Some New Closed Functions in Topological Spaces

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Abstract

A set A in a topological space (X ,τ) is said to be a g*p-closed(briefly, g*p-closed) if pcl(A)⊂U whenever A⊂U and U is g-open in X. In this paper, we introduce quasi g*p-closed maps from a space X into a space Y as the image of every g*p-closed set is closed. Also, we obtain its characterizations and its basic properties.

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1. Introduction

The notion of pre-open set [5] plays a significant role in general topology. The most important generalizations of regularity (resp. normality) are the notions of pre-regularity [1] and strong regularity (resp. pre-normality [6], strong normality [6]).Levine [3] in 1963, started the study of generalized open sets with the introduction of semi-open sets. Then Njatsad [7] studied a-open sets; In 1982,Mashhour et.' al [5] introduced preopen sets and pre-continuity in topology.

In 2002, Veerakumar [12] have defined the notion of g*p-closed sets, g*p-continuity and g*p-irresolute maps. In this paper, we will continue the study of related functions by involving g*p-open sets. We introduce and characterize the concept of quasi g*p-open functions.

2. Preliminaries

Throughout this paper, by spaces we mean topological spaces on which no separation axioms are assumed unless otherwise mentioned and f: (X, τ)→(Y, a) (or simply f: X→Y) denotes a function f of a space (X , τ) into a space (Y, a). For a subset A of X , cl(A) and int(A) denote the closure and interior of A, respectively.

Definition 2.1.A subset A of X is said to be **pre-open** [5] if A ⊂int(cl(A)). The complement of pre-open set is said to be **pre-closed** [5]. The family of all pre-open sets (respectively pre-closed sets) of (X , τ) is denoted by pO(X, τ) [respectively pCL (X, τ)].

Definition 2.2. Let A be a subset of X. Then (i) **pre-interior** [2,5] of A is the union of all pre-open sets contained in A. (ii) **pre-closure** [2,5] of A is the intersection of all pre-closed sets containing A.

The pre-interior [respectively pre-closure] of A is denoted by pint(A) [respectively pcl(A)].

Definition 2.3. A subset A of a space X is called a **g-closed set** [4] (briefly, g-closed) if cl (A) ⊂U whenever A ⊂U and U is open in X. The complement of g-open set is said to be **g-open set** [4].

Definition 2.4. A subset A of a space X is called a **g*p-closed set** [12](briefly, g*p-closed) if pcl (A)⊂U whenever A ⊂ U and U is g-open in X. The complement of g*p-open set is said to be g*p-closed **set** [8]. The family of all g*p-open sets (respectively g*p-closed sets) of (X, τ) is denoted by G*pO (X, τ) [respectively G*pCL(X, τ)].

Definition2.5. Let A be a subset of X . Then

(i) **g*p-fnterior**[12] of A is the union of all g*p-open sets contained in A.
(ii) **g*p-closure** [12] of A is the intersection of all g*p-closed sets containing A.

The g*p-interior [respectively g*p-closure] of A is denoted by g*pint(A) [respectively g*pcl(A)].

Definition 2.6. A subset S is called a g*p-neighbourhood [12] of a point *x* of X, if there exists a g*p-open set U such that $x \in U \subset S$.

Definition 2.7. A function f: (X, τ) →(Y, σ) is called a g*p-open function [12] if the image f (A) is g*p-open in (Y, σ) for each open set A in (X, τ).

Definition 2.8. A function f: (X, τ)→(Y, σ) is called a **g*p-continuous** [12] if the inverse image of every closed set in (Y, σ) is g*p-closed in (X, τ).

Definition 2.9. A function f: (X, τ)→((Y, a) is called a **g*p-irresolute** [12] if the inverse image of every g*p-closed in (Y, a) is g*p-closed in (X, τ).

3. Quasi g*p-closed Functions

Definition3.1.A function f : X →Y is said to **quasi g*p-closed** if the image of each g*p-closed set in X is closed in Y .Clearly, every quasi g*p- closed function is closed as well as g*p-closed.

Lemma 3.2. If a function f: X →Y is quasi g*p-closed, then f^{-1} (int (B)) ⊂g*p-int (f^{-1}(B)) for every subset B of Y .

Proof. This proof is similar to the proof of Lemma on quasi g*p-open maps [13]

Theorem 3.3.A function f: X →Y is quasi g*p-closed iff for any subset B of Y and for any g*p-open set G of X containing f^{-1} (B), there exists an open set U of Y containing B such that f^{-1} (U) ⊂ G.

Proof This proof is similar to that Theorem 3.5. [13]

Definition 3.4.A function f: X →Y is called **g*p*-closed** if the image of g*p-closed subset of X is g*p-closed set in Y .

Theorem 3.5. If f : X→Y and g : Y→Z are two quasi g*p-closed function, then g°f is quasi g*p-closed function.

Proof. Obvious.

Furthermore, we have the following Theorem:

Theorem 3.6. Let f: X →Y and g : Y →Z be any two functions. Then

(i) If f is g*p-closed and g is quasi g*p-closed, then g ° f is closed.

(ii) If f is quasi g*p-closed and g is g*p-closed, then g ° f is g*p*-closed.

(iii) If f is g*p*-closed and g is quasi g*p-closed, then g ° f is quasi g*p- closed.

Proof. Obvious.

Theorem 3.7. Let f: X →Y and g : Y →Z be two functions such that

g°f: X →Z is quasi g*p-closed.

(i) If f is g*p-irresolute surjective, then g is closed.

(ii) If g is g*p-continuous injective, then f is g*p*-closed.

Proof (i) Suppose that F is an arbitrary closed set in Y. As f is g*p- irresolute.f^{-1} (F) is g*p-closed in X . Since g°f is quasi g*p-closed and f is surjective, g°f (f^{-1}(F)) = g (F) which is closed in Z. This implies g is a closed function.

(ii) Suppose F is any g*p-closed set in X . Since g°f is quasi g*p-closed,(g°f)

(F) is closed in Z. Again g is a g*p-continuous injective function,g^{-1} (g°f (F)) = f (F), which is g*p-closed in Y . This shows that f is g*p*-closed.

Theorem 3.8. Let X and Y be topological spaces. Then the function g : X →Y is a quasi g*p-closed if and only if g (X) is closed in Y and g (V) \g (X \V) is open in g (X) whenever V is g*p-open in X.

Proof. Necessity: Suppose g: X→Y is a quasi g*p-closed function. Since X is a g*p-closed, g (X) is closed in Y and g (V) \g (X \V) = g (V)Hg (X)\g (X \V) is open in g (X) when V is g*p-open in X .

Sufficiency: Suppose g (X) is closed in Y , g (V) \g (X \V) is open in g (X) when V is g*p-open in X, and let C be closed in X . Then g (C) = g (X) \g (X \C) \g (C) is closed in g (X) and hence, closed in Y .

Corollary 3.9. Let X and Y be topological spaces. Then a surjective function g : X→Y is quasi g*p-closed iff g (V) \g (X \V) is open in Y whenever U is g*p-open in X.

Proof Obvious.

Corollary 3.10. Let X and Y be topological spaces and let g : X →Y be a g*p-continuous quasi g*p-closed surjective function. Then the topology onY is **{g(V**)\g(X \V): V is g*p-open in X}.

Proof Let W be open in Y . Then g^{-1} (W) is g*p-open in X , and g (g^{-1} (W))\g (X \g^{-1} (W)) = W. Hence, all open sets in Y are of the form g (V) \g (X \V), V is g*p-open in X . On the other hand, all sets of the form g (V) \g (X \V), V is g*p-open in X , are open in Y from Corollary 3.9.

Definition 3.11. A topological space (X, T) is said to be **g*p-normal,** if for any pair of disjoint g*p-closed subsets F1 and F2 of X, there exist disjoint open sets U and V such that F1⊂Uand F2 ⊂V.

Theorem 3.12. Let X and Y be topological spaces with X is g*p-normal. If g : X→*Y* is g*p-continuous quasi g*p-closed surjective function. Then Y is normal.
Proof. Let K and M be disjoint closed subsets of Y. Then g^{-1}(K), g^{-1}(M) are disjoint g*p-closed subsets of X. Since X is g*p-normal, there exists disjoint open sets V and W such that g^{-1}(K) ⊂ V and g^{-1}(M) ⊂W. Then K ⊂g(V)\g(X\V) and M⊂g(W)\g(X\W). Further by Corollary 3.9., g(V)\g(X \V) and g(W)\g(X\W) are open sets in Y and clearly (g(V)\g(X\V))∩(W)\g(X \W)) = ∅. This shows that Y is normal.

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