Super Magic of Some Connected Graphs $P_n * 2nP_2$, $P_n * 2nP_3$, $P_n * 2nP_4$, and $P_n * 2nP_5$

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Abstract: Neelam Kumari, and Seema Mehra [2013] gave the contributions (1) P_t admits n-edge magic labeling for all positive integer t; (2) C_t admits n-edge magic labeling when t is even; (3) A sun graph S_t is n-edge magic only when t is even; (4) If G admits n-edge magic labeling, then G + K_1 admits n-edge magic labeling; and (5) Let $S_{m,t}$ be a double star graph, then the graph $S_{m,t}$ admits n-edge magic labeling. Antony Xavier [2014] verified (1) Let G be a graph. If G has a vertex magic total labeling then it has a modular super vertex magic labelling. But converse need not be true;(2) Km,m has no modular super vertex magic total labeling; (3) IfG is a star graph with odd number of vertices then there exist a modular super vertex magic total labeling with k = 1. Ponnappan, Nagaraj, and Prabakaran [2014] found (1)Let G be a nontrivial graph G with $m \geq n-1$ is odd vertex magic labeling then the magic constant k is given by k =3n-2 when m = n-1 otherwise k = 1 + 2m + otherwise $k = 1 + 2m + (m^2+m)/n$; (2) A cycle Cn is odd vertex magic labeling iff n is odd;(3) A path Pn is odd super vertex magic labeling if and only if n is odd and $n \geq 3$; (4) All n-suns are not odd vertex magic labelling. **The aim of the paper** is to find super magic labelings for the graphs$2nP_2$, $P_n * 2nP_3$, $P_n * 2nP_4$, and $P_n * 2nP_5$.
Keywords: Magic graph, super magic graph

Section 1: Introduction and definitions:

Neelam Kumari, Seema Mehra**[**2014] proved (1)**the g**raph C_n admits V-super vertex magic labeling and E-super vertex magic labeling only if n is odd positive integer; (2) The path P_n admits E-super vertex magic labeling for all $n \geq 3$, but not admits V- super vertex magic labeling corresponding to this E-super vertex magic labeling of Pn; (3) mC_n admits V-super vertex magic labelingand E-super vertex magic labeling if and only if m and n are odd positive integers.

Jayapal Baskar Babujee, Babitha Suresh [2011] established the results (1)If G has super edge edge-magic total labeling, then GôP$_n$, admits edge bi-magic total labeling; (2) If G has super edge edge-magic total labeling then GôF$_{1,n}$ admits edge bi-magic total labeling; (3) GôK$_{1,n}$ is total edge bi-magic for any arbitrary super edge edge-magic Graph G; (4) If G has super edge edge-magic total labeling then, GôF$_{1,n}$ admits edge bimagic total labeling; (5) If G has super edge edge-magic total labeling then, G + K_1 admits edge bimagic total labeling.

Petr Kovar [2007] analyzed the findings (1)LetGbe a2r-regular graph with vertex set $\{x_1, x_2, \ldots, x_n\}$. Let s be an integer,$s \in \{(rn+1)(r+1) + tn : t = 0,1,\ldots, r\}$. Then there exists an(s,1)-VAT labelling λ of G such that $\lambda(x_i) = s + (i-1)$; (2) LetGbe a(2 +s)-regular graph such that it contains an s-regular factor G′which allows a VMT labeling with magic constant h and vertex labels being consecutive integers starting at k. Then G has VMT labeling with magic constants h =14(s+ 4)(n(s+ 4) + 2)−12(n−1)−t,where t∈k, and 12n(s+ 2) + 1; **(3):**LetGbe anr-regular graph on n vertices. If G has a VMT labelling such that the vertex labels constitute an arithmetic progression with odd difference, theneither r is even and n is odd or r is odd and n≡0 (mod 4).

Section 2: Some magic graph related paths

Definition 2.1: P_n * $2nP_2$is a connected graph whose vertex set is { $V_1, V_2, \ldots, V_n, U_1, U_2, \ldots, U_{2n}$} and edge set is {$V_iV_{i+1}$: i = 1 to n} ∪ {$V_iU_{2i-1}$, V_iU_{2i}: i = 1 to n} . The edge set of P_n is {V_iV_{i+1}: i = 1 to n}, and edge set of $2nP_2$ is {V_iU_{2i-1}, V_iU_{2i}: i = 1 to n}.

Theorem 2.2: The graph P_n * $2nP_2$ is super magic.

Proof: Due to the definitiom (2.1), the graph P_n * $2nP_2$ is drawn as follows in figure 1:

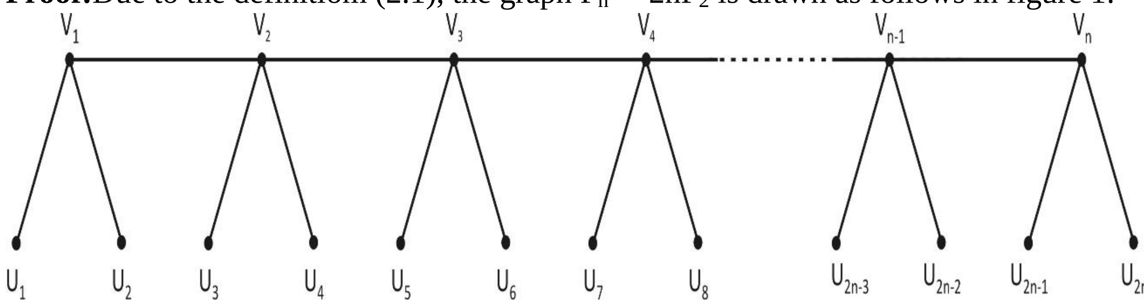

Figure 1: One of the arbitrary labelings of vertices for P_n * nP_2

Here p = 3n; q = 3n-1;

(Vertex rule): Define f : V(G) → {0,1,2,…, p} by

$f(V_2) = \left(\frac{P}{2}+3\right)$ *or* $\left(\frac{P-1}{2}+3\right)$ if p is even (or) odd respectively;

$f(U_1) = f(V_2) - 2$; $f(U_2) = f(V_2) - 1$; $f(U_3) = 2$; $f(U_4) = 3$

$f(V_i) = (3i-1)/2$; i is odd ; $f(V_i) = f(V_2) + \frac{3(i-2)}{2}$; i is even where i varies 1 to n.

$f(U_i) = f(U_1) + \frac{3(i-1)}{4}$; $i \equiv 1 \pmod 4$ (i = 5, 9,…, n);

$= f(U_2) + \frac{3(i-2)}{4}$; $i \equiv 2 \pmod 4$ (i = 6, 10 ,…, n);

$= 2 + \frac{3(i-3)}{4}$; $i \equiv 3 \pmod 4$ (i = 7, 11 ,…, n);

$= 3 + \frac{3(i-3)}{4}$; $i \equiv 0 \pmod 4$ (i = 8, 12 ,…, n);

(Edge rule): Define f: E(G) {1, 2,…., q} by

$f(V_iV_i+1) = p+q+1-3i$: I =1, 2 ,…, (n-1);

$f(V_1U_1) = p+q$;

$f(V_1U_2) = p+q-1$;

$f(V_iU_{2i-1}) = (p+q)-3(i-1)$; i =1,2,…n;

$f(V_iU_{2i}) = (p+q-1) - 3(i-1)$;i =1,2….n;

The map f satisfies the requirements for super magic labeling to both vertex set and edge set of the given graph (P_n * $2nP_2$) with the magic number (7n+4). Therefore the graph (P_n * $2nP_2$) is super magic.

Example 2.3: The graphs P_6*$12P_2$ and P_7*$14P_2$ are super magic graphs as given in the following figures(2)and (3).

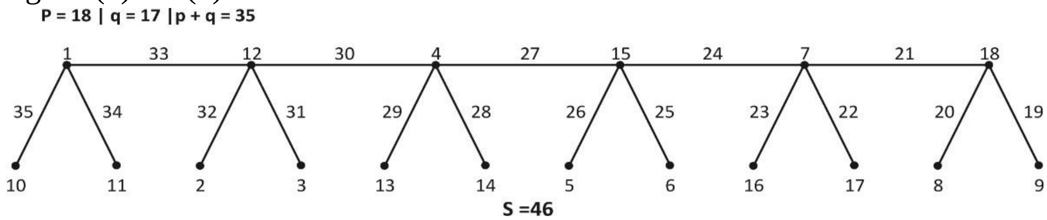

Figure 2 – super magic labeling of the graph P_6*$12P_2$

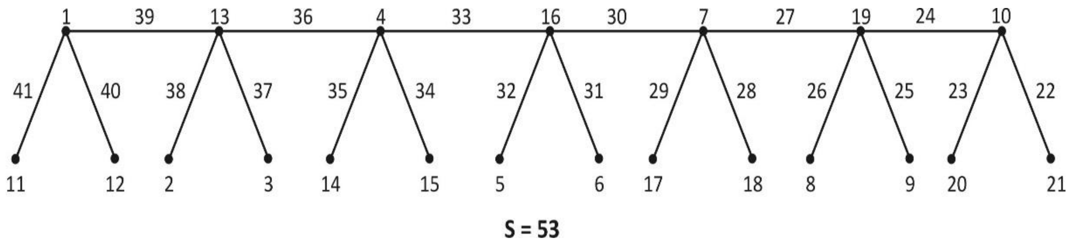

Figure 3 – super magic labeling of the graph P_6*$14P_2$

Definition 2.4: P_n * $2nP_3$is a connected graph whose vertex set is { $V_1,V_2,\ldots, V_n, U_1, U_2,\ldots,U_{2n}, t_1,t_2\ldots,t_{2n}$ } and edge set is $\{V_iV_{i+1}: i = 1 \text{ to } n\} \cup \{V_iU_{2i-1}, V_iU_{2i}: i = 1 \text{ to } n\} \cup \{U_it_{2i-1}, U_it_{2i}: i = 1 \text{ to } n\}$. The edge set of P_n is $\{V_iV_{i+1}: i = 1 \text{ to } n\}$, and edge set of $2nP_3$ is $\{V_iU_{2i-1}, V_iU_{2i}: i = 1 \text{ to } n\} \cup \{U_it_{2i-1}, U_it_{2i}: i = 1 \text{ to } n\}$.

Theorem 2.5: The graph P_n * $2nP_3$ is super magic.

Proof: According to the definition (2.4), the graph P_n * $2nP_3$ is given as follows in figure 4:

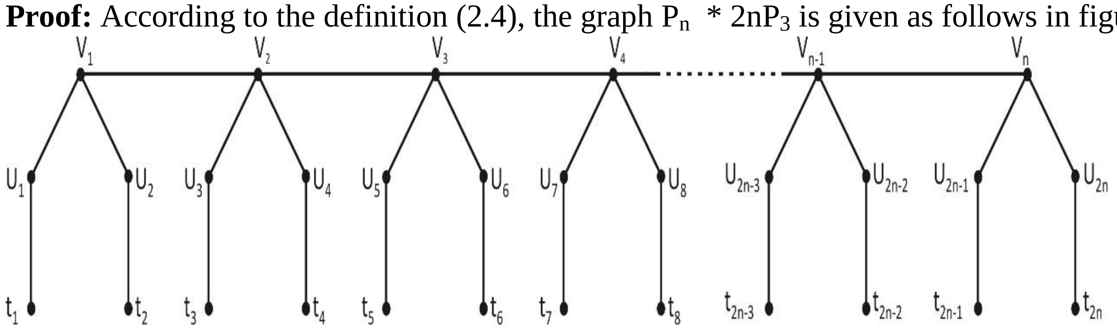

Figure 4: One of the arbitrary labelings of vertices for P_n * nP_3

(Vertex rule): Define a map f : V(G) $\to$ {0, 1, 2 ,…, p} by

$f(V_i) = (p-2n+1) + \frac{5(i-2)}{2}$ ('i' is even);

$f(V_i) = 2 + \frac{5(i-1)}{2}$; (i is odd);where i varies from 1 to n;

$f(U_1) = f(V_2)-3$; $f(U_2) = f(V_2)-2$; $f(U_3) = 4$; $f(U_4) = 5$

$f(t_1) = 1$; $f(t_2) = 3$; $f(t_3) = f(v_2)-1$; $f(t_4) = f(v_2)+1$

For i varying from 1 to n,

$f(U_i) = f(U_1)+5 (i-1)/4$; i Ξ [1(mod 4)] ; $= f(U_2)+5(i-2)/4$; i Ξ [2(mod 4)] ;
$= f(U_3)+5(i-3)/4$; i Ξ [3(mod 4)] ; $= f(U_4)+5(i-4)/4$; i Ξ [0(mod 4)] ;

$f(t_i) = f(t_1)+5(i-1)/4$; i Ξ [1(mod 4)] ; $= f(t_2)+5(i-2)/4$; i Ξ [2(mod 4)]

$= f(t_3)+5(i-3)/4$; $i \equiv [3(\text{mod } 4)]$; $= f(t_4)+5(i-4)/4$; $i \equiv [0(\text{mod } 4)]$

(Edge rule): Define the map $f: E(G) \to \{p + 1, p + 2, \ldots, p + q\}$ by

$f(V_iV_{i+1})=(p+q-4)-5(i-1)$; $i = 1,2\ldots\ldots(n-1)$;

$f(U_iV_{2i-1})=(p+q-1)-5(i-1)$; $i = 1,2\ldots\ldots n$; $f(U_iV_{2i}) = (p+q-2) - 5(i-1)$; $i = 1,2,\ldots, n$;

$f(U_it_i) = p+q - 5(i-1)$ where i is odd; $i = 1, 2,\ldots, n$;

$f(U_it_i) = (p+q-3) - 5(i-2)$; (i is even); $i = 1, 2,\ldots, n$;

The map f satisfies the requirements for super magic labeling to both vertex set and edge set of the given graph ($P_n * 2nP_3$) with the magic number $(13n - 2)$. Therefore the graph ($P_n * 2nP_3$) is super magic.

Example 2.6: The graphs $P_6 * 6P_3$ and $P_7 * 7P_3$ are super magic as given in figures 5 & 6.

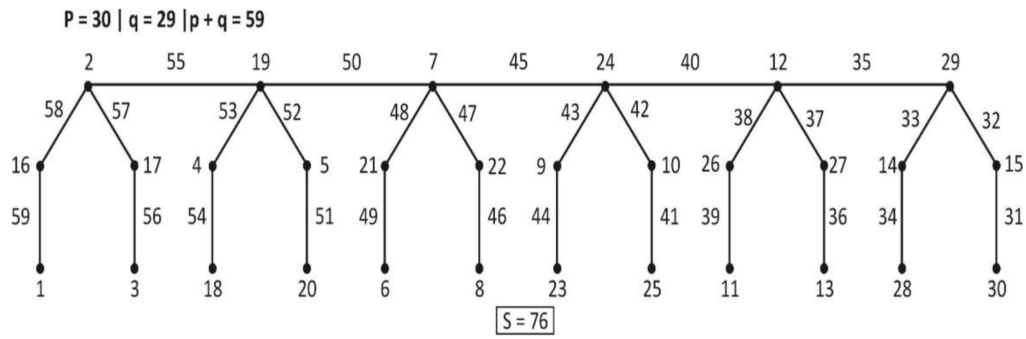

Figure 5 – super magic labeling of the graph $P_6 * 6P_3$

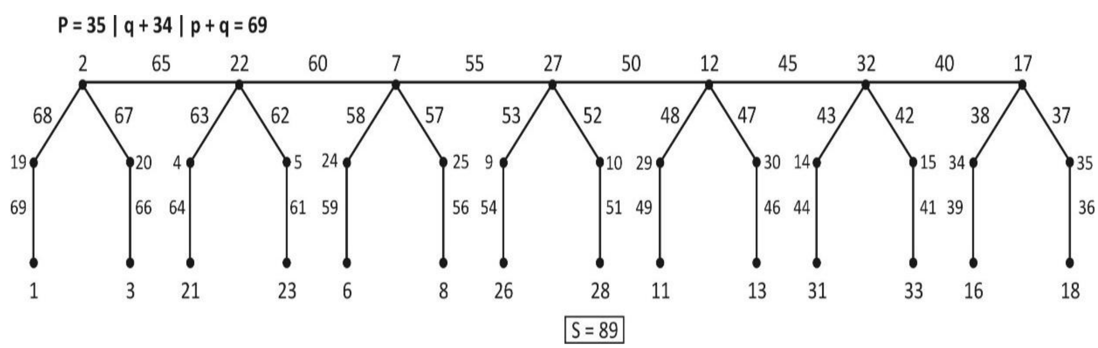

Figure 6 – super magic labeling of the graph $P_6 * 7P_3$

Definition 2.7: $P_n * 2nP_4$ is a connected graph whose vertex set is $\{V_1,V_2,\ldots, V_n, U_1, U_2,\ldots,U_{2n}, s_1,s_2\ldots,s_{2n}, t_1,t_2\ldots,t_{2n}\}$ and edge set is $\{V_iV_{i+1}: i = 1 \text{ to } n\} \cup \{V_iU_{2i-1}, V_iU_{2i}: i = 1 \text{ to } n\} \cup \{U_is_{2i-1}, U_is_{2i}: i = 1 \text{ to } n\} \cup \{s_it_{2i-1}, s_it_{2i}: i = 1 \text{ to } n\}$. The edge set of P_n is $\{V_iV_{i+1}: i = 1 \text{ to } n\}$, and edge set of $2nP_4$ is $\{V_iU_{2i-1}, V_iU_{2i}: i = 1 \text{ to } n\} \cup \{U_is_{2i-1}, U_is_{2i}: i = 1 \text{ to } n\} \cup \{s_it_{2i-1}, s_it_{2i}: i = 1 \text{ to } n\}$.

Theorem 2.8: The graph $P_n * 2nP_4$ is super magic.

Proof: As per definition (2.7), the graph $P_n * 2nP_4$ is given as follows in the figure 7:

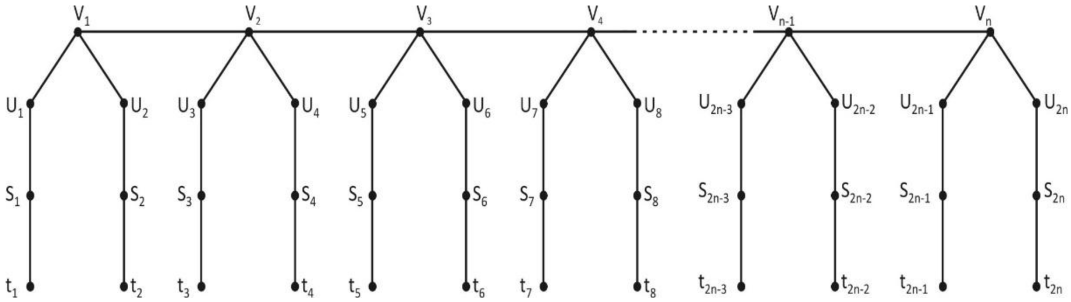

Figure 7: On eeof the arbitrary labelings of vertices for the graph (P_n * nP_4)

(Vertex rule): Define a map $f : V(G) \to \{0, 1, 2,\dots, p\}$ by

$f(V_1) = (p-3n-1)$

For i varying from 1 to n,

$f(V_i) = f(V_1) + \frac{7(i-1)}{2}$; i is odd;

$f(V_i) = 6 + \frac{7(i-2)}{2}$; i is even;

$f(U_1) = 2$; $f(U_2) = 3$; $f(U_3) = f(V_1) + 3$; $f(U_4) = f(V_1) + 4$;

$f(S_1) = f(V_1)-1$; $f(S_2) = f(V_1)+1$; $f(S_3) = 5$; $f(S_4) = 7$;

$f(t_1) = 1$; $f(t_2) = 4$; $f(t_3) = f(V_1)+2$; $f(t_4) = f(V_1)+5$

For i varying from 1 to n,

$f(U_i) = f(U_1) + 7(i-1)/4$; $i \equiv 1 \pmod 4$; $= f(U_2) + 7(i-2)/4$; $i \equiv 2 \pmod 4$

$= f(U_3) + 7(i-3)/4$; $i \equiv 3 \pmod 4$; $= f(U_4)+ 7(i-4)/4$; $i \equiv 0 \pmod 4$

$f(S_i) = f(S_1) + 7(i-1)/4$; $i \equiv 1 \pmod 4$; $= f(S_2) + 7(i-2)/4$; $i \equiv 2 \pmod 4$

$= f(S_3) + 7(i-3)/4$; $i \equiv 3 \pmod 4$; $= f(S_4) + 7(i-3)/4$; $i \equiv 0 \pmod 4$

$f(t_i) = f(t_1) + 7(i-1)/4$; $i \equiv 1 \pmod 4$; $= f(t_2) + 7(i-2)/4$; $i \equiv 2 \pmod 4$

$= f(t_3) + 7(i-3)/4$; $i \equiv 3 \pmod 4$; $= f(t_4) + 7(i-4)/4$; $i \equiv 0 \pmod 4$

(Edge Rule): Define the map $f: E(G) \to (p+1, p+2, \dots, p+q\}$ by

$f(V_iV_{i+1}) = p+q-6-7(i-1)$; i=1,2……..(n-1); $f(V_iU_{2i-1}) = p+q-2-7(i-1)$; i=1 to n

$f(V_iU_{2i}) = f(V_iU_{2i-1})-1$; i = 1 to n;

$f(U_iS_i) = f(V_iU_{2i-1})+1$; (i is odd); $f(U_iS_i) = f(V_iU_{2i})-1$; (i is even)

$f(S_it_i) = f(U_iS_i)+1$; (i is odd); $f(S_it_i) = f(U_iS_i)-1$; (i is even)

The map f satisfies the requirements for super magic labeling to both vertex set and edge set of the given graph (P_n * $2nP_4$) with the magic number (18n – 2). Therefore the graph (P_n * nP_4) is super magic.

Example 2.9: The graphs P_6 * $12P_4$ and P_7 * $14P_4$are super magic in the firgures8 & 9.

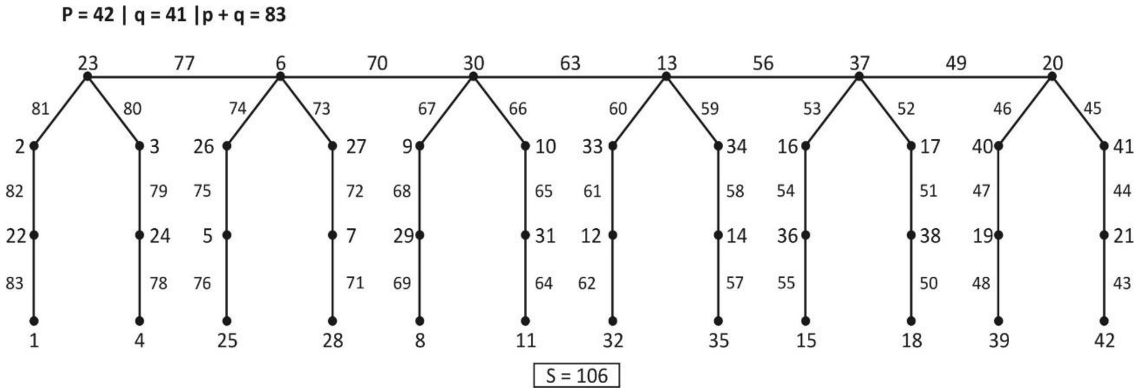

Figure 8 – super magic labeling of the graph ($P_6 * 12P_4$)

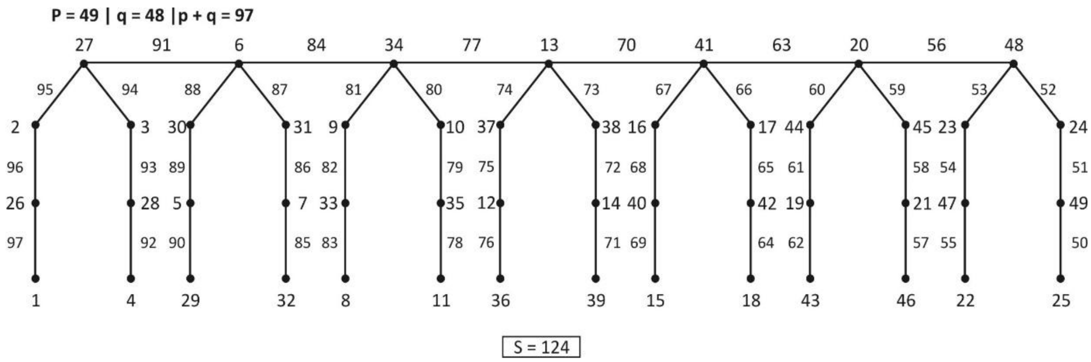

Figure 9 – super magic labeling of the graph ($P_6 * 14P_4$)

Definition 2.10: $P_n * 2nP_5$is a connected graph whose vertex set is { $V_1,V_2,\ldots, V_n$, $U_1, U_2,\ldots,U_{2n}$, $w_1,w_2\ldots,w_{2n}$, $s_1,s_2\ldots,s_{2n}$, $t_1,t_2\ldots,t_{2n}$ } and edge set is $\{V_iV_{i+1}: i = 1 \text{ to } n\} \cup \{V_iU_{2i-1}, V_iU_{2i}: i = 1 \text{ to } n\} \cup \{U_iw_{2i-1}, U_iw_{2i}: i = 1 \text{ to } n\} \cup \{w_is_{2i-1}, w_is_{2i}: i = 1 \text{ to } n\} \cup \{s_it_{2i-1}, s_it_{2i}: i = 1 \text{ to } n\}$. The edge set of P_n is $\{V_iV_{i+1}: i = 1 \text{ to } n\}$, and edge set of $2nP_5$ is $\{V_iU_{2i-1}, V_iU_{2i}: i = 1 \text{ to } n\} \cup \{U_iw_{2i-1}, U_iw_{2i}: i = 1 \text{ to } n\} \cup \{w_is_{2i-1}, w_is_{2i}: i = 1 \text{ to } n\} \cup \{s_it_{2i-1}, s_it_{2i}: i = 1 \text{ to } n\}$.

Theorem 2.11: The garph **(**$P_n * 2nP_5$) is super magic..

Proof: The tgraph ($P_n * 2nP_5$) is given as follows in the figure 10:

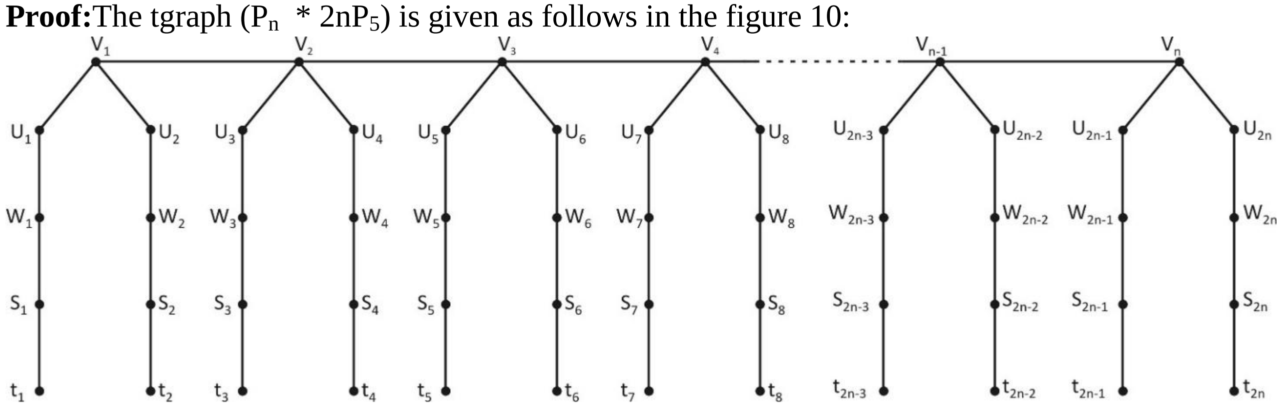

Figure 10: Arbitrary labelings of vertices for the graph ($P_n * nP_5$)

(Vertex set): Define $f : V(G) \rightarrow \{0,1 , 2,\ldots, p\}$ by

$f(V_2) = \frac{(P+1)}{2} + 7$ (or) $\frac{P}{2} + 7$; if "n" is odd (or) even respectively.

$f(V_i) = 3 + \frac{9(i-1)}{2}$; i is odd and i =1, 2 ,.., n);

$f(V_i) = f(V_2) + \frac{9(i-2)}{2}$; (i > 2 is even) and i = 1, 2 ,…, n)

$f(U_1) = f(V_2)-5$; $f(U_2) = f(V_2)-4$; $f(U_3) = 7$; $f(U_4) = 8$;
$f(W_1) = 2$; $f(W_2) = 4$; $f(W_3) = f(V_2)-1$; $f(W_4) = f(V_2)+1$
$f(S_1) = f(V_2)-6$; $f(S_2) = f(V_2)-3$; $f(S_3) = 6$; $f(S_4) = 9$;
$f(t_1) = 1$; $f(t_2) = 5$; $f(t_3) = f(V_2) - 2$; $f(t_4) = f(V_2) + 2$;

For i varying from 1 to n,

$f(U_i) = f(U_1) + 9(i-1)/4$; $i \equiv 1 \pmod 4$; $= f(U_2) + 9(i-2)/4$; $i \equiv 2 \pmod 4$
$= f(U_3)+9(i-3)/4$; $i \equiv 3 \pmod 4$; $= f(U_4)+9(i-4)/4$; $i \equiv 0 \pmod 4$
$f(W_i) = f(W_1)+9(i-1)/4$; $i \equiv 1 \pmod 4$; $= f(W_2)+9(i-2)/4$; $i \equiv 2 \pmod 4$
$= f(W_3)+9(i-3)/4$; $i \equiv 3 \pmod 4$; $= f(W_4)+9(i-4)/4$; $i \equiv 0 \pmod 4$
$f(S_i) = f(S_1)+9(i-1)/4$; $i \equiv 1 \pmod 4$; $= f(S_2)+9(i-2)/4$; $i \equiv 2 \pmod 4$
$= f(S_3)+9(i-3)/4$; $i \equiv 3 \pmod 4$; $= f(S_4)+9(i-4)/4$; $i \equiv 0 \pmod 4$
$f(t_i) = f(t_1)+9(i-1)/4$; $i \equiv 1 \pmod 4$; $= f(t_2)+9(i-2)/4$; $i \equiv 2 \pmod 4$
$= f(t_3)+9(i-3)/4$; $i \equiv 3 \pmod 4$; $= f(t_4)+9(i-4)/4$; $i \equiv 0 \pmod 4$

(Edge rule): Define the map f on edge set of the graph **(**$P_n * 2nP_5$) as follows:

$f(t_1S_1) = p + q$; $f(S_1W_1) = p+q-1$;
$f(W_1U_1) = p+q-2$; $f(U_1V_1) = p+q-3$;
$f(V_1U_2) = p+q-4$; $f(U_2W_2) = p+q-5$;
$f(W_2S_2) = p+q-6$; $f(S_2t_2) = p+q-7$;
$f(V_1V_2) = p+q-8$;

For i varying from 1 to n, define the map f as follows:

$f(t_iS_i) = f(t_1S_1) - (\frac{i-1}{2})9$; ("i" is odd)
$= f(t_2S_2) - (\frac{i-2}{2})9$; ("i" is even)
$f(S_iW_i) = f(S_1W_1) - (\frac{i-1}{2})9$; ("i" is odd)
$= f(S_2W_2) - (\frac{i-2}{2})9$; ("i" is even)
$f(W_iU_i) = f(W_1U_1) - (\frac{i-1}{2})9$; ("i" is odd)
$= f(W_2U_2) - (\frac{i-2}{2})9$; ("i" is even)

For i varying from 2 to n, define the map f on edge set of the graph **(**$P_n * 2nP_5$) as follows:

$f(V_iV_{i+1}) = f(V_1V_2) - 9(i-1)$;
$f(V_iU_{2i-1}) = f(V_1U_1) - 9(i-1)$;
$f(V_iU_{2i}) = f(V_1U_2) - 9(i-2)$;

The map f satisfies the requirements for super magic labeling to both vertex set and edge set of the given graph (P_n * $2nP_5$) with the magic number (23n – 2). Therefore the graph (P_n * nP_5) is super magic.

Example 2.12: The graphs P_6*$12P_5$and P_7*$14P_5$ are super magic in the figures (11)and (12).

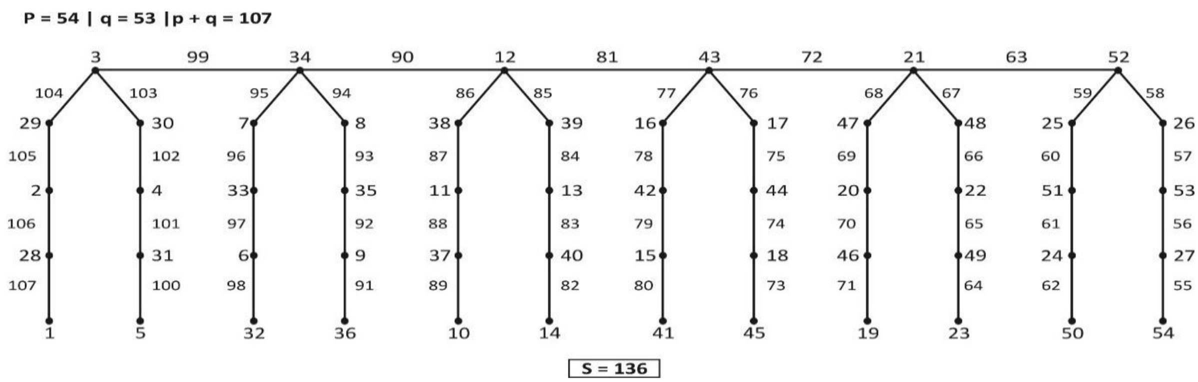

Figure 11 – super magic labeling of the graph (P_6 * $12P_5$)

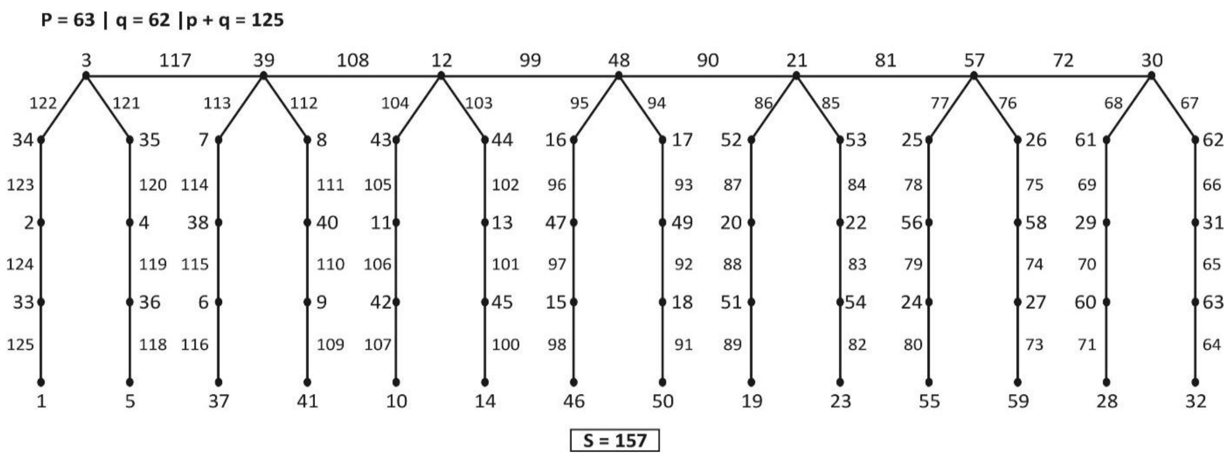

Figure 12 – super magic labeling of the graph (P_6 * $14P_5$)

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