Integral Root Labeling of Graphs

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ABSTRACT

Let $G = (V, E)$ be a graph with p vertices and q edges. Let $f: V \to \{1,2, \dots q + 1\}$ is called an **Integral Root labeling** if it is possible to label all the vertices $v \in V$ with distinct elements from $\{1,2, \dots q + 1\}$ such that it induces an edge labeling $f^+: E \to \{1,2, \dots q\}$ defined as

$f^+(uv) = \left\lceil \sqrt{\frac{(f(u))^2+(f(v))^2+f(u)f(v)}{3}} \right\rceil$ is distinct for all $uv \in E$. (i.e.) The distinct vertex labeling induces a distinct edge labeling on the graph. The graph which admits Integral Root labeling is called an **Integral Root Graph**.

In this paper, we introduce Integral Root labeling and investigate Integral Root labeling of Path, Comb, Ladder, Triangular Snake and Quadrilateral Snake.

KEY WORDS

Integral Root labeling, Integral Root graph, Path, Comb, Ladder, Triangular Snake, and Quadrilateral Snake.

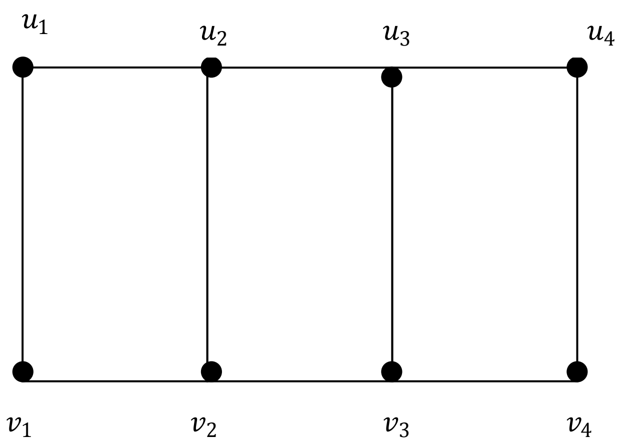
INTRODUCTION

By a graph *G= (V (G), E (G))* we mean a finite undirected graph without loops or parallel edges. For all detailed survey of graph labeling we refer to Gallian[1]. For all other standard terminology and notations we follow Harary[2]. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions.

In this paper we investigate the Integral Root labeling of Path, Comb, Ladder, Triangular Snake, and Quadrilateral Snake.

2. BASIC DEFINITIONS

Definition: 2.1
A walk in which $u_1, u_2, \dots u_n$ are distinct is called a **path**. A path on n vertices is denoted by P_n .
Definition: 2.2
A Closed Path is called a Cycle. A cycle on n vertices is denoted by C_n.
Definition: 2.3
The graph obtained by joining a single pendent edge to each vertex of a path is called a **Comb.**
Definition: 2.4
The Cartesian product of two graphs $G_1=(V_1,E_1)$ and $G_2=(V_2,E_2)$ is a graph $G=(V,E)$ with $V=V_1\times V_2$ and two vertices $u=(u_1u_2)$ and $v=(v_1v_2)$ are adjacent in $G_1\times G_2$ whenever ($u_1=v_1$and u_2 is adjacent to v_2) or ($u_2=v_2$and u_1 is adjacent to v_1) .It is denoted by $G_1\times G_2$.
Definition: 2.5
The Corona of two graphs G_1 and G_2 is the graph $G=G_1\odot G_2$ formed by taking one copy of G_1 and $|(G_1)|$ copies of G_2 where the i^{th} vertex of G_1 is adjacent to every vertex in the i^{th} copy of G_2.
Definition: 2.6
The product graph $P_2 \times P_n$ is called a **ladder** and it is denoted by L_n.
Example:
Ladder graph of L_4 is given below

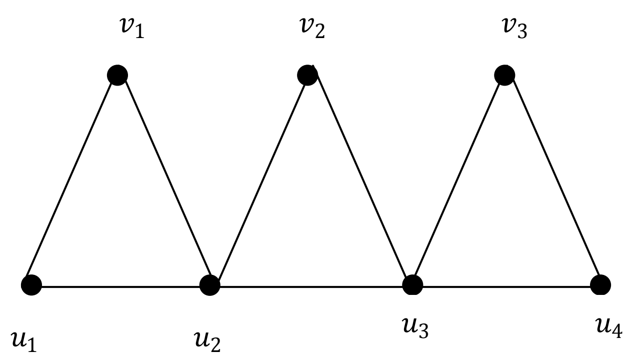


Definition: 2.7

A **Triangular Snake T_n** is obtained from a path $u_1, u_2, \ldots u_n$ by joining u_i and u_{i+1} to a new vertex v_i for $1 \leq i \leq n-1$.That is every edge of a path is replaced by a triangle C_3.

Example:

Triangular Snake T_4 is given below

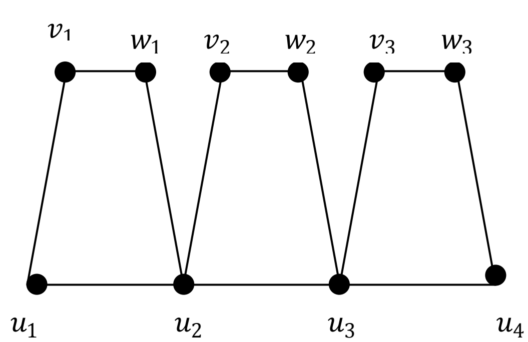


Definition: 2.8

A **Quadrilateral Snake Q_n** is obtained from a path $u_1, u_2, \ldots u_n$ by joining u_i and u_{i+1} to two new vertices v_i and w_i respectively and then joining v_i and w_i. That is every edge of a path is replaced by a cycle C_4.

Example:

Quadrilateral Snake Q_4 is given below



3. MAIN RESULTS

Definition: 3.1

Let $G = (V, E)$ be a graph with p vertices and q edges. Let $f: V \to \{1,2, \dots q+1\}$ is called an **Integral Root labeling** if it is possible to label all the vertices $v \in V$ with distinct elements from $\{1,2, \dots q+1\}$ such that it induces an edge labeling $f^+: E \to \{1,2, \dots q\}$ defined as

$f^+(uv) = \left\lceil \sqrt{\frac{(f(u))^2+(f(v))^2+f(u)f(v)}{3}} \right\rceil$ is distinct for all $uv \in E.$ (i.e.) The distinct vertex labeling induces a distinct edge labeling on the graph. The graph which admits Integral Root labeling is called an **Integral Root Graph.**

Theorem: 3.2

Any path P_n is an Integral Root graph.

Proof:

Let G be a path graph P_n.

Let $\{u_1, u_2, \dots . u_n\}$ be the vertices of path P_n and $\{e_1, e_2, \dots . e_{n-1}\}$ be the edges of path P_n

The path P_n consists of n vertices and $n-1$ edges.

Define $f: V(P_n) \to \{1,2, \dots . q+1\}$ by $f(u_i) = i$; $1 \le i \le n.$

Then we find the edge labels $f^+(e = u_i u_{i+1}) = \left\lceil \sqrt{\frac{i^2+i(i+1)+(i+1)^2}{3}} \right\rceil$; $1 \le i \le n-1$

$$= \left\lceil \sqrt{\frac{i^2+i^2+i+i^2+1+2i}{3}} \right\rceil$$

$$= \left\lceil \sqrt{\frac{3i^2+3i+1}{3}} \right\rceil$$

$= i$ are distinct.

Hence P_n is an Integral Root graph.

Example: 3.3

The Integral Root labeling of P_6 is given below

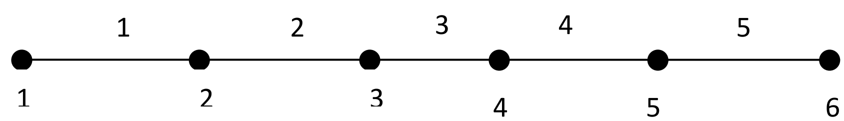


Figure: 1

Theorem: 3.4

Any Comb $P_n \boldsymbol{\Theta k_1}$ is an Integral Root graphs. $n \ge 2$

Proof:

Let G be a comb graph $P_n \boldsymbol{\Theta k_1}$.

Let $\{u_1, u_2, \dots . u_n, v_1, v_2, \dots v_n\}$ be the vertices of comb.

The comb has $2n-1$ edges.

Define a function $f: V(P_n \boldsymbol{\Theta k_1}) \to \{1,2, \dots . q+1\}$ by

$f(u_i) = 2i - 1$; $1 \le i \le n,$

$f(v_i) = 2i$; $1 \le i \le n.$

Then we find the edge labels

$f^+(e = u_i u_{i+1}) = 2i;$ $1 \le i \le n-1$;

$f^+(e = u_i v_i) = 2i + 1;$ $1 \le i \le n-1$ are distinct.

Hence $P_n \boldsymbol{\Theta k_1}$ is an Integral Root graph.

Example: 3.5

The Integral Root labeling of $P_7 \boldsymbol{\Theta k_1}$ is given below.

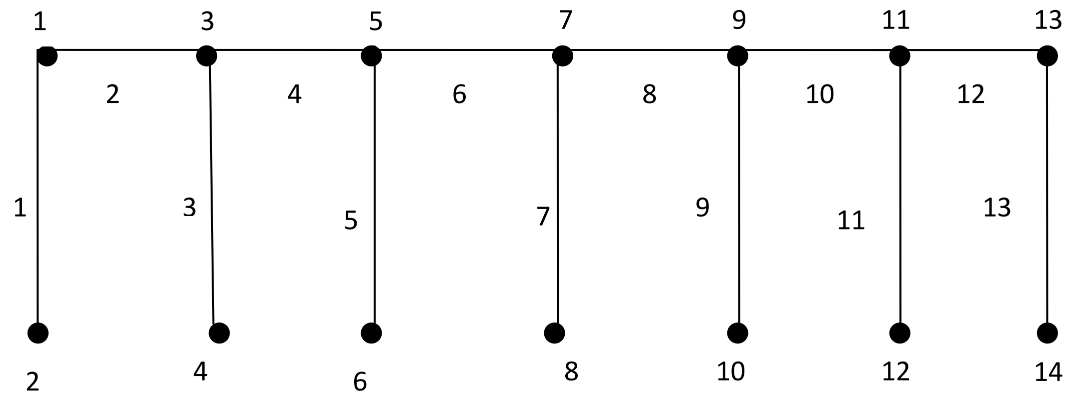


Figure: 2

Theorem: 3.6

Any Ladder L_n is an Integral Root graph.

Proof:

Let G be a Ladder graph L_n.

Let $\{u_1, u_2, \dots . u_n, v_1, v_2, \dots v_n\}$ be the vertices of ladder.

Define a function $f: V(L_n) \to \{1,2, \dots . q+1\}$ as

$f(u_i) = 3i - 2\,; \quad 1 \le i \le n,$

$f(v_i) = 3i - 1\,; \quad 1 \le i \le n.$

Then we find the edge labels

$f^+(u_i u_{i+1}) = 3i - 1\,; \quad 1 \le i \le n-1,$

$f^+(u_i v_i) = 3i - 2; \quad 1 \le i \le n-1,$

$f^+(v_i v_{i+1}) = 3i\,; \quad 1 \le i \le n-1$ are distinct.

Hence L_n is an Integral Root graph.

Example: 3.7

The Integral Root labeling of L_6 is given below

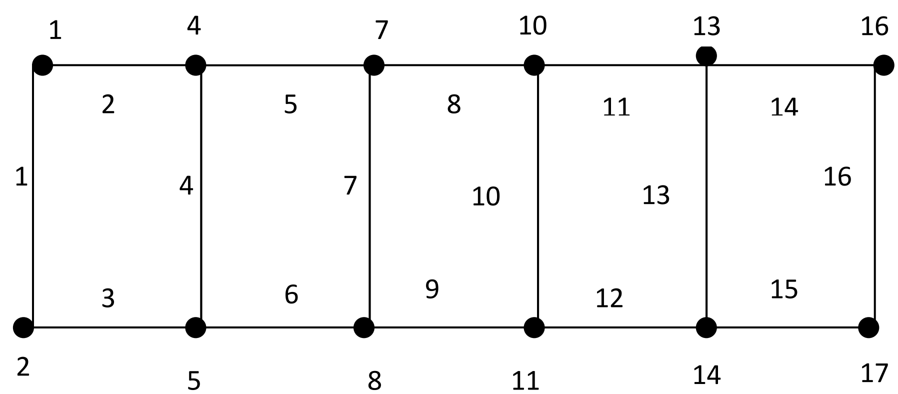


Figure: 3

Theorem: 3.8

Any Triangular Snake T_n is an Integral Root graph.

Proof:

Let G be a triangular snake T_n.

Define a function $f: V(T_n) \to \{1,2, \dots, q+1\}$by

$f(u_i) = 3i - 2;\ 1 \le i \le n,$

$f(v_i) = 3i - 1;\ 1 \le i \le n - 1.$
Then we find the edge labels
$f^+(u_i u_{i+1}) = 3i - 1;\ \ 1 \le i \le n - 1,$
$f^+(v_i u_{i+1}) = 3i;\ \ \ \ \ 1 \le i \le n - 1\ ,$
$f^+(u_i v_i) = 3i - 2;\ \ \ 1 \le i \le n - 1$ are distinct.
Hence T_n is an Integral Root graph.

Example: 3.9

The Integral Root labeling of T_6 is given below

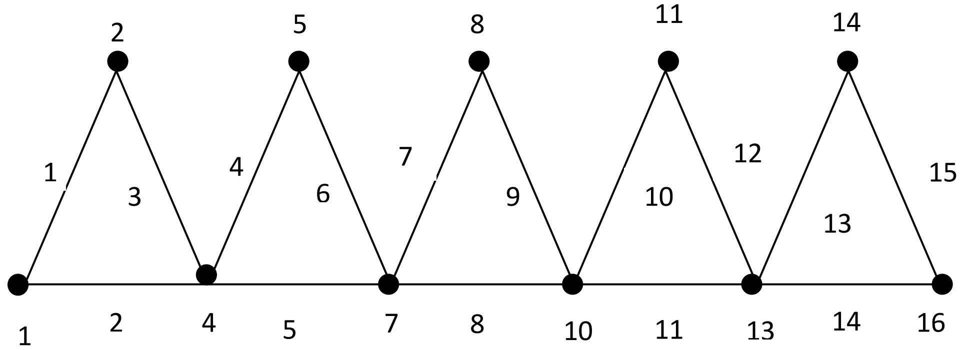


Figure 4

Theorem: 3.10

Any Quadrilateral Snake Q_n is an Integral Root graph.

Proof:

Let G be a Quadrilateral Snake Q_n.
Define a function $f: V(Q_n) \to \{1,2, \ldots . q + 1\}$ by
$f(u_i) = 4i - 3;\ \ 1 \le i \le n,$
$f(v_i) = 4i - 2;\ \ \ 1 \le i \le n - 1,$
$f(w_i) = 4i - 1;\ \ 1 \le i \le n - 1.$
Then the edge labels
$f^+(u_i v_i) = 4i - 3;\ \ \ \ 1 \le i \le n - 1,$
$f^+(u_i u_{i+1}) = 4i - 1;\ \ 1 \le i \le n - 1,$
$f^+(u_{i+1} w_i) = 4i;\ \ \ \ \ \ 1 \le i \le n - 1,$
$f^+(v_i w_i) = 4i - 2;\ \ \ \ \ 1 \le i \le n - 1$ are distinct.
Hence Q_n is an Integral Root graph.

Example: 3.11

The Integral Root labeling of Q_5 is given below.

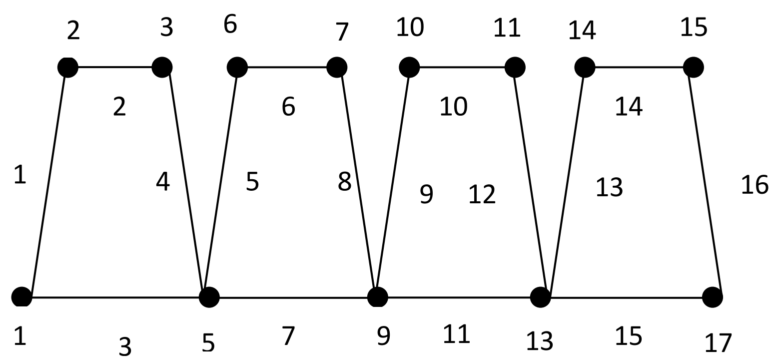


Figure: 5

Theorem: 3.12

Any cycle C_n is not a Integral Root graph $n \geq 3$.

Proof:

Let G be a graph of cycle C_n .

Let $\{u_1, u_2, \ldots . u_n\}$ be the vertices of cycle C_n and $\{e_1, e_2, \ldots . e_{n-1}\}$ be the edges of cycle C_n.

The cycle C_n consists of n vertices and $n - 1$ edges.

Define $f: V(C_n) \rightarrow \{1,2, \ldots . q + 1\}$ by

$f(u_i) = i\ ; 1 \leq i \leq n$

Then we find the edge labels

$f^+(e = u_i u_{i+1}) = i; 1 \leq i \leq n - 1$ are distinct., and $f(u_n u_1)$ is not distinct.

Hence C_n is not a Root graph.

Example: 3.13

Cycle C_7 is not Integral Root Labeling of graph is given below

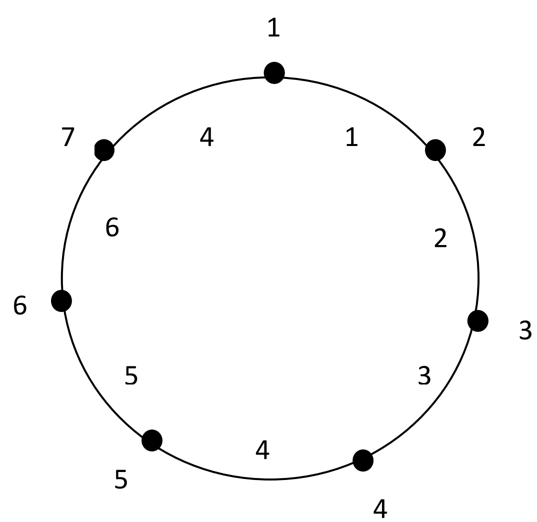


Figure 6

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