New Structures of Fuzzy Soft R-Ideal and R-Idealistic Soft Bci-Algebra

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Abstract: Molodtsov [1999] discussed Soft set theory, and introduced the concept of soft set as a new mathematical tool for dealing with uncertainties. Aygünoglu, and Aygün [2009] introduced fuzzy soft group, and verified few algebraic properties. Ahmad and Athar Kharal [2009] found some results on fuzzy soft sets. Aktas and Cagman [2007] investigated usual properties on soft sets and soft group. Kim and Yuan [1995] gave an application on fuzzy soft theory in decision making problems. Jun [2008] analyzed soft BCK/BCI-algebras. In this paper we introduce the notion of soft R-ideals and a-idealistic soft BCI-algebras, and then investigate their basic properties. Using soft sets, we give characterizations of (fuzzy) R-ideals in BCI algebras. We provide relations between fuzzy R-ideals and R-idealistic soft BCI-algebras.

Keywords: Soft set (R-idealistic) soft BCI-algebra, Soft ideal, soft R-ideal

Section - 1 Introduction: Ghosh [1998] introduced fuzzy k-ideals of semi-rings. Mukherjee and Sen [1991] discussed rings with chain conditions for ideals as subrings. Kim and Park [1996] studied on *k*-fuzzy ideals in semi-rings. Molodtsov [1999] and Maji et al. [2002] suggested that one reason for these difficulties may be due to the inadequacy of the parameterization tool of the theory. Maji et al. [2002] described the application of soft set theory to a decision making problem. Maji et al. [2003] also studied several operations on the theory of soft sets. Jun [2008] applied the notion of soft sets by Molodtsov to the theory of BCK/BCI-algebras. He introduced the notion of soft BCK/BCI-algebras and soft sub algebras, and then derived their basic properties.

Jun and Park [2008] dealt with the algebraic structure of BCK/BCI-algebras by applying soft set theory. They discussed the algebraic properties of soft sets in BCK/BCI-algebras. They introduced the notion of soft ideals and idealistic soft BCK/BCI-algebras, and gave several examples. They investigated relations between soft BCK/BCI-algebras and idealistic soft BCK/BCI-algebras.

Tang and Zhang [2001] got results on Q-fuzzy R-submodule by a given arbitrary Q-fuzzy set. They also proved the lattice of all Q-fuzzy R-submodule of a module can be embedded into a lattice of Q-fuzzy R-submodule of the given module, and gave characterization of Q-fuzzy left R-submodule with respect to t-norm.

Kim and Yun [2000] discussed basic algebraic properties on fuzzy R-subgroup of near ring. They further found its homomorphic image & preimage, union, intersection, its power and its primary decomposition. Kim and Jun [2001] studied on the basic algebraic properties of normal fuzzy R-subgroup in near ring. They also explained its normalizer, its conjugate classes, its memberships, the intersection of finite number of fuzzy normal R-subgroups, direct product of two fuzzy normal R-subgroups, Q-fuzzy normal R-subgroup, Q-fuzzy normalizer, and homomorphic images and preimages of all such above fuzzy normal R-subgroups.

In this paper, the notion of soft sets by Molodtsov is applied to R-ideals in BCI-algebras. The notion of soft R-ideals and R-idealistic are introduced in soft BCI-algebras, and then derived their basic properties. Using soft sets, few characterizations of (fuzzy) a-ideals in BCI-algebras are given. Relations between fuzzy R-ideals and R-idealistic soft BCI-algebras are provided.

Section 2 -Basic results on BCI-algebra

A **BCK-algebra** is an algebra (X; *, 0) of type (2, 0) satisfying the following axioms:(1) ((x * y) *(x*z)*(z*y))=0; (2) (x*(x*y))*y) =0; (3) x*x =0; (4) x*y = 0 and y*x = 0 imply x = yfor all x, y, z   X.

If a BCI-algebra X satisfies the following identity: (5) 0 * x = 0 for all in X, then X is called a **BCK-algebra**. In any BCK/BCI-algebra X one can define a partial order by putting x ≤ y if and only if x *y = 0.Every BCK/BCI-algebra's Xsatisfies(x * y) * z = (x * z) *y for all x, y, z in X

A non-empty subset S of a BCI-algebra X is called **sub algebra** of X if x * y in S, for all x, y in S.

A subset H of a BCI-algebra X is called **an ideal of X** if it satisfies the following axioms: (I1) $0 \in H$, (I2) $\forall \ x \in X, y \ \in H$ and $x * y \in H \rightarrow x \in H$.

Any ideal H of a BCI-algebra X satisfies the following implication: $\forall x \in X, \forall y \in H \ \& x \leq y \rightarrow x \in H$. A subset H of a BCI-algebra X is called an **a-ideal** of X if it satisfies (I1) and (I3), $\forall \ x \in X, z \in X, y \in H$ and $(x * z) *(0 * y) \in H \rightarrow x *y \in H$. It knows that every a-ideal of a BCI-algebra X is also an ideal of X.

Section 3 – Basic definitions on soft sets

Definition 3.1: Let U is an initial universe set and E is a set of parameters. Let P (U) denotes the power set of U and A subset E. A pair (F, A) is called a soft set over U, where F is a mapping given by F: A → P (U). In other words, a soft set over U is a parameterized family of subsets of the universe U. For $\varepsilon \in A$, F (ε) may be considered as the set of ε-approximate elements of the soft set (F, A). Clearly, a soft set is not a set. For illustration, Molodtsov considered several examples in [1].

Definition 3.2: Let (F, A) and (G, B) be two soft sets over a common universe U. The intersection of (F, A) and (G, B) is defined to be the soft set (H, C) satisfying the following conditions: (i)$C = A \cap B$, (ii) for all$e \in C$, H (e)=F (e) or G (e), (as both are same sets). In this case, we write $(F, A) \cap (G, B) = (H, C)$.

Definition 3.3: Let (F, A) and (G, B) be two soft sets over a common universe U. The intersection of (F, A) and (G, B) is defined to be the soft set (H, C) satisfying the following conditions: (i) $C = A \cup B$, (ii) $\forall e \in C$, H (e) = F (e), if $e \in A \setminus B$, G (e), if $e \in B \setminus A$, $F (e) \cup G (e)$, if $e \in A \cap B$. In this case, write this as $(F, A) \cup (G, B) = (H, C)$.

Definition 3.4: If (F, A) and (G, B) are two soft sets over a common universe U, then (F, A) AND (G , B), denoted by (F, A)v^(G , B) is defined by (F, A)v^(G , B) = (H , (A × B), where $H(\alpha, \beta) = F(\alpha) \cap G(\beta)$, $\forall (\alpha, \beta) \in A \times B$.

Definition 3.5: If (F, A) and (G , B) are two soft sets over a common universe U, then ``(F, A) OR (G , B)" denoted by (F, A)v^(G ,) is defined by (F, A)v^(G , B)= (H, A × B), where $H(\alpha, \beta)= F(\alpha) \cup G(\beta)$, $\forall (\alpha, \beta) \in A \times B$.

Definition 3.6: For two soft sets (F, A) and (G, B) over a common universe U, we say that (F, A) is a soft subset of (G, B) denoted by $(F, A) \subset (G, B)$, if it satisfies: (i) $A \subset B$, (ii) for every "$\varepsilon \in A$, F (ε) and G(ε) are identical approximations.

Section 4 - Soft R-ideal

Let X and A be a BCI-algebra and a nonempty set, respectively, and R will refer to an arbitrary binary relation between an element of A and an element of X, that is, a is a subset of A× X without otherwise specified. A set valued function F: A →P (U) can be defined as F(x) = {y in X :(x, y) in R} for all $x \in A$. The pair (F, A) then a soft set over X

Definition 4.1: Let S is sub algebra of X. A subset I of X is called an ideal of X related to S (S-ideal of X), denoted by I Δ S, if it satisfies: (i) $0 \in I$, (ii) $\forall \ x \in S, \forall \ y \in I, (x * y) \in I \rightarrow x \in I$.

Definition4.2: Let S is sub algebra of X. A subset I of X is called a R-ideal of X related to S denoted by $I\Delta_R S$, if it satisfies: (i) $0 \in I$, (ii) $\forall x, z \in S, \forall y$ in I, (x * z) * (z* y)in I impliesx*z$\in$I.

Example4.3(R-ideal related S): Let X= {0,1,2, a, b} be a BCI-algebra with the following Cayley table:

*	0	1	2	a	b
0	0	1	2	a	b
1	1	0	2	a	b
2	2	1	0	b	b
a	a	b	b	0	0
b	b	b	a	0	0

Then S = {0, a, b} is sub-algebra of X, and I_1= {0} and I_2= {0, 1} are R-ideal of X related to S.They are also S-ideals of X. Note that **every R-ideal of X related to S is an S-ideal of X in BCK-algebra.**

Definition 4.4: Let (F, A) be a soft set over X. Then (F, A) is called a soft BCI-algebra over X if F(x) is sub algebra of X for all x in A.

Definition 4.5: Let (F, A) be a soft BCI-algebra over X. A soft set (G,I) over X is called a soft ideal of (F,A) denoted by (G,I) Δ (F,A), if it satisfies: (i)I $\subset$ A, (ii) For all x in I, G(x) Δ F(x).

Definition4.6: Let (F,A) be a soft BCI-algebra over X. A soft set (G,I) over X is called a soft R-ideal of (F,A), denoted by (G,I) Δ_R (F,A) if it satisfies: (i)I $\subset$ A, (ii)forall x in I, G (x) Δ_R F (x).

Let us illustrate this definition using the following example.

Example 4.7 (soft R-ideal): Consider a BCI-algebra X = {0, 1, 2, a, b} which is given in (4.3). Let (F,A) be a soft set over X, where A = {0, 1, 2, a}$\subset$ X and F: A → P (U) is a set-valued function defined by F(x) ={yin X: y*(y * x) in {0, 1} for all x in A.

Then F (0) = F (1) =X, F (2) = {0, 1, a, b} and F (a) = f (0), which are sub algebras of X. Hence (F, A) is as oft BCI-algebra over X. Let I = {0, 1, 2}$\subset$ A and G: I→ P (U) be a set-valued function defined by G (x) = Z [{0, 1}], &if x = 2,[0] if x in {0, 1}where Z(0, 1) = { x in X :0*(0 * x) in {0, 1}}$ Then G(0) Δ_R F(0), G(1) Δ_R F(1) and G(2) Δ_R F(2). Hence (G, I) is a soft R-ideal of (F, A).

Note that every soft R-ideal is a soft ideal. But the converse is not true as seen in the following example.

Example 4.8 (Soft ideal but nor soft R-ideal): Let X = {0, a, b, c, d} be a BCK-algebra, and hence a BCI-algebra, with the following Cayley table:

c	0	a	b	c	d
0	0	0	0	0	0
a	a	0	a	a	0
b	b	b	0	b	0
c	c	c	c	0	0
d	d	d	d	d	0

For A = X, define a set-valued function F: A → P(X) by F(x) = {y $\in$ X: y * (y*x) $\in$ {a, 0}}, for all x $\in$ A. Then (F, A) is a soft BCI-algebra over X (see [16]). (1)Let (G, I) be a soft set over X, where I = {a, b, c, d} andG: I → P(X) is a set-value function defined by G(x) ={y $\in$ X: y*(y*x) $\in$ {0, d}}, for all x $\in$ I. Then G(a)= {0, b, c, d} ΔX = F(a), G(b)={0, a, c, d} Δ{0, a, c, d}= F(b) and G (c)={0, a, b, d}Δ{ 0, a, b, d}= F(c), G(d) = {0, a ,b, d} Δ {0, a, b, c} F(d). Hence (G, I) is a soft ideal of (F,A). But (G,I) is not a soft R-ideal of (F,A), since (a *a)*(a*a) = 0 in G (a) and a$\notin$ G (a).

(2) For I = {a, b, c, d}, let H: I → P(X) be a set-valued function defined by H(x) = {0} $\cup${y $\in$ X: x $\le$ y}, for all x $\in$ I. Then H(a)= {0, a} Δ X = F(a),H(b)={0, b}Δ{0, a, c, d}= F(b) and H(c)={0, c} Δ{0, a, b, d}= F(c) ,H(d)={0, d} Δ {0, a, b, c} =F(d). Therefore (H, I) is a soft ideal of (F,A). But. (H, I) is not a soft R-ideal of (F, A) since. Since (b *b)*(b*b) = 0 in H (a) and b$\notin$ H (a).

Theorem 4.9: Let (F, A) be a soft BCI-algebra over X. Then (G_1, I_1) Δ_R (F, A), (G_2, I_2) ΔR(F, A) $\Rightarrow$$(G_1, I_1)$$\cap$ (G_2, I_2) Δ_R (F, A) For any soft sets (G_1, I) and (G_2, I) over X.

Proof: Using (3.2), write thus as $(G_1, I_1)$$\Delta$ (G_2, I_2)= (G , I), where I = $I_1 \Delta I_2$ and G(x)= $G_1(x)$ or $G_2(x)$ for all x $\in$ I. Obviously, I$\subset$A and G : I → P(X) is a mapping. Hence (G, I) is a soft set over X. Since (G_1, I_1) Δ_R(F, A) and (G_2, I_2) Δ_R (F, A), itknows that G (x) = $G_1(x)$ Δ_R F(x) or G (x) = $G_2(x)$ Δ_RF(x) for all x $\in$ I. Hence (G_1, I_1) Δ (G_2, I_2) = (G, I) Δ_R (F, A). This completes the proof.

Corollary 4.10: Let (F, A) be a soft BCI-algebra over X. For any soft sets (G, I) and (H, I) over X, it follows that (G, I) Δ_R (F,A), (H, I) Δ_R (F, A) → (G, I) $\cap$ (H, I) Δ_R(F, A)

Proof: Straightforward.

Theorem4.11: Let (F, A) be a soft BCI-algebra over X,for any soft sets (G, I) and (H, J) over X in which I and J are disjoint, we have (G , I) Δ_R(F, A), (H , J) Δ_R (F, A) →(G , I) (H , J) Δ_R(F, A).

Proof: Assume that (G, I) Δ_R (F, A) and (H, J) Δ_R (F, A). By (3.3), write thus as (G, I) $\cup$ (H, J) = (K, U), where U = I $\cup$J and for every x $\in$ U.K(x) = G (x), if x $\in$ I\ J; H (x),if x $\in$ J \ I; G (x) $\cup$ H (x), if x$\in$ I $\cap$ J. Since I$\cap$ J =0; either x $\in$ I \ J or x $\in$J \ I for all x$\in$ U.

If $x \in I \setminus J$, then $H(x) = G(x)\ \Delta_R = F(x)$ since$(G,I)\Delta_R (F,A)$. If $x \in J\setminus I$, then $K(x) = H(x)\ \Delta_R\ F(x)$, since. $(H, J)\Delta_R (F, A)$. Thus $H(x)\Delta_R F(x)$ for all $x \in U$, and $(G, I)\ \Delta\ (H, J)=(H , U)\Delta_R(F, A)$.

Example 4.12: If I and J are not disjoint in (4.11), the conclusion of the result (4.11) is not true. Let (F, A) be a soft BCI-algebra over X, for any soft sets (G, I) and (H, J) over X in which I and J are not disjoint, then $(G , I)\ \Delta_R\ (F, A)$, $(H , J)\ \Delta_R\ (F, A)$ does not imply $(G , I) \cup (H , J)\ \Delta_R\ (F, A)$ explained in the following example. Let X = {0, 1, a, b, c} be a BCI-algebra with the following Cayley table:

*	0	1	a	b	c
0	0	0	a	b	c
1	1	0	a	b	c
a	a	a	0	a	a
b	b	b	a	0	a
c	c	c	a	a	0

For $A = \{0, 1\} \subset X$, let $F: A \to P(X)$ be a set-valued function defined by$F(x) = \{y \in\ x: y *x =y\}$, for all$x \in A$. Then F (0) = X and F (1) = {0, a, b, c}, which are sub algebras of X, and hence (F, A) is a soft BCI-algebra over X. If we take I = A and define a set-valued function $G: I \to P(X)$ by $G(x) = \{y\in X: x *(x *y) \in \{0, b\}\}$, for all $x \in I$, then we obtain that $G(0)= \{0, 1, b\}\Delta_R F(0)$ and$G(1)=\{ 0, 1, b\}\ \Delta_R\ F(1)$, This means that $(G , I)\Delta_R(F, A)$.

Now, consider J = {0} which is not disjoint with I, and let $H: J \to P(X)$ be a set-valued function defined by $H(x) =\{y\in X: x *(x *y) \in \{0, c\}\}$, for all $x\in J$. Then $H(0) = \{0, 1, c\}\ \Delta_R = F(0)$, and so $(H, J)\ \Delta_R\ (F, A)$. But if $(H , U) = (G , I) \cup (H , J)$, then $H(0)= G(0) \cup H(0)= \{0, 1, b, c\}$, which is not a R-ideal of X related to F(0) since (a *0)*(b*0)= c in H(0) and $a \in H(0)$.Thus $(H, U)= (G , I) \cup (H , J)$ is not a soft R-ideal of (F, A).

Section 5 -R-idealistic soft BCI-algebra:

Definition 5.1: Let (F, A) be a soft set over X. Then (F, A) is called an idealistic soft BCI-algebra over X if F(x) is an ideal of X for all $x\in A$.

Definition 5.2: Let (F,A) be a soft set over X. Then (F, A) is called an a-idealistic soft BCI-algebra over X if F(x) is a R-ideal of X for all $x\in A$.

Example 5.3 (R-idealistic soft BCI-algebra): Consider a BCI-algebra X ={0, 1, 2, a, b} which is given in Example 4.3. Let (F, A) be a soft set over X, where A = X and $F: A\to P(X)$ is a set-valued function defined by$F(x) = Z\{0, 1\}$, if $x\in\{2, a, b\}$,X, if $x\in \{0, 1\}$,where $Z\{0, 1\} =\{x \in X: 0 *(0 *x) \in \{0, 1\}\}$. Then (F, A) is an R-idealistic soft BCI-algebra over X. For any element x of a BCI-algebra X, we define the order of x is o(x)=min { $n\in N$:0*x^{n} = 0},where 0*x^{n} =(. . . {0 *x}*x . . .) *x in which x appear n-times

Example 5.4 (not R-idealistic soft BCI-algebra): Let X ={0, a, b, c, d, e, f, g} and consider the following Cayley table:

*	0	a	B	c	d	e	f	g
0	0	0	0	0	e	e	e	e
a	a	0	0	0	f	e	e	e
b	b	b	0	0	g	f	e	e
c	c	b	A	0	d	g	f	e
d	d	e	E	e	0	0	0	0
e	e	f	E	e	a	0	0	0
f	f	g	C	e	b	a	0	0
g	g	f	D	e	c	b	a	0

Then (x;*, 0) is a BCI-algebra. Let (F, A) be a soft set over X, where $A =\{a, b, c\}\subset X$and $F: A \to P(X)$ is a set-valued function defined as follows. $F(x) = \{y \in X: o(x) = o(y)\}$, for all $x \in A$. ThenF (a) = F (b) = F(c) = {0, a, b, c}is an R-ideal of X. Hence (F, A) is an R-idealistic soft BCI-algebra over X. But, if we take $B = \{a ,b ,d, f\}\subset X$

and define a set-valued function G: B → P(X) by G (x)= {0}∪{ y ∈ X: o(x)= o(y)}, ∀x ∈ B, then (G , B) is not a R-idealistic soft BCI-algebra over X since G (d)={0, d, e, f, g} is not a R-ideal of X.

Example 5.5 (R-idealistic soft BCI-algebra): Consider a BCI-algebra X = {0, a, b, c} with the following Cayley table:

*	0	A	b	c
0	0	A	b	c
a	a	0	c	b
b	b	C	0	a
c	c	B	a	0

LetA=X and F: A →P(X) is a set-valued function defined as follows F(x) = {0, x}, for all x in A. Then F(0)=(0);F(a)={0, a};F(b)= {0, b} and F(c)={0, c} which are ideals of X . Hence (F, A) is an idealistic soft BCI-algebra over X (see [17]). Note that F(x) is a a-ideal of X for all x ∈A. Hence (F, A) is a R-idealistic soft-BCI-algebra over X. Obviously, every R-idealistic soft BCI-algebra over X is an idealistic soft BCI-algebra over X, but the converse is not true in general as seen in the following example

Example 5.6 (idealistic soft BCI-algebra, but not R-idealistic soft BCI-algebra): Consider a BCI-algebra X= Y x Z, where {Y, *, 0} is a BCI-algebra and (Z, -, 0) is the ad joint BCI-algebra of the additive group (Z,+,0) of integers. Let F: X → P(X) be a set-valued function defined as follows f{y, n} = Y× N_0, if n in N_0,{0, 0}, otherwise ,∀(y, n)∈X, where N_0 is the set of all non-negative integers. Then (F, X) is an idealistic soft BCI-algebra over X (see [17]).But it is not an R-idealistic soft BCI-algebra over X since {(0,0)} may not be an R-ideal of X.

Theorem 5.7: Let (F, A) and (F, B) be soft sets over X where B ⊂ A ⊂ X. If (F, A) is an R-idealistic soft BCI-algebra over X, then so is (F, B).

Proof: Straightforward.

The converse of (5.7) is not true in general as seen in the following example.

Example 5.8: Let (F, A) and (F, B) be soft sets over X where B ⊂ A ⊂ X. If (F, B) is an R-idealistic soft BCI-algebra over X, then so is not (F, A).Consider anR-idealistic soft BCI-algebra (F, A) over X which is described in(5.4). If we take B = {a, b,c, d} ⊇ A, then (F, B) is not a R-idealistic soft BCI-algebra over X since F (d) = {d, e, f, g} is not a R-ideal of X.

Theorem 5.9: Let (F, A) and (G, B) be two R-idealistic soft BCI-algebras over X. If A ∩ B ≠0, then the intersection (F, A) ∩ (G, B) is an R-idealistic soft BCI-algebra over X.

Proof: Using (3.2), we can write (F, A) ∩ (G, B) = (H, C), where C = A∩ B and H (x) = F(x) or G (x) for all x ∈ C. Note that H: C → P(X) is a mapping, and therefore (H, C) is a soft set over X. Since (F, A) and (G, B) are a-idealistic soft BCI-algebras over X, it follows that H (x) = F(x) is an R-ideal of X, or H (x) = G (x) is an R-ideal of X for all x ∈C. Hence (H, C) = (F, A) ∩(G, B) isR-idealistic soft BCI-algebra over X.

Corollary 5.10: Let (F, A) and (G, A) be two R-idealistic soft BCI-algebras over X. Then their intersection (F, A)∩(G, A) is an R-idealistic soft BCI-algebra over X.

Proof: Straightforward.

Theorem 5.11: Let (F, A) and (G, B) be two R-idealistic soft BCI-algebras over X. If A and B are disjoint, then the union (F, A)∪ (G, B) is an R-idealistic soft BCI-algebra over X.

Proof: Using (3.3), write this as (F, A) ∪ (G, B) = (H, C), where C = A ∪B and for every x∈C,

H (x) = F(x), if x ∈ A\ B, G(x),if x ∈ B\A,F(x) ∪G (x), if x ∈A ∩ B

Since A ∩ B =0; either x ∈ A\ B or x ∈B\ A for all x ∈ C. If x∈ A \ B, then H (x) = F(x) is an R-ideal of X since (F, A) is anR-idealistic soft BCI-algebra over X. If x ∈ B \ A, then H (x) = G(x) is an R-ideal of X since (G, B) is an R-idealistic soft BCI-algebra over X. Hence. (H, C) = (F, A)∪ (G, B) is an R-idealistic soft BCI-algebra over X.

Theorem 5.12: If (F, A) and (G, B) are R-idealistic soft BCI-algebras over X, then (F, A) ∩ (G, B) is an R-idealistic soft BCI-algebra over X.

Proof: By (3.4), it knows that (F, A) Λ (G, B) = {H, A x B}, where H (x, y) = F(x)∩ G (y) for all (x, y) ∈ A x B. Since F(x) and G (y) are R-ideals of X, the intersection F(x)∩ G (y) is also anR-ideal of X. Hence H (x, y) is an R-ideal of X for all (x, y) ∈Ax B, and therefore (F, A)Λ (G, B) =(H , Ax B) is an R-idealistic soft BCI-algebra over X .

Definition 5.13: A R-idealistic soft BCI-algebra (F, A) over X is said to be trivial (resp., whole) if F(x) ={0} (resp., F(x) = X) for all x∈A.

Example 5.14(Trivial R-idealistic soft BCI-algebra and whole R-idealistic soft BCI-algebra):Let X be a BCI-algebra which is given in (5.5), and let F: X →P(X) be a set-valued function defined by F(x) = {0} ∪{y ∈ X: o(x) = o(y)}; for all x ∈ X. Then F (0) = {0}and F (a) = F (b) = F(c) = X. We can check that {0} Δ_RX and X Δ_R X. Hence (F,{0}) is a trivial R-idealistic soft BCI-algebra over X and(F, X\ {0} is a whole R-idealistic soft BCI-algebra over X. The proofs of the following three lemmas are straight forward, so they are omitted.

Lemma 5.15: Let f: X → Y is an onto homomorphism of BCI-algebras. If I is an ideal of X, then f (I) is an ideal of Y.
Lemma 5.16: Let f: X→Y is an isomorphism of BCI-algebras. If I is an R-ideal of X, then f (I) is an R-ideal of Y. Let f: X →Y is a mapping of BCI-algebras. For a soft set (F, A) over X, (f (F), A) is a soft set over Y where f (F): A →P(Y) is defined by f (F) (x) = f (F(x)) for all x ∈A.

Lemma 5.17: Let f: X → Y is an isomorphism of BCI-algebras. If (F, A) is an R-idealistic soft BCI-algebra μover X, then (f (F), A) is an R-idealistic soft BCI-algebra over Y.

Theorem 5.18: Let f: X → Y is an isomorphism of BCI-algebras and let (F, A) be an R-idealistic soft BCI-algebra over X.(1) If F(x) ⊆kern (f) for all x ∈A, then (f (F), A) is a trivial R-idealistic soft BCI-algebra over Y.(2) If (F, A) is whole, then (f (F), A) is a whole R-idealistic soft BCI-algebra over Y.
Proof: (1) Assume that F(x) ⊆kern (f) for all x ∈A. Then f (F) (x) = f (F(x)) = {0y} for all x ∈A. Hence (f (F), A) is a trivial R-idealistic soft BCI-algebra over Y by (5.17)and (5.13); (2) Suppose that (F, A) is whole. Then F(x) = X,∀x ∈A, and so {f (F) x) = f (F(x)) = f(X) = Y,∀x ∈ A. It follows from (5.17) and (5.13) that (f (F), A) is a whole R-idealistic soft BCI-algebra over Y.

Definition 5.19: A fuzzyμin X is a fuzzy R-ideal of X if it satisfies the following assertions:
(i)(∀x ∈X) (μ(0) ≥μ(x), (ii) (∀x, y, z ∈ X) (μ(x *z) ≥min {μ{(x * z)*(z *y)), μ(y)})

Lemma 5.20: .A fuzzy set μ in X is a fuzzy R-ideal of X if and only if it satisfies:(∀ t ∈[0, 1])(U (μ ; t) ≠0⇒U (μ; t) is a R-ideal of X)

Theorem 5.21: There exists anR-idealistic soft BCI-algebra (F,A) over X for every fuzzy a-ideal μof X.
Proof: Letμbe a fuzzy R-ideal of X. Then U(μ; t)={x∈X : μ(x) ≥ t} is an R-ideal of X for all t∈Im(μ).
If we take A = Im (μ) and consider a set-valued function F: A → P(X) given by$ F (t) = U (μ; t) for all t∈A, then F (f,A) isan R-idealistic soft BCI-algebra over X.

Conversely, the following theorem is straightforward.
Theorem 5.22: For any fuzzy set μ in X, if a a-idealistic soft BCI-algebra (F, A) over X is given by A = Im (μ) and F (t) = U (μ; t),∀t ∈ A, then is a fuzzy R-ideal of X.
Proof: Let μ be a fuzzy set in X and (F, A) be a soft set over X in which A = Im (μ) and F: A → P(X) is a set-valued function defined by (∀ t ∈ A) (F (t) ={x∈ X: |μ(x) + t > 1}.Then there exists t∈ A such that F (t) is not an R-ideal of X as seen in the following example.

Example 5.23 (non - R-ideal): For any BCI-algebra X, define a fuzzy set μ in X by μ(0) = t_0< 0.5 andμ(x) =1 – t_0 for all x ≠ 0. Let A = Im (μ) and F: A→ P(X) is a set-valued function given by (5.2). Then F(1 – t_0) = X \{0}, which is not R-ideal of X.

Theorem 5.24: Let μ be a fuzzy set in X and let (F, A) be a soft set over X in which A = [0, 1] andF: A → P(X) is given by (5.2). Then the following assertions are equivalent:(1) μ is a fuzzy R-ideal of X,(2) For every t ∈A with F (t)≠ 0, F (t) is anR-ideal of X.
Proof: Assume thatμ is a fuzzy R-ideal of X. Let t ∈ A be such that F (t) ≠0. If we select x ∈F (t), then μ(0) + t ≥μ(x) +t > 1 and so 0 ∈F (t). Let t ∈ A and x, y, z ∈A be such that y ∈F (t) and(x*z)*(z *y)∈F (t). Then μ (y) +t> 1

and μ ((x *z) *(z *y)) t > 1. Since μ is a fuzzy R-ideal of X, it follows that μ (x) + t ≥ min {μ((x *z)*(z *y)), μ (y)} + t= min {μ((x *z)*(z *y)) +t, μ(y) + t}>1.
So that x∈ F (t), Hence F (t) is anR-ideal of X for all t∈ A with F (t)≠ 0.

Conversely, suppose that (2) is valid. If there exists an∈ X such that μ(0) <μ(a), then we can select t_a ∈ A such that μ(0) +t_a ≤ 1 <μ(a) +t_a. It follows that a∈F (t_a) and 0 ≠F (t_a), which is a contradiction. Hence μ(0)≥μ (x), ∀x ∈X. Now, assume that μ (a) < min {μ((a *c)*(b *c)), μ (b)}, for some a, b, c in X. Then μ(a) +S_0 ≤ 1< min μ ((a *c)*(c *b)), μ(b)} +$S_0$for some S_0∈A, which implies that (a *c)*(c*b) ∈F (S_0) and b ∈F (S_0), but a∉F (S_0). This is a contradiction. Therefore μ(x*c) ≥ min {μ((x *z)*(z *y)), μ(y)}, for all x, y, z ∈ X, and thusμ is a fuzzy R-ideal of X.

Corollary 5.25: Let μ be a fuzzy set in X such thatμ (x) > 0.5 for some x ∈ X, and let (F, A) be a soft set over X in which A= {t ∈ Im(μ)|t > 0.5} and F: A→ P(X)$ is given by (5.2). If μ is a fuzzy R-ideal of X, then (F, A) is anR-idealistic soft BCI-algebra over X.
Proof: Straightforward

Theorem 5.26: Let μ be a fuzzy set in X and let (F, A) be a soft set over X in which A = (0.5, 1] andF: A → P(X) is defined by (∀ t ∈ A) (F (t) = U (μ; t)). Then F (t) isanR-ideal of X for all t ∈ A with F (t)≠0 if and only if the following assertions are valid X.(1)(∀ x ∈ X)(Max {μ(0), 0.5} ≥μ(x)); and (2) (∀x, y, z ∈ X) (max {μ(x), 0.5} ≥min {μ {(x*z)*(z *y), μ(y)}).
Proof: Assume that F (t) is anR-ideal of X for all t ∈ A with F (t) ≠0. If there exists X_0∈X such that max { μ(0),0.5} <μ(X_0),then we can select t_0∈ A such that max{ μ(0), 0.5} < t_0≤μ(X_0) . It follows that μ(0) <t_0 so that X_0∈F (t_0) and 0∉F (t_0).This is a contradiction, and so (1) is valid. Suppose that there exist a, b, c ∈ X such thatmax {μ(a), 0.5} < min (μ(a *c)*(b *c)), μ(b)}.Thenmax {μ(a), 0.5} < u_0≤ Min {μ((a *c)*(c *b)), μ(b)}, forsome u_0∈ A. Thus (a *c)*(c *b)∈ F (u_0) and b ∈ F (u_0) but a ∉ F (u_0). This is a contradiction, and so (2) is valid.

Conversely, suppose that (1) and (2) are valid. Let t ∈ A with F (t) ≠0, for any x ∈ F (t), we have Max {μ(0), 0.5} ≥μ(x) ≥ t > 0.5and so μ(0)≥ t, (ie) 0 ∈ F (t). Let x, y, z ∈X be such that y ∈ F (t) and (x *z)*(z *y)∈ F (t). Thenμ(y) ≥ t and μ(x *z)*(z* y))>t. It follows from the second condition that Max { μ(x), 0.5}≥min { μ((x * z)*(z * y)), μ(y)}≥ t > 0.5 , so that μ(x) ≥t, i.e., x ∈ F (t). Therefore F(t) is anR-ideal of X,∀t ∈ Awith F (t)≠ 0.

References:

01. Aygünoglu, A., Aygün, H., Introduction to fuzzy soft groups, Computers and Mathematics with Applications 58,(2009) 1279_1286
02. Ahmad, B., and Athar Kharal, On Fuzzy Soft Sets, Advances in Fuzzy Systems, (2009).
03. Aktas. H and Cagman, N., Soft sets and soft group, Information Science, Volume 177, (2007), 2726-2735.
04. Ghosh, S., Fuzzy k-ideals of semirings, Fuzzy sets and systems, 95(1998),103-108.
05. Jun, Y.B., Soft BCK/BCI-algebras, Compute. Math, Appl. 56 (2008) 1408-1413.
06. Jun, Y.B., and Park, C.H., Applications of soft sets in ideal theory of BCK/BCI-algebras, Inform, Sci. 178 (2008) 2466-2475
07. Kim, C.B., and Park, M., *k*-fuzzy ideals in semirings, Fuzzy sets and systems, Volume 81, (1996),281-286.
08. Klir, G.J., and Yuan, B., Fuzzy sets and fuzzy logic theory and applications, PretticeHelltic, New Jersey (1995).
09. Maji, P.K., Roy, A.R., and Biswas,R., An application of soft sets in a decision making problem, Compute. Math. Appl. 44 (2002) 1077-1083
10. Maji,P.K.,Roy,A.R.,and Biswas,R., Soft set theory, Compute.Math.Appl.45 (2003), 555-562.
11. Molodtsov, D., Soft set theory - First results, Compute. Math, Appl. 37 (1999) 19-31.
12. Mukherjee, T.K., and Sen, M.K., Rings with chain conditions, Fuzzy sets and systems, Volume 39, (1991),117-123.
13. Tang, J., and Zhang, X., Product operations in the category of L-fuzzy groups, Journal of Fuzzy Math. Volume 9, (2001), 1-10.
14. Zadeh, L.A., Fuzzy sets, Inform. Control, Volume 8, (1965), 338-353.