕏. If it satisfies the following a𝕏ioms.
(i) $\emptyset_\sim$ ∉ F.
(ii) If F ∈ F and F ∈ H then H ∈ F .
(iii) If F ∈ F and H ∈ F then F ∩ H ∈ F

An intuitionistic symmetric filter F on 𝕏 is called an intuitionistic symmetric ultra filter on 𝕏 if and only if F is not properly contained in any other intuitionistic symmetric filter on 𝕏.

NOTATION 3.5. An intuitionistic filter uniform structure space is an intuitionistic uniform structure space (𝕏, ξ) with an intuitionistic symmetric filter F on 𝕏 and is denoted by (𝕏, ξ, F).

DEFINITION 3.12. Let (𝕏, ξ, F) be an intuitionistic filter uniform structure space. (𝕏, ξ, F) is said to be $IUB_{\theta F}$ -adhere at (𝕏, 𝕏) ∈ 𝕏 if for each F ∈ F and each U ∈ IBO(𝕏, 𝕏), F ∩ IUBcl(U) ≠$\emptyset_\sim$.

DEFINITION 3.13. Let (𝕏, ξ, F) be an intuitionistic filter uniform structure space. (𝕏, ξ, F) is said to be $IUB_{\theta F}$ -converge at (𝕏, 𝕏) ∈ 𝕏 to each U ∈ IBO(𝕏; 𝕏), there corresponds F ∈ F, such that F ⊆ IUBcl(U).

DEFINITION 3.14. Let (𝕏, ξ, $\mathcal{G}$) be an intuitionistic grill uniform structure space. Then the intuitionistic uniform section grill is defined and denoted by IUsec $\mathcal{G}$ = {A ⊆ 𝕏 : A ∩ G ≠ $\emptyset_\sim$ for all G ∈ $\mathcal{G}$}.

DEFINITION 3.15. Let (𝕏, ξ, F) be an intuitionistic filter uniform structure space. Then the intuitionistic uniform section filter is defined and denoted by IUsec F = {A ⊆ 𝕏 : A ∩ F ≠ $\emptyset_{\sim}$ for all F ∈ F }.

PROPOSITION 3.1. (i) Let (𝕏, ξ, $\mathcal{g}$) be an intuitionistic grill uniform structure space. Then IUsec $\mathcal{g}$ is an intuitionistic filter uniform structure space.
(ii) Let (𝕏, ξ, F) be an intuitionistic filter uniform structure space. Then IUsec F is an intuitionistic grill uniform structure space.

PROPOSITION 3.2. Let (𝕏, ξ, $\mathcal{g}$) and (𝕏, ξ, F) be any two intuitionistic grill uniform structure space and intuitionistic filter uniform structure space with F ⊆ $\mathcal{g}$, then there is an intuitionistic symmetric ultra filter U on 𝕏 such that F ⊆ U⊆ $\mathcal{g}$.

PROPOSITION 3.3. If(𝕏, ξ, $\mathcal{g}$) be an intuitionistic grill uniform structure space, IUB_{θ}-adheres at some (𝕏, 𝕏) ∈ 𝕏, then(𝕏, ξ, $\mathcal{g}$) is IUB_{θ}-convergent to (𝕏, 𝕏).

PROPOSITION 3.4. Let(𝕏, ξ, $\mathcal{g}$) be an intuitionistic grill uniform structure space, IUB_{θ}-adheres at some (𝕏, 𝕏) ∈ 𝕏, if and only if $\mathcal{g}$ ⊆ $\mathcal{g}$($IUB_{\theta,}$ (𝕏, 𝕏)).

PROPOSITION 3.5. Let(𝕏, ξ, $\mathcal{g}$) be an intuitionistic grill uniform structure space, IUB_{θ}-convergent at some (𝕏, 𝕏) ∈ 𝕏,, if and only if IUsec $\mathcal{g}$ ($IUB_{\theta,}$ (𝕏, 𝕏)) ⊆ $\mathcal{g}$.

IV AN INTUITIONISTIC UNIFORM B-CLOSEDNESS IN TERMS OF INTUITIONISTIC UNIFORM GRILL

DEFINITION 4.1. Let(𝕏, ξ) be an intuitionistic uniform structure space and A = $\langle x, A^1, A^2 \rangle$ be an intuitionistic symmetric member in (𝕏, ξ). Then A is said to be an intuitionistic uniform B-closed relative to 𝕏 if for every cover U of A by intuitionistic B-open symmetric members of 𝕏, there exists a finite family of intuitionistic symmetric member U_0 of U such that A ⊆ {IUBcl(U) : U ∈ Uo }. If, in addition, A = 𝕏, then 𝕏 is called an IUB-closed space.

DEFINITION 4.2. Let(𝕏, ξ) be an intuitionistic uniform structure space. A nonempty collection B of nonempty subsets of 𝕏 is called an intuitionistic uniform filter base if for B_1 , B_2 ∈ B, there is B_3 ∈ B such that $B_3 \subset B_1 \cap B_2$.

DEFINITION 4.3. An intuitionistic uniform filter base F on an intuitionistic uniform structure space(𝕏, ξ) is said to be IUB_{θ}-adhere or IUB_{θ}-accumulate at (x, x) ∈ 𝕏 if for each F ∈ F and each U ∈ IBO(x, x), F ∩ IUBcl(U) ≠ $\emptyset_{\sim}$. The set of all IUB_{θ}-adherent points of A is called the intuitionistic uniform B-closure(A) and denoted by IUBcl(A).

DEFINITION 4.4. An intuitionistic uniform filter base F on an intuitionistic uniform structure space(𝕏, ξ) is said to be IUB_{θ}-converge at (x, x) ∈ 𝕏 to each U ∈ IBO(x, x), there exists F ∈ F, such that F ⊆IUBcl(U).

PROPOSITION 4.1. Let(□, ξ) be an intuitionistic uniform structure space. Then the following statements are equivalent:
(i)(□, ξ) is an IUB-closed;
(ii) Every maximal intuitionistic uniform filter base on □, IUB_{θ}-converges to (x, x) ∈ □;
(iii) Every intuitionistic uniform filter base on □, IUB_{θ} -adhere at (x, x) ∈ □;
(iv) For every family {$U_{\propto}$: ∝ ∈ I} of IB-closed symmetric member such that ∩{$U_{\propto}$: ∝ ∈ I} = $\emptyset_{\sim}$, there is a finite family of intuitionistic symmetric members I_0 of I such that $\bigcap_{i=1}^{n} IUB\ \mathrm{int}(U_{\alpha_i}) = \emptyset_{\sim}$.

PROPOSITION 4.2. An intuitionistic uniform structure space (□, ξ) is IUB-closed if and only if every intuitionistic grill uniform structure space(□, ξ, $\mathcal{g}$) is IUB_{θ}-convergent in(□, ξ).

PROPOSITION 4.3. Let □ be any intuitionistic uniform structure space such that every intuitionistic uniform grill $\mathcal{g}$ on □ with the property that $\bigcap_{i=1}^{n} IUB_{\theta} cl(G_i) \neq \emptyset_{\sim}$ for every finite subfamily {G_1,G_2,……,G_n} of $\mathcal{g}$, IUB_{θ}-adheres in □, then □ is an IUB-closed space.

DEFINITION 4.5. Let(□, ξ, $\mathcal{G}$) be an intuitionistic grill uniform structure space.(□, ξ, $\mathcal{G}$) is said to be IUB_θ-linked if for any two intuitionistic symmetric members A, B ∈ $\mathcal{G}$, $IUB_\theta cl(A) \cap IUB_\theta cl(B) \neq \emptyset_\sim$.

DEFINITION 4.6. Let(□, ξ, $\mathcal{G}$) be an intuitionistic grill uniform structure space.(□, ξ, $\mathcal{G}$) is said to be IUB_θ-conjoint if for every finite subfamily $A_1, A_2, \ldots, A_n$ of $\mathcal{G}$, $IUBint[\bigcap_{i=1}^{n} IUB_\theta cl(A_i)] \neq \emptyset_\sim$.

PROPOSITION 4.4. In an intuitionistic uniform B-closed space □, every IUB_θ-conjoint grill IUB_θ-adheres in □.

DEFINITION 4.7. An intuitionistic symmetric member A of an intuitionistic uniform structure space □ is called IUB-regular open if A = IUBint(IUBcl(A)). The complement of IUB-regular open symmetric member is IUB-regular closed.

DEFINITION 4.8. An intuitionistic uniform structure space □ is called IUB-almost regular if for each (x, x) ∈ □ and each IUB-regular open symmetric member V in □ with (x, x) ∈ V , there is an IUB-regular open symmetric member U in □ such that (x, x) ∈ U ⊆ IUBCl(U) ⊆ V .

PROPOSITION 4.5. In an IUB-almost regular IUB-closed space □, every intuitionistic uniform grill $\mathcal{G}$ on □ with the property $\bigcap_{i=1}^{n} IUB_\theta cl(G_i) \neq \emptyset_\sim$ for every finite subfamily $\{G_1, G_2, \ldots, G_n\}$ of $\mathcal{G}$, IUB_θ-adheres in □.

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