Irresolute Supra Pre-Open Continues Map on Topological Spaces

[1]Taha.H.Jasem
College of computer science & mathematics Dept of mathematics

[2]Dr.V.Amarendra Babu[4]
Universty Of Acharya Nagarjuna Nagarjuna Nagar-522510, India

[3] Abdulsattar Abdullah Hamad
University of Acharya nagarjuna department of mathematics nagarguna nagar Guntur 522510

Abstract
in this paper we will present new class from the (irresolute supra pre-open continues maps on topological spaces) and Respectively are introduced and give Two new definitions example

Keywords: *, irresolute supra pre-open continues ,Supra pre-open set, supra pre-continuity, "supra pre-open map(sp-pre-on), "supra pre-closed(sp-pre-cd) map and supra topological"(sp-tp-space)*

Introduction
In [10] O. R. SAYED" he is introduce" a new type of continuous maps called a "supra pre-continuous" maps and obtain some of their "properties and "characterizations. Also he "introduce the concepts of supra pre-"continuous" maps, sp-pre-on maps and "sp-pre-closed maps and "investigate "several properties for this class of maps. In particular, we study the relation between supra" pre-continuous" maps and sp-pre-on" maps (sp-pre-cd maps). Throughout his paper[10] simply $(\tilde{X},\tilde{T})(\tilde{Y},\tilde{\sigma}), and\ (\tilde{Z},\tilde{\upsilon}) or$ simply, X , Y,and Z) denote "topological spaces on which no "separation" axioms are Assumed" unless explicitly stated. All sets are assumed to be "subset of "topological spaces. The "closure and the "interior" of a set A. are denoted by $Cl_p^{\mu}(\tilde{A})\ and Int_p^{\mu}(\tilde{A})$, respectively". A "subcollection $\tilde{\mu} \subset 2^x$ is called a "supra "topology [5] on X if $X \in \mu$ and $\tilde{\mu}$ is closed under arbitraryunion. $(\tilde{X},\tilde{\mu})$ is called a sp-topological" space ."General "topology is important in many fields of" applied sciences as well as branches of mathematics. In "reality it is used in data mining,computational topology for geometric design and molecular design,computer-aided design, "computer-aided geometric design and engi-neering design (briey CAGD), digital topology, information systems,non-commutative geometry and its application to particle physics and"quantum" physics etc. We "recommend that the" reader should" refer tothe "following papers, "respectively: [2-4, 6, 7, 8]. One can observe the inuence made in these realms of "applied research by general "topological spaces, "properties and structures. Rosen and Peters [8] have used "topology as a body of" mathematics that could unify diverse areas of CAGD and "engineering design "research. They have presented several examples of the" application of "topology to "CAGD and design.

2. "Supra pre-open sets"

Denition 2.1. A set A is sp-pre-on if $A\ I\tilde{n}t^{\mu}(\tilde{C}l^{\mu}(\tilde{A}))$. The complement of sp-pre-on is called sp-pre-cd. Thus A is sp-pre-cd if and only if $\tilde{C}l^{\mu}(\tilde{A})) \subseteq A$

Theorem 2.2. (1) "Every sp-σ open is sp-pre-open"
(2) "Every sp-pre-on is sp- b-open.
Proof. Obvious.
The following examples show that sp-pre-ON)) is placed strictly
Between" supra _α open and supra b-open.

Example 2.3. Let (X, $\tilde{\mu}$) be a sp-tp –space". where X $=\{\tilde{a},\tilde{b},\tilde{c}\}$ and $\tilde{\mu}=\{X,\Phi,\{\tilde{a}\},\{\tilde{a},\tilde{b}\},\{\tilde{b},\tilde{c}\}\}$

Here $\left\{\tilde{a},\tilde{b}\right\}$ is sp-pre-on", but it is not "supra -α open.

Example 2.4. Let (X, $\tilde{\mu}$) be a sp-top space, where X ={a,b,c,d}

And $\tilde{\mu}=\{X,\phi,\{\tilde{a}\},\{\tilde{b}\},\{\tilde{a},\tilde{b}\}\}$ Here $\left\{\tilde{b},\tilde{c}\right\}$ is a supra b-open, but it is not sp- pre-on.

The following diagram show how sp-pre-on sets are related to some similar types of "supra-open sets.

$supra\,\alpha - open \to supra\,\alpha - open \to supra\,\alpha\ pre - open \to supra\,\alpha b - open$

Theorem 2.5. (i) Arbitrary union of sp-pre-on sets is always sp- pre-on.
(ii) Finite intersection of sp-pre-on sets may fail to be sp-pre-on.
(iii) X is a sp-pre-on set

Proof. (i) Let $\{\tilde{A}_\lambda : \lambda \in \tilde{A}\}$ be a family of sp-pre-on sets in a "topological space X. Then for any $\lambda \in A$ we have $\tilde{A}_\lambda \subseteq Int^\mu(Cl^\mu(\tilde{A}_\lambda)).Hence$

$$\bigcup_{\lambda\in A}\tilde{A}_\lambda \subseteq \bigcup_{\lambda\in A}(I\tilde{n}t^\mu(\tilde{C}l^\mu(\tilde{A}_\lambda))) \subseteq I\tilde{n}t^\mu(\tilde{A}_\lambda))$$

$$\subseteq I\tilde{n}t^\mu(\bigcup_{\lambda\in A}(\tilde{C}l^\mu(\tilde{A}_\lambda)) \subseteq I\tilde{n}t(\tilde{C}l(\bigcup_{\lambda\in A}\tilde{A}_\lambda)$$

Therefore $\bigcup_{\lambda\in A}\tilde{A}_\lambda$ is a sp-pre-on set.

(ii) In Example 2.1 both $\left\{\tilde{a},\tilde{b}\right\}$ and $\left\{\tilde{b},\tilde{c}\right\}$ are sp-pre-on, but their intersection {e} is not sp- pre-on.

Theorem 2.6. (i) Arbitrary "intersection of supra pre-closed sets is always sp-pre-cd.
(ii) Finite union of sp-pre-cd sets may fail to be sp- pre-cd.
Proof. (i) This follows immediately from Theorem 2.2.

(ii) In Example 2.1 both $\{\tilde{a}\}$ and $\left\{\tilde{b}\right\}$ are sp-pre-closed", but their union {ab} is not sp-pre-cd.

Denition 2.7. The sp-pre-cd of a set A, denoted by $Cl_p^\mu(\tilde{A})$ is the "intersection" of the sp-pre-cd sets including A. The "supra pre-interior of a set A;denoted by $Int_p^\mu(\tilde{A})$, is the union of the sp-pre-on sets "included in A.

Remark 2.8. It is clear that $Int_p^\mu(\tilde{A})$, is a sp-pre-on set and $Cl_p^\mu(\tilde{A})$ is sp- pre-cd.

Theorem 2.9. $(i)\,\tilde{A} \subseteq Cl_p^\mu(\tilde{A}) : and\,\tilde{A} = Cl_p^\mu(\tilde{A})$ iff A is a sp-pre-cd set.

$(ii)\,Int_p^\mu(\tilde{A}) \subseteq \tilde{A};and\,Int_p^\mu(\tilde{A}) = \tilde{A}$ iff A is a sp-pre-on set.

$(iii)\,\tilde{X} - Int_p^\mu(\tilde{A}) = Cl_p^\mu(\tilde{X} - \tilde{A}):$

$(iv)\,\tilde{X} - Cl_p^\mu(\tilde{A}) = Int_p^\mu(\tilde{X} - \tilde{A}).$

$(v)\ If\tilde{A} \subseteq \tilde{B},\ then\ "\tilde{C}l_p^\mu(\tilde{A}) \subseteq \tilde{C}l_p^\mu(\tilde{B})and\ :$

$Int_p^\mu(\tilde{A}) \subseteq Int_p^\mu(\tilde{B})$

Proof. Obvious.

Theorem 2.10.

$(\mathbf{a})\, Int_p^{\mu}(\tilde{A}) \cup Int_p^{\mu}(\tilde{B}) \subseteq Int_p^{\mu}(\tilde{A} \cup \tilde{B});$

$(b)\, Cl_p^{\mu}(\tilde{A} \cap \tilde{B}) \subseteq Cl_p^{\mu}(\tilde{A}) \cap Cl_p^{\mu}(\tilde{B}).$

Proof. obvious. The inclusions in (a) and (b) in Theorem 2.5 can not replaced by equalities by Example 2.1. Where, if A = {b} and B = {c};then

$Int_p^{\mu}(\tilde{A}) = Int_p^{\mu}(\tilde{B}) = \Phi; and; Int_p^{\mu}$

$(\tilde{A} \cup \tilde{B}) = \{\tilde{a}, \tilde{b}\}: Also, if \tilde{C} = \{\tilde{a}\ \tilde{b}\}$

$and, D = \{a, c\}, then, Cl_p^{\mu}(\tilde{C}) = Cl_p^{\mu}(\tilde{D}) = X, and,$

$Cl_p^{\mu}(\tilde{C} \cap \tilde{D}) = \{\tilde{a}\}$

Proposition 2.11. (1) The "intersection of sp- on and sp-pre-on is supra pre-open.
(2) The "intersection of sp-on and sp-pre-on is sp-pre-on.

3. Supra pre-continuous maps

Denition 3.1. Let $(X, \tilde{\tau})and\ (Y, \tilde{\sigma})$) be two topological spaces and be an associated supra topology with τ . A map $f : (X, \tilde{\tau}) \to (Y, \tilde{\sigma})$ is called" supra pre-continuous map if the inverse image of each open set in Y is a sp-pre-on set in X.

Theorem 3.2. Every "continuous map is "supra pre-continuous.

Proof. Let $f : \tilde{X} \to \tilde{Y}$ be a "continuous map and A is open set in Y . Then $f^{-1}(\tilde{A})$ is an "open set in X. Since μ associated with τ. then $\tau \in \mu$.

Therefore $f^{-1}(\tilde{A})$ is a "supra open set in X which is a sp-pre-on set in X . Hence f is sp-pre-"continuous map.The converse of the above theorem is not true as shown in the following example.

Example 3.3. $Let, \tilde{X} = \{\tilde{a}, \tilde{b}, \tilde{c}\}; and; \tilde{\tau} = \{X, \Phi, \{\tilde{a}, \tilde{b}\}\}; be$ a topology on X. The"supra "topology μ is defined as follows:

$\mu = \{X, \Phi, \{\tilde{a}\}, \{\tilde{a}, \tilde{b}\}\}; and;$

$let, f : (X, \tilde{\tau}) \to (X, \tilde{\tau})$

$be\ a\ map\ defined\ as$

$follows: f(\tilde{a}) = b;\ f(\tilde{b}) = c;\ f(\tilde{c}) = a.$

$Since$ The inverse image of the open set fa; bg is fa; cg which is not an open set but it is a sp-pre-on set. Then f is supra pre-continuous" map but not "continuous map The following example shows that supra pre-continuous map need not be supra α – continuous" map.

Example 3.4. Consider the set

$X = \{\tilde{a}, \tilde{b}, \tilde{c}, \tilde{d}\}\ with\ the\ topology$

$\tilde{\tau} = \{X, \Phi, \{\tilde{a}, \tilde{c}\}\{\tilde{b}, \tilde{d}\}\} and\ the\ supra$

$topology$

$\tilde{\mu} = \{X, \Phi\{\tilde{a}, \tilde{c}\}, \{\tilde{b}, d\}, \{\tilde{a}, \tilde{c}, \tilde{d}\};$

$Also,\ let \tilde{Y} = \{\tilde{x}, \tilde{y}, \tilde{z}\}$

with the $topology\ \sigma = \{Y, \Phi, \{\tilde{z}\}, \{\tilde{y}, \tilde{z}\}.$

$"Define\ the\ map$

$f : (X, \tilde{\tau}) \to (Y, \tilde{\sigma})$

$by f(a) = y;\ f(b) = f(c) = z;\ f(d) = x.$

Clearly, f is a supra pre continuous map but it is not" supra -α continuous.
The following example shows that a supra b-continuous map need

not be supra pre-continuous".

Example 3.5. Consider the set

$\tilde{X} = \{\tilde{a}, \tilde{b}, \tilde{c}\}$; with the "topology ; $\tilde{\tau} = \{X, \Phi, \{\tilde{a}, \tilde{b}\}\}$

and the supra topology ; $\mu = \{X, \Phi, \{\tilde{b}\}, \{\tilde{c}\}, \{\tilde{b}, \tilde{c}\}\}$ "Define

.Also,

let $\tilde{Y} = \{\tilde{x}, \tilde{y}, \tilde{z}\}$; with the topology ; $\sigma = \{Y, \Phi, \{\tilde{x}, \tilde{z}\}\}$

the map

$f : (X, \tilde{\tau}) \to (Y, \tilde{\sigma})$ by $f(\tilde{a}) = x$; $f(\tilde{b}) = y$; Then the inverse image of the open set $\{x, z\}$ is $\{a, c\}$

$f(\tilde{c}) = z$.

which is not sppre-on but it is a supra b-open set. Therefore f is a supra b-"continuous map but it is not sp-pre-"continuous".

4 - main result

Denition 4.1. . Let (X, η) and $(\tilde{X}, \zeta)$) be two topological spaces and

be an associated supra topology with η . A map $f : (X, \eta) \to (\tilde{X}, \zeta)$ is

called irresolute supra--open "continuos map if the inverse image of each supra-open set in $\tilde{X}$ is a sp-pre-op) set in X.

Denition 4.2. . Let (X, η) and $(\tilde{X}, \zeta)$) be two" topological spaces and

be an "associated sp-topology with η . A map $f : (X, \eta) \to (\tilde{X}, \zeta)$ is

called Contra-continuos map if the inverse image of each supra closed set in $\tilde{X}$ is a supra pre-op set in X

Theorem 4.3. Every irresolute supra continuous map is (supra-open)continuous.

Proof. Let $f : X \to \tilde{X}$ be a irresolute supra continuous map and A is su-open set in $\tilde{X}$ .Then $f^{-1}(\tilde{A})$ is an su-open set in X. Since α – associated with then Therefore $f^{-1}(\tilde{A})$ is a su-open set in X. which is a supra-open set in X . Hence f is irresolute supra-open continuos map.The converse of the above theorem is not true as shown in the following example

Example :4.4

$Let, X = \{\tilde{a}, \tilde{b}, \tilde{c}, \tilde{d}\}$; and ; $\tau = \{X, \Phi, \{\tilde{a}, \tilde{b}\}\}$; be a topology on X. The "supra topology μ is defined as follows:

$\mu = \{X, \Phi, \{\tilde{a}\}, \{\tilde{a}, \tilde{b}\}\}$; and ;

let, $f : (X, \eta) \to (\tilde{X}, \zeta)$

be a map defined " as

follows : $f(\tilde{a}) = b$; $f(\tilde{b}) = c$; $f(\tilde{c}) = d$.

$f(\tilde{d}) = a$

Since The inverse image of

The "supra open set {a,b} is {a,c}which is not an "supra open set but it is a irresolute supra-open set. Then f is supra -"continuous map but not continuous" map

The following example shows that irresolute supra-open "continuos map need not be supra continuous" map

Example: 4.5 Consider the set

$X = \{a, b, c, d\}$ with the topology

$\tilde{X} = \{X, \Phi, \{a, c\}\{b, d\}\}$ and the supra

topology

$\mu = \{X, \Phi\{a, c\}, \{b, d\}, \{a, c, d\};$

Also, let $Y = \{x, y, z\}$

with the topology $\zeta = \{Y, \Phi, \{z\}, \{y, z\}$.

Define the map

$f : (X, \eta) \to (\tilde{X}, \zeta)$

by $f(a) = y$; $f(b) = f(c) = z$; $f(d) = x$.

Clearly, f is a irresolute supra-open continuos map is supra-open set

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Example 4.6. Consider the set

$X = \{a, b, c\}$; with the topology; $\eta = \{X, \Phi, \{a, b\}\}$

and the supra topology; $\mu = \{X, \Phi, \{b\}, \{c\}, \{b, c\}\}$

.Also,

let $\tilde{X} = \{x, y, z\}$; with the topology; $\zeta = \{Y, \Phi, \{x, z\}\}$

Define the map

$f : (X, \eta) \to (\tilde{X}, \zeta)$; by $f(\tilde{a}) = x$; $f(\tilde{b}) = y$; $f(\tilde{c}) = z$. Then the inverse image of the open set

$\{\tilde{x}, \tilde{z}\}$ is $\{\tilde{a}, \tilde{c}\}$ which is not supra pre-open but it is a supra b-open set. Therefore f is a supra b-continuous map but it is not supra pre-continuous.map but it is not irresolute su-pre-open continuos

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