On τ*- generalized pre connectedness and τ*- generalized pre compactness in Topological spaces

[1]C.Aruna, [2]R.Selvi,

[1]*Research Scholar, Research Centre of Mathematics,*

Sri Parasakthi College for Women, Courtallam-627802

[2]*Sri Parasakthi College for Women,Courtallam-627802.*

ABSTRACT: In this paper, the authors introduce a new type of connected spaces called τ*-gp-connectedness in topological spaces. The notion of τ*-gp -compact space is also introduced and their properties are studied. Some characterizations and several properties related to contra τ*-gp-continuous functions are obtained.

Keywords: τ*-gp-open set, τ*-gp-closed set τ*-gp-connectedness, τ*-gp-compactness, τ*-gp-Lindelof space.

1. INTRODUCTION

In 1970, Levine introduced the concept of generalized closed sets in topological spaces. Dunham introduced the concept of the closure operator cl* and a new topology τ* and studied some of their properties. A.Pushpalatha, S.Eswaran and P.RajaRubi introduced a new class of sets called τ* - generalized closed sets and studied some of their properties. The authors introduced the concepts of τ* - generalized pre closed sets and contra τ* - generalized pre continuous and studied some of their properties in topological spaces. Connectedness and Compactness is one of the most important and fundamental concepts in topology. The aim of this paper is to introduce the concept of τ* - generalized pre connectedness and τ* - generalized pre compactness in topological spaces.

2. PRELIMINARIES

Definition 2.1: A subset A of a space X is called

(i) pre-open [4] if A $\subseteq$ Int(cl (A))

(ii) generalized closed [2] (briefly g- closed) if cl(A) $\subseteq$U whenever A $\subseteq$U and U is open in X

Definition: 2.2: Let A subset A of a topological space (X, τ), is called a generalized pre closed set (briefly gp-closed) if pcl (A) $\subseteq$U whenever A $\subseteq$U and U is open in X. The complement of gp-closed set is called gp-open. The family of all gp-open [respectively gp-closed] sets of (X, τ) is denoted by gp-O(X, τ) [respectively gp-CL(X, τ)].

Definition: 2.3[10]:A subset A of a topological space (X, τ*), is called a τ*-generalized pre closed set (briefly, τ*- gp-closed) if $cl^*_p(A) \subseteq$U whenever A $\subseteq$U and U is gp τ* open in X. The complement of τ*- gp-closed set is called τ*-gp-open. The family of all τ*-gp -open [respectively τ*-gp- closed] sets of (X, τ*) is denoted by τ*- gp -O(X, τ*) [respectively τ* -gp -CL(X, τ*)].

Definition: 2.4: The τ*- closure of a set A, denoted by $cl^*_p(A)$ is the intersection of all τ*-gp -closed sets containing A.

Definition: 2.5: The τ*-gp interior of a set A, denoted by $int^*_p(A)$ is the union of all τ*-gp -open sets contained in A.

Lemma 2.6 [10]: Every open set is τ*-gp open.

3. τ*-gp–CONNECTEDNESS

Definition: 3.1 A topological space X is said to be τ*-gp -connected if X cannot be expressed as a disjoint union of twonon- empty τ*-gp -open sets in X. A set A of X is τ*-gp -connected if it is τ*-gp -connected as a subspace

Example: 3.2 Let X = {a, b} and let $\tau=\{X,\phi,\{b\}\}$. Here (X, τ*) is τ*- gp connected.

Theorem: 3.3 Fora topological space X, the following are equivalent

(i) X is τ*-gp-connected.

(ii) X and φ are the only subsets of X which are both τ*-gp-open and τ*-gp-closed.

(iii) Each τ*-gp-continuousmapofXintoadiscretespaceY withatleasttwopoints is constant a map.

Proof:

(i) ⇒(ii): Suppose X is τ*-gp-connected. Let S be a proper subset which is both τ*-gp-open and τ*-gp-closed in X. Its complement X\S is also τ*-gp-open and τ*-gp-closed. X = S ∪ (X\S), a disjoint union of two nonempty τ*-gp-open sets which is contradicts (i). Therefore S = φ orX.

(ii) ⇒ (i): Suppose that X = A ∪B where A and B are disjoint nonempty τ*-gp-open subsets of X. Then A is both τ*-gp-open and τ*-gp-closed. By assumption A = φ or X. Therefore X is τ*-gp-connected.

(ii) ⇒ (iii): Let f: X →Y be a τ*-gp-continuous map. X is covered by τ*-gp-open and τ*-gp-closed covering $\{f^{-1}(y): y\in(Y)\}$. By assumption $f^{-1}(y)$ = φ or X for each $y\in(Y)$. If $f^{-1}(y) = \varphi$ for all $y\in(Y)$, then f failsto be a map. Then there exists only one point y∈(Y) suchthat $f^{-1}(y) \neq \varphi$ and hence $f^{-1}(y)$ = X. This shows that f is a constant map.

(iii) ⇒ (ii): Let S be both τ*-gp-open and τ*-gp-closed in X. Suppose $S\neq\varphi$. Let f: X →Y be a τ*-gp-continuous function defined by f(S) = {y} and f(X\S) = {w} for some distinct points y and w in Y. By (iii) f is a constant function. Therefore S =X.

Theorem: 3.4 Every τ*-gp-connected space is connected.

Proof: Let X be τ*-gp -connected. Suppose X is not connected. Then there exists a proper non empty subset B of X which is both open and closed in X. Since every closed set is τ*-gp -closed, B is a proper non empty subset of X which is both τ*-gp -open and τ*-gp -closed in X. Using by Theorem 3.3, X is not τ*-gp -connected. This proves the theorem.

The converse of the above theorem need not be true as shown in the following example.

Example: 3.5 Let X = {a, b, c} and let τ= {X,φ, {b},{a, b},{b, c}}. X is connected but not τ*-gp -connected. Since {b},{a, c} are disjoint τ*-gp -open sets and X = {b} ∪{a, c}

Theorem: 3.6 If f: X →Y is a τ*-gp -continuous and X is τ*-gp -connected, then Y is connected.

Proof: Suppose that Y is not connected. Let Y = A ∪B where A and B are disjoint non-empty open set in Y. Since f is τ*-gp -continuous and onto, X = $f^{-1}(A) \cup f^{-1}(B)$ where $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint non-empty τ*-gp -open sets in X. This contradicts the fact that X is τ*-gp -connected. Hence Y isconnected.

Theorem: 3.7 If f: X →Y is a τ*-gp -irresolute and X is τ*-gp -connected, then Y is τ*-gp -connected.

Proof: Suppose that Y is not τ*-gp -connected. Let Y = A $\cup$B where A and B disjoint non-empty τ*-gp -open set in Y. Since f is τ*-gp -irresolute and onto, X = f^{-1}(A) $\cup f^{-1}$(B) where f^{-1}(A) and f^{-1}(B) are disjoint non-empty τ*-gp –open sets in X. This contradicts the fact that X is τ*-gp -connected. Hence Y is τ*-gp -connected.

Definition: 3.8 A topological space X is said to be $T_{\tau^*\text{-gp}}$-space if every τ*-gp -closed subset of X is closed subset of X.

Theorem: 3.9. Suppose that X is $T_{\tau^*\text{-gp}}$ -space then X is connected if and only if it is τ*-gp -connected.
Proof: Suppose that X is connected. Then X cannot be expressed as disjoint union of two non-empty proper subsets of X. Suppose X is not a τ*-gp -connected space. Let A and B be any two τ*-gp -open subsets of X such that X = A $\cup$B, where A $\cap$B=$\varphi$and A $\subset$X, B $\subset$X. Since X is $T_{\tau^*\text{-gp}}$ -space and A, B are τ*-gp -open. A, B are open subsets of X, which contradicts that X is connected. Therefore X is $T_{\tau^*\text{-gp}}$-connected.
Conversely, every open set is τ*-gp -open. Therefore every $T_{\tau^*\text{-gp}}$ -connected space is connected.

Theorem: 3.10 If the τ*-gp -open sets C and D form a separation of X and if Y is τ*-gp -connected subspace of X, then Y lies entirely within C or D.
Proof: Since C and D are both τ*-gp -open in X , the sets C $\cap$Y and D $\cap$Y are τ*-gp -open in Y. These two sets are disjoint and their union is Y. If they were both non-empty, they would constitute a separation of Y. Therefore, one of them is empty. Hence Y must lie entirely C or D.

Theorem: 3.11 Let A be a τ*-gp -connected subspace of X. If A $\subset$B $\subset\tau$*-gp -cl(A) then B is also τ*-gp -connected.
Proof: Let A be τ*-gp -connected and let A $\subset$B $\subset\tau$*-gp -cl(A). Suppose that B = C $\cup$D is a separation of B by τ*-gp -open sets. By using theorem 3.10, A must lie entirely in C or D. Suppose that A $\subset$C, then gp*-cl(A) $\subset\tau$*-gp -cl(B). Since τ*-gp -cl(C) and D are disjoint, B cannot intersect D. This contradicts the fact that C is non empty subset of B. So D=φ which implies B is τ*-gp -connected.

Theorem: 3.12 A contra τ*-gp -continuous image of an τ*-gp -connected space is connected.
Proof: Let f: X $\rightarrow$Y is a contra τ*-gpcontinuous function from τ*-gp –connected space X on to a space Y. Assume that Y is disconnected. Then Y =A $\cup$B, where A and B are non emptyclopen sets in Y with A $\cap$B=φ. Since f is contra τ*-gp –continuous, we have f^{-1}(A) and f^{-1}(B) are non empty τ*-gp open sets in X with f^{-1}(A) $\cup f^{-1}$(B) = f^{-1}(A $\cup$B) = f^{-1}(Y)= X and f^{-1}(A) $\cap f^{-1}$(B) = f^{-1}(A $\cap$B) = f^{-1}(φ) =φThis shows that X is not τ*-gp -connected, which is a contradiction. This proves the theorem.

4. τ*-gp COMPACTNESS

Definition: 4.1. Acollection$\{A_\alpha:\alpha\in\Lambda\}$ of τ*-gp -open sets in a topological space X are called a τ*-gp-open cover of a subset B of X if $B\subset\bigcup\{A_\alpha:\alpha\in\Lambda\}$ holds.

Definition: 4.2 A topological space X is τ*-gp– compact if every τ*-gp-open cover of X has a finite sub-cover.

Definition: 4.3 A subset B of a topological space X is said to be τ*-gp compact relative to X, if for every collection $\{A_\alpha:\alpha\in\Lambda\}$of τ*-gp-open subsets of X such that $B\subset\bigcup\{A_\alpha:\alpha\in\Lambda\}$ there exists a finite subset Λ_0 of Λ such that $B\subset\bigcup\{A_\alpha:\alpha\in\Lambda_0\}$

Definition: 4.4 A subset B of a topological space X is said to be τ*-gp- compact if B is τ*-gp – compact as a subspace of X.

Theorem: 4.5 Every τ*-gp -closed subset of τ*-gp -compact space is τ*-gp -compact relative to X.

Proof: Let A be τ*-gp- closed subset of a τ*-gp- compact space X. Then A^C is τ*-gp- open in X.Let $M=\{G_\alpha:\alpha\in\Lambda\}$ bea cover of A by τ*-gp -open sets inX. Then$M^* = MUA^C$is a τ*-gp -open cover of X. Since X is τ*-gp -compact, M^* isreducible to a finite sub cover of X, say X= $G_{\alpha 1} U\, G_{\alpha 2} U \ldots\ldots U G_{\alpha m} U\, A^c, G_{\alpha k} \in M$But A and A^C aredisjoint.Hence$A\subset G_{\alpha 1} U G_{\alpha 2} U \ldots\ldots U G_{\alpha m}, G_{\alpha k}\in M$,this implies that any τ*-gp open cover M of A contains a finite subcover. Therefore A is τ*-gp -compact relative to X. That is, Every τ*-gp -closed subset of a τ*-gp -compact space X is τ*-gp - compact.

Definition: 4.6 A function f: X $\rightarrow$Y is said to be τ*-gp -continuous if f^{-1}(V) is τ*-gp -closed in X for every closed set V of Y.

Theorem: 4.7 A τ*-gp -continuous image of a τ*-gp -compact space is compact.

Proof: Let f: X $\rightarrow$Y be a τ*-gp–continuous map from a τ*-gp compact space X onto a topological space Y. Let $\{A_\alpha : \alpha \varepsilon \wedge\}$ be an open cover of X. since X is τ*-gp compact, it has a finite sub-cover say $\{f^{-1}(A_1), f^{-1}(A_2) \ldots f^{-1}(A_n) : i\in\Lambda\}$.Since f is onto $\{A_1, A_2,\ldots..A_n\}$ is a cover of Y,which is finite.Therefore Y is Compact.

Definition: 4.8 A function f: X $\rightarrow$Y is said to be τ*-gp -irresolute if f^{-1}(V) is τ*-gp -closed in X for every τ*-gp -closed set V of Y.

Theorem: 4.9 If a map f : X $\rightarrow$Yis τ*-gp –irresolute and a subset B of X is τ*-gp compact relative to X, then the image f(B) is τ*-gp -compact relative to Y.

Proof: Let$\{A:\alpha\in\Lambda_0\}$be any collection of τ*-gp -open subsets of Y such that $f(B)\subset\bigcup\{A_\alpha:\alpha\in\Lambda\}$. Then $B\subset\bigcup\{f^{-1}(A_\alpha):\alpha\in\Lambda\}$Since by hypothesis B is τ*-gp -compact relative to X, there exists a finite subset $\Lambda_0\in\Lambda$ suchthat$B\subset\bigcup\{f^{1}(A_\alpha):\alpha\in\Lambda_0\}$. Thereforewehave$f(B)\subset\bigcup\{(A_\alpha):\alpha\in\Lambda_0\}$,itshows thatf(B)is τ*-gp -compact relative to Y.

Theorem: 4.10 A space X is τ*-gp -compact if and only if each family of τ*-gp -closed subsets of X with the finite intersection property has a non-empty intersection.

Proof: Given a collection $\mathbb{A}$ of subsets of X, let $\mathbb{C}$ = {X – A: A$\in\mathbb{A}$} be the collection of their complements. Then the following statements hold.

(a) $\mathbb{A}$isacollectionof τ*-gp-opensetsifandonlyifCisacollectionof τ*-gp closedsets.

(b) The collection A covers X if and only if the intersection $\bigcap_{c\in C} C$of all the elements of C is non empty.

(c) The finite sub collection {A1,A2…An} of A covers X if and only if the intersection of the correspondingelements $C_i = X - A_i$ of C isempty.

The statement (a) istrivial,whilethe(b) andfollowfromDeMorgan'slaw.

$X-(D\bigcup_{\alpha\in J} A_\alpha)= D\bigcap_{\alpha\in J}(X-A_\alpha)$.

The proof of the theorem nowproceeds in two steps, taking contra positive of the theorem and then thecomplement. The statement X is τ*-gp-compact is equivalent to : Given any collection $\mathbb{A}$ of τ*-gp -open subsetsof X, if$\mathbb{A}$ covers X, then some finite sub collection of $\mathbb{A}$ covers X. This statement is equivalent to itscontra positive, which is the following.

Given any collection $\mathbb{A}$ of τ*-gp -open sets, if no finite sub-collection of $\mathbb{A}$ of covers X, then $\mathbb{A}$ does not cover X. Let $\mathbb{C}$ be as earlier, the collection equivalent to the following:

Given any collection ℂ of τ*-gp closed sets, if every finite intersection of elements of ℂ is not-empty, then the intersection of all the elements of ℂ is non-empty.

Definition: 4.11 A space X is said to be τ*-gpLindelof space if every cover of X by τ*-gp -open sets contains a countable sub cover.

Theorem: 4.12 Let f: X⟶Y be a τ*-gp -continuous surjection and X be τ*-gp -Lindelof, Then Y is Lindelof Space.

Proof: Let *f:* X⟶Y be a τ*-gp - continuous surjection and X be τ*-gp -Lindelof. Let {Vα} be an open cover for Y. Then $\{f^{-1}(V\alpha)\}$ is a cover of X by τ*-gp -open sets. Since X is τ*-gp -Lindelof, $\{f^{-1}(V\alpha)\}$ contains a countable sub cover, namely $\{f^{-1}(V\alpha_n)\}$. Then $\{V\alpha_n\}$ is a countable sub cover for Y. Thus Y is Lindelof space.

Theorem: 4.13 Let f: *X*⟶Y be a τ*-gp -irresolute surjection and X be τ*-gp -Lindelof, Then Y is τ*-gp -Lindelof space.

Proof: Let f: X⟶Y be a τ*-gp -irresolute surjection and X be τ*-gp -Lindelof. Let {Vα} be a τ*-gp -open cover for Y. Then $\{f^{-1}(V\alpha)\}$ is a cover of X by τ*-gp -open sets. Since X is τ*-gp - Lindelof, $\{f^{-1}(V\alpha)\}$ contains a countable sub cover, namely $\{f^{-1}(V\alpha_n)\}$. Then $\{V\alpha_n\}$ is a countable sub cover for Y. Thus Y is τ*-gp -Lindelof space.

Theorem: 4.14 If f: X⟶Y is a τ*-gp -open function and Y is τ*-gp -Lindelof space, then X is Lindelof space.

Proof: Let{Vα}be an open cover for X. Then {f (Vα)} is a cover of Y by τ*-gp -open sets. Since Y is τ*-gp – Lindelof, {f (Vα)} contains a countable sub cover, namely $(\{f(V\alpha_n)\}$. Then
$\{V\alpha_n\}$is a countable sub cover for X. Thus X is Lindelof space.

REFERENCES

[1] DunhamW., A new closure operator for non-T_1 topologies, Kyungpook Math.J.22(1982),55-60.

[2]. N.Levine, Generalized closed sets in topology, Tend Circ., Mat. Palermo (2) 19 (1970), 89- 96

[3]. O.Njastad, On some classes of nearly open sets, Pacific J Math., 15(1965), 961-970.

[4]. A.S.MashorAbd.El-Monsef.M.E and Ei-Deeb.S.N., On Pre continuous and weak pre-continuous mapping, Proc.Math.,Phys.Soc.Egypt, 53 (1982), 47-53.

[5]. Sundaram.P. and A.Pushpaplatha.2001.Strongly generalized closed sets in topological spaces.Far East J.Math.Sci., 3(4): 563-575.

[6]. S.Sekar and P.Jayakymar, On gp*-closed map in Topological Spaces- Applied Mathematical Sciences, Vol.8, 2014, no. 9, 415-422

[7]. Somasundaram.S.,Murugalingam.M. andPalaniammal.S. 2005. A generalized Star Sets. Bulletin of Pure andApplied Sciences.Vol.24E (No.2): 233-238.

[8]. P.Jayakymar, K.Mariappa and S.Sekar, On generalized gp*- closed set in Topological Spaces Int.Journal of Math.Analysis, Vol. 7, 2013, no.33, 1635 - 1645.

[9] Pushpalatha A.S.Eswaran and P.RajaRubi, -generalized closed sets in topological spaces, WCE 2009, July 1-3, 2009, London, U.K.

[10]C.Aruna and R.Selvi, τ*Generalized Preclosed Sets In Topological Spaces International Journal of Technical Research and Applications e-ISSN: 2320-8163, Volume 4, Issue 4 (July-Aug, 2016), PP. 99-103

[11] C.Aruna and R.Selvi, Contra τ* - Generalized Pre continuity – Communicated.