Radio Antipodal Mean Number of Certain Graphs

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Abstract: *Let $G(V,E)$ be a graph with vertex set V and edge set E. Let d denote the diameter of G and $d(u,v)$ denote the distance between the vertices u and $v$in G. In this paper, we introduce a new labeling called radio antipodal mean labeling. An radio antipodal mean labeling of G is a function f that assigns to each vertex a non-negative integer such that $f(u) \neq f(v)$ if $d(u,v) < d$and $d(u,v) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \geq d$, for any two distinct vertices $u, v \in V(G)$. The radio antipodal mean number of f denoted by $r_{amn}(f)$, is the maximum number assigned to any vertex of G. The radio antipodal mean number of G, denoted by $r_{amn}(G)$ is the minimum value of $r_{amn}(f)$ taken over all radio antipodal mean labelings f of G. We determine the antipodal mean number of path, circle, wheel, mesh and enhanced mesh.*

Keywords-*Labeling, radio antipodal mean numbering, diameter.*

.1.Introduction

Let G be a connected graph and let $k \geq 1$,be an integer. A radio k- labeling f of G is an assignment of positive integers to the vertices of G such that $d(u,v) + |f(u) - f(v)| \geq k + 1$ for every two distinct vertices u and $v$of G. The span of such a function f, denoted by $\text{sp}(f) = max\{|f(u) - f(v)|: u, v \in V(G)\}$. Radio k −labeling was motivated by the frequency assignment problem [5]. The maximum distance among all pairs of vetices in G is the diameter of G. The radio labeling is a radio k- labeling when $k = diam(G)$. When $k = diam(G) - 1$, a radio k- labeling is called a radio antipodal labeling. In otherwords, an antipodal labeling for a graph G is a function,$f: V(G) \to \{1,2, \ldots\}$such that $d(u,v) + |f(u) - f(v)| \geq diam(G)$. The radio antipodal number for G, denoted by $an(G)$, is the minimum span of an antipodal labeling admitted by G. A radiolabelingis a one-to-one function, while in an antipodal labeling, two vertices of distance $diam(G)$ apart may receivethe same label. The antipodal labeling for graphs was first studied by Chartrand et al.[10], in which, among other results, general bounds of $an(G)$were obtained. Khennoufa and Togni [12] determined the exact value of $an(P_n)$ for paths P_n. In this paper we introduce a new labeling called radio antipodal mean labeling and obtained the lower bound for radio antipodal mean number. Also, we have completely determined the radio antipodal mean number for mesh and its derived architectures.

Definition 1.1:Let $G(V,E)$ be a graph with vertex set V and edge set E. Let $d$denote the diameter of G. An radio antipodal mean labeling of $G$is a function f that assigns to each vertex a non-negative integer such that $f(u) \neq f(v)$ if $d(u,v) < d$and $d(u,v) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \geq d$, for any two distinct vertices $u, v \in V(G)$. The radio antipodal mean number of f denoted by $r_{amn}(f)$,is the maximum number assigned to any vertex of G. The radio antipodal mean number of G, denoted by $r_{amn}(G)$ is the minimum value of $r_{amn}(f)$ taken over all radio antipodal mean labelings$f$ of G.

we determine an upper bound for the radio antipodal mean number of path, circle, wheel, mesh and enhanced mesh.

Theorem1.1: Let G be a path P_n. Then the radio antipodal mean number of $r_{amn}(G)$ is $3n - 10$.

Theorem1.2: The radio antipodal mean number of circle is $r_{amn}(C_n) = \begin{cases} n-3, & n \text{ is even, where } n > 7 \\ n-3, & n \text{ is odd, where } n > 6 \end{cases}$

Theorem1.3: The radio antipodal mean number of the wheel is given by

$$r_{amn}(W_n) = \begin{cases} 4, & n \text{ is even} \\ 3, & n \text{ is odd} \end{cases}$$

2. Radio Antipodal Mean Number of Mesh and its derived architectures.

2.1 Radio Antipodal Mean Number of Mesh

Definition 2.1: The $m \times n$ mesh denoted $M_{m\times n}$ is defined as the Cartesian product $P_m \times P_n$ of paths on m and n vertices respectively. The number of vertices in $M_{m\times n}$ is mn and its diameter is $m + n - 2$. See Fig 1.

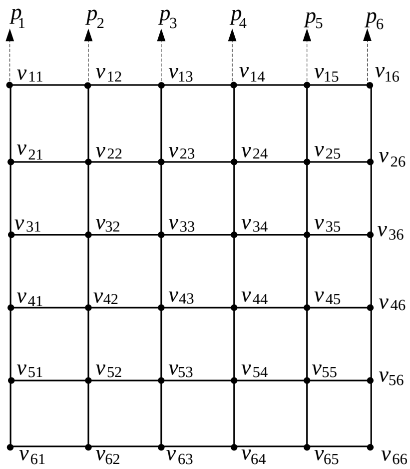

Fig 1. Mesh $M_{6\times6}$

Theorem 2.1: Let G be a mesh $M_{n\times n}$. Then the antipodal mean number of $r_{amn}(G) \leq n^2+2n-7$.

Proof.Label the vertices of M_{nxn} from left to right as $v_{11,}v_{12},\dots v_{1n,}$ $v_{21,}v_{22},\dots v_{2n,}\dots$ $v_{n-11,}v_{n-12},\dots v_{n-1n,}v_{n1,}v_{n2},\dots v_{nn,}$ shown in the Fig 1. Now partition the vertex set V of $M_{n\times n}$ into three disjoint subsets $V_1=\{v_{1j,}v_{2j},\dots v_{nj}\}, 1\leq j\leq n, V_2=\{v_{2n,}v_{3n},\dots v_{n-1n}\}$and $V_3=\{v_{1n},v_{nn}\}$. Clearly $V=V_1\cup V_2\cup V_3$. .

Define a mapping $f: V(M_{nxn}) \to N$ as follows:

$f(v_{ij}) = 2n-4, \ i=j=1; \ \ i=j=n.$

$f(v_{i1}) = 2n-5+i, \qquad i=2,3,\dots n$

$f(v_{ij}) = 2n-5+n(j-1)+i, \qquad i=1,2,3,\dots n, \qquad j=2,3,\dots n-1.$

$f(v_{in}) = n^2+n+i-6, \qquad i=2,3,\dots n-1.$

$f(v_{i1}) = f(v_{1j}) = 3n-5, \ \ i=n, \ j=n.$

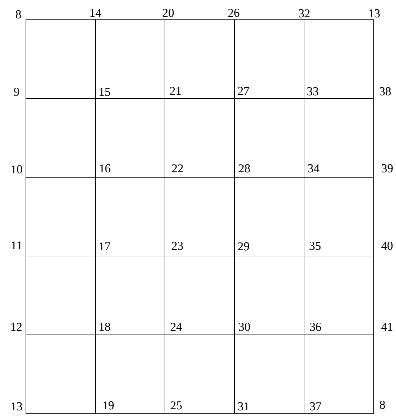

Fig 2. Radio Antipodal Mean number of grid M_{6x6}

Claim: To prove that $f(u) \neq f(v)$.

Case (i): If $v_{ij} \in V$ is not a boundary vertex then it is adjacent to the vertices $v_{i-1j-1}, \ v_{i+1j+1}, \ v_{i-1j,} \ v_{i+1j}$. Since the length of distance between them is one and $d(u,v) < d$.

Clearly the adjacent vertices of v_{ij} which are adjacent to each other does not receive the same map under f.

Case (ii): If $v_{ij} \in V$ is a boundary vertex then it is adjacent to the vertices $v_{i-1j}, \ v_{i+1j}, \ v_{i+1j+1}$ or v_{i+1j}, v_{ij+1}, then the adjacent vertices of v_{ij} which are adjacent to each other does not receive the same map under f. Clearly $f(u) \neq f(v)$, for all $(u,v) \in E(G)$. Hence the given labeling are distinct when $d(u,v) < d$.

Next we claim that $d(u,v) + \left\lceil\frac{f(u)+f(v)}{2}\right\rceil \geq 2(n-1)$ for all $u,v \in V(M_{nxn})$.

Case (iii): Suppose u and w are any two vertices in V_1. Then $u=v_{kl}$ and $w=v_{st}$ for some k,l,s and t where $1\leq k,s\leq n, 2\leq l,t\leq n-1, k\neq s, l\neq t$. Therefore $f(u)=2n-5+n(l-1)+k$, $f(w)=2n-5+n(t-1)+s$ and $d(u,w)\geq 1$.

Hence$d(u,w)+\left\lceil\frac{f(u)+f(v)}{2}\right\rceil \geq 1+\left\lceil\frac{2n-5+n(l-1)+k+2n-5+n(t-1)+s}{2}\right\rceil$

$\geq 2(n-1).$

Case (iv): Suppose u and w are any two vertices in V_2. Then $u=v_{kn}$ and $w=v_{sn}$ for some k and s where$2\leq k,s\leq n-1, k\neq s$. Therefore $f(u)=n^2+n+k-6, f(w)=n^2+n+s-6$ and $d(u,w)\geq 1$.

Hence $d(u,w)+\left\lceil\frac{f(u)+f(v)}{2}\right\rceil \geq 1+\left\lceil\frac{n^2+n+k-6+n^2+n+s-6}{2}\right\rceil$
$\geq 2(n-1).$

Case (v): $u \in V_1, w \in V_3,$

If $u=v_{11}$ and $w=v_{nn}$, then $d(u,w)\geq 2(n-1)$. Therefore $f(u)=2n-4,\ f(w)=2n-4.$

Hence $d(u,w)+\left\lceil\frac{f(u)+f(v)}{2}\right\rceil \geq 2(n-1)1+\left\lceil\frac{2n-8}{2}\right\rceil$
$\geq 2(n-1).$

If $u=v_{kl}$ and $w=v_{nn}$, where $1\leq k\leq n, 2\leq l\leq n-1$, then $d(u,w)\geq 1$ and
$f(u)=n^2+n+k-6, \quad f(w)=2n-4.$

Now $d(u,w)+\left\lceil\frac{f(u)+f(v)}{2}\right\rceil \geq 1+\left\lceil\frac{n^2+n+k-6+2n-4}{2}\right\rceil$
$\geq 2(n-1).$

Similarly, we can prove the other cases.

Hence $d(u,v)+\left\lceil\frac{f(u)+f(v)}{2}\right\rceil \geq 2(n-1).$

3. Radio Antipodal Mean Number of Extended Mesh

Definition 3.1: The mesh $M_{m\times n}$ is defined as the Cartesian product $P_m \times P_n$ of paths. The architecture obtained by making each 4-cycle in $M_{m\times n}$ into a complete graph is called an extended mesh. It is denoted by $EX_{m\times n}$. The number of vertices in $EX_{m\times n}$ is mn and its diameter is $min\{m,n\}-1$. See Fig 3.

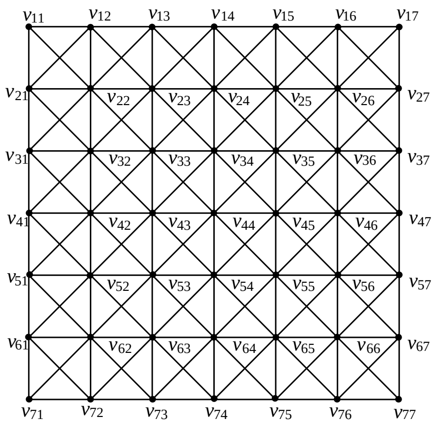

Fig 3. An Extended mesh $EX_{7\times 7}$

Theorem 3.1: Let G be an extended mesh $EX_{n\times n}$. Then the antipodal mean number of $r_{amn}(G) \leq n^2-n-3$.

Proof. Label the vertices of EX_{nxn} from left to right as $v_{11,}v_{12},\dots v_{1n,}\ v_{21,}v_{22},\dots v_{2n,}\dots\ v_{n-11,}v_{n-12},\dots v_{n-1n,}v_{n1,}v_{n2},\dots v_{nn,}$ shown in the Fig 3. Now partition the vertex set V of $EX_{n\times n}$ into three disjoint subsets

$$V_1=\begin{Bmatrix} v_{11,}v_{12},\dots v_{1n-1} \\ v_{21,}v_{22},\dots v_{2n-1} \\ v_{n-11,}v_{n-12},\dots v_{n-1n-1} \end{Bmatrix}, 1\leq i\leq n-1, 1\leq j\leq n-1,$$

$V_2=\{v_{n1,}v_{n2},\dots v_{nn-1}\}$ and $V_3=\{v_{1n},v_{2n},\dots v_{nn}\}$. Clearly $V=V_1\cup V_2\cup V_3$. .

Define a mapping $f: V(EX_{nxn})\to N$ as follows:

$f(v_{ij})=n-3,\ i=1\,j=1;\ \ i=nj=n; i=n\,j=1; i=1\,j=n.$

$f(v_{ij})=n-4+n(j-1)+i, \quad i=1,2,3,\dots n-1, \quad j=1,2,3,\dots n-1.$

The labelingof $f(v_{1j}),\ j=\ 2,3,\dots n-1$, is same as $f(v_{nj}),\ j=\ 2,3,\dots n-1.$

Similarly the labelingof $f(v_{i1}),\ i=\ 2,3,\dots n-1$, is same as $f(v_{in}),\ i=\ 2,3,\dots n-1.$

Conjuture : Radio antipodal mean number the lower bounded attained is $(n-1)^2+n-1$. If we label the vertices with 1 or 2 we will not attain the lower bounded,therefore we start labeling the vertices with $n-3$.

Example

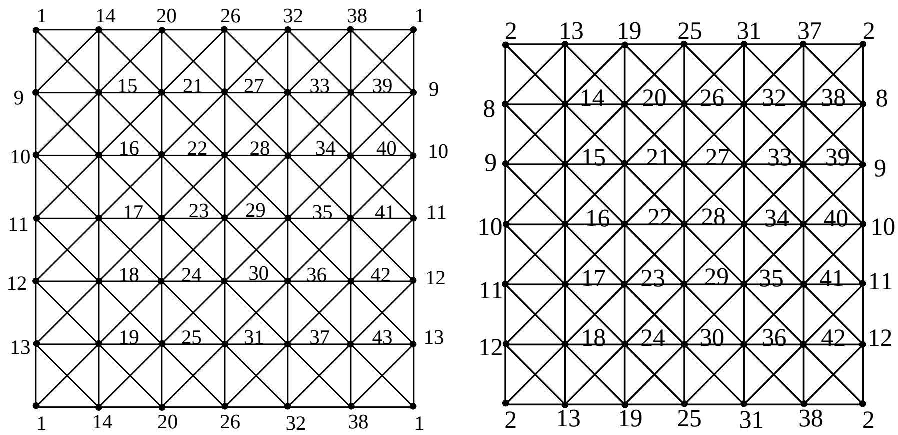

Fig 4. Radio Antipodal mean labeling with 1 and 2 of an Extended mesh $EX_{7\times 7}$

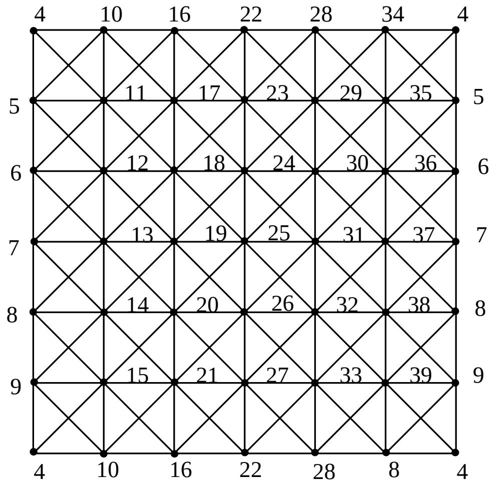

Fig 5. Radio Antipodal mean labeling of an Extended mesh $EX_{7\times 7}$

Proof is similar to the above theorem 2.1.

Conclusion:

In this paper, we introduced a new labeling called radio antipodal mean labeling and weobtainedthe radio antipodal mean number for path, circle, wheel, mesh and its derived architectures. Futher, we investigate the problems in various interconnection networks such as butterfly, benes, honeycomb etc.

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