ALTERNATIVE METHODS OF ORDINARY DIFFERENTIAL EQUATIONS

AUTHOR'S INTRODUCTION

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Abstract

In this paper we study about alternative method's by which we can solve the problem of ordinary differential equation. In this method very useful to Engineering students, we can easily solve any ordinary differential equation problem in this method.

Keywords

Differentiation, Integration.

Solve :

$$y'' + 3y' + 2y = x^2 \qquad - \qquad (1)$$

To Solve Previous Method

$$(D^2 + 3D' + 2)\, y = x^2$$

The Auxiliary equation is $m^2 + 3m + 2 = 0$

$$(m + 2)\,(m + 1) = 0$$

$$m = -1, -2$$

$\therefore$ C.F is $Ae^{-x} + Be^{-2x}$

$$\text{PI} = \frac{1}{D^2 + 3D + 2} x^2$$

$$= \frac{1}{2\left[1 + \frac{D^2 + 3D}{2}\right]} x^2$$

$$= \frac{1}{2}\left[1 + \frac{D^2 + 3D}{2}\right]^{-1}$$

$$= (1 - x)^{-1} = 1 - x + x^2$$

$$= \frac{1}{2}\left[1 - \left(\frac{D^2 + 3D}{2}\right) + \left(\frac{D^2 + 3D}{2}\right)^2\right]$$

$$= \frac{1}{2}\left[1 - \frac{D^2}{2} - \frac{3D}{2} + \frac{D^4 + 6D^4 + 9D^2}{4}\right] x^2$$

$$= \frac{1}{2}\left[x^2 - \frac{2}{2} - \frac{6x}{2}\right] + \frac{9}{2}$$

$$= \frac{x^2}{2} - \frac{1}{2} - \frac{3x}{2} + \frac{9}{4}$$

$$= \frac{x^2}{2} - \frac{3}{2}\,x - \frac{1}{2} + \frac{9}{4}$$

$$= x^2 = \frac{x^2}{2} - \frac{3}{2}x + \frac{7}{4}$$

$$y = \text{C.F.} + \text{P.I}$$

$$= Ae^{-x} + Be^{-2x} + \frac{x^2}{2} - \frac{3}{2}x + \frac{7}{4}$$

The same problem to solve by new methods

$$y'' + 3y' + 2y = x^2 \qquad - \qquad (1)$$

$$(D^2 + 3D' + 2)\, y = x^2$$

$$m = -1, -2$$

3

1 2

$\therefore$ C.F is $Ae^{-x} + Be^{-2x}$

$$\text{PI} = [C_0 + C_1 x + C_2 x^2]$$

$$= (D^2 + 3D + 2)\,(C_0 + C_1 x + C_2 x^2) = x^2$$

$$= D^2C_0 + D^2C_1x + D^2C_2x^2 + 3DC_0 + 3DC_1x + 3DC_2x^2$$

$$+ 2C_0 + 2C_1x + 2C_2x^2 = x^2$$

$$= 0 + 0 + 2C_2 + 0 + 3C_1 + 6C_2x + 2C_0 + 2C_1x$$

$$+ 2C_2x^2 = x^2$$

Equating Co-efficient of x^2, x & constant

$$2C_2 = 1 \qquad 6C_2 + 2C_1 = 0$$

$$C_2 = \frac{1}{2} \qquad 3 + 2C_1 = 0$$

$$2C_1 = -3$$

$$C_1 = -\frac{3}{2}$$

$$2C_2 + 3C_1 + 2C_0 = 0 \qquad C_0 = \frac{7}{2x2}$$

$$2\,x\frac{1}{2} + 3\,x - \frac{3}{2} + 2C_0 = 0 \qquad C_0 = \frac{7}{4}$$

$1-\frac{9}{2}+2C_0 \quad = \quad 0$

$-\frac{7}{2}+\ 2C_0 \quad = \quad 0$

$\therefore$ P.I. $=\frac{7}{4}-\frac{3}{2}x+\frac{1}{2}x^2$ y $=Ae^{-x}+\ Be^{-2x}+\frac{7}{4}-\frac{3x}{2}+\frac{x^2}{2}$

Solve

$(D^2-\ 2D+5)y\ =\ e^x\ sin2x$

To Solve P.I. of previous method

P.I. $=\frac{1}{D^2-2D+5}\ e^x\ sin2x$

$=e^x\ \frac{1}{(D+1)^2-2\ (D+1)+5}\sin 2x$

$=e^x\ \frac{1}{D^2+\ 2D+1-2D-2+5}\sin 2x$

$=\frac{e^x}{D^2+4}\ Sin\ 2x \qquad D^2=-4$

$=\frac{e^x}{-4+4}\ Sin\ 2x$

= Differentiate with respect to D

$=\frac{xe^x}{2D}\ Sin\ 2x \qquad \frac{1}{D}=intergration$

$=\frac{xe^x}{2}\frac{1}{D}Sin\ 2x$

$=\frac{xe^x}{2}\left(-\frac{Cos2x}{2}\right)$

P.I. $=\frac{-x\ e^x Cos2x}{4}$

To Solve New method

P.I. $=\frac{1}{D^2-2D+5}\ e^x\ sin2x$

$=\frac{1}{D^2-2D+5}\ e^x\ e^{i2x}$

$=\frac{1}{D^2-2D+5}\ e^{(1+i2)x}$

$=\frac{1}{(1+2i)^2-2(1+2i)+5}\ e^{(1+i2)x}$

$=\frac{1}{1+4i-4-2-4i+5}\ e^{(1+i2)x}$

$=\frac{1}{0}\ e^{(1+i2)x}$

Differentiate with respect to D

$=\frac{x}{2D-2}\ e^{(1+i2)x}$

$= \frac{x}{2(D-1)} \; e^{(1+i2)x}$

$= \frac{x}{2(1+i2)-1)} \; e^{(1+i2)x}$

$= \frac{x}{2+4i-2} \; e^{(1+i2)x}$

$= \frac{x}{4i} \; e^{x} \, sin2x$

$= \frac{x}{4} \; e^{x} \int 2\, sin2x$

$= \frac{x}{4} \; e^{x} \left(-\frac{2Cos2x}{2}\right)$

P.I. $= -\frac{x}{4} \; e^{x} \, Cos2x$

i =1/n*d($Cosnx$ or $sinnx$)

1/i =n* intergeral($Cosnx$ or $sinnx$)

To Solve previous method of variation of parameters

Solve = $y'' + y = \sec x$

$= (D^2 + 1)\, y = 0$

$= m^2 = -1$

$= m = i^2 \qquad m = \pm i$

C.F. $= (C_1 cosx + C_2 sinx)$

$= C_1 f_1 + C_2 f_2$

$f_1 = Cosx \qquad f_2 = Sinx \qquad x = Secx$

$f_1^1 = -Sinx \qquad f_2^1 = Cosx$

$f_1 f_2^1 - f_2 f_1^1 = Cos^2 x + Sin^2 x = 1$

P $= -\int \frac{f_{2x}}{f_1 f_2^1 - f_2 f_1^1} \, dx$

$= -\int \frac{sinx \;\; secx}{1} \, dx$

$= -\int \frac{sinx}{Cosx} \, dx$

$= -\int tanx \, dx$

$= \log cosx$

Q $= \int \frac{f_{1x}}{f_1 f_2^1 - f_2 f_1^1} \, dx$

$= \int \frac{Cosx \ secx}{1} \, dx$

$= \int \frac{\cos x}{Cosx} \, dx$

$= \int dx$

$= x$

P.I. $= Pf_1 + Qf_2$

$= (\log cosx) \ cosx + xsinx$

y = C.F. + P.I.

y $= (C_1 cosx + C_2 sinx) + log \ cosx + xsinx$

To solve above problem by new method

Solve = $y'' + y = \sec x$

$= (D^2 + 1) \ y = 0$

$= m^2 = -1$

$= m = i^2 \qquad m = \pm i$

C.F. $= (C_1 cosx + C_2 sinx)$

By wronskian method

$W(y_1, y_2) = y_1 y_2^1 - y_2 y_1^1$

$y_1 = Cosx \qquad y_2 = sinx$

$y_1^1 = - Sinx \qquad y_2^1 = cosx$

$W(y_1, y_2) = y_1 y_2^1 - y_2 y_1^1$

$= cos^2 x + sin^2 x = 1$

P $= -\int \frac{y_2 x}{W(y_1, y_2)} \, dx$

$= -\int \frac{sinx \ \sec x}{1} \, dx$

$= -\int \frac{sinx}{cosx} \, dx = \log cosx$

Q $= \int \frac{y_1 x}{W(y_1, y_2)} \, dx$

$= \int \frac{cosx \ \sec x}{1} \, dx$

$= \int \frac{cosx}{cosx} \, dx = x$

y = C.F. + P.I.

y = $(C_1 cosx + C_2 sinx) + log\ cosx + xsinx$

CONCLUSION

In this method is very easily solve to ordinary differentially equations. It is easily understanding to students.

Thanking You,

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Thanking You,