Some New Results of the Differentiation in the Groups Mod-n

AtefaJalilSalh Abdullah

Mustansiriyah University, College of Basic Education, Dept. of Math.

Abstract *In [1] by H. M. A. Abdullah ,Atefa J. S. Abdullah and Rawaa. E. E. Ibrahim, it was showna new results of the integrals in the group Z_n , by giving a new definition of the integrals of all elements of Z_n (n≥2) .In this paper, we shall study a new properties of the definition of the derivativeof all cyclic elements in Z_n .*

K**eywords**: *Cyclic elements, neat subgroup, derivative subgroup***.**

Introduction

Let G= Z_n be abelian group Mod n , n≥2 . we shall start with the definition of the integrals.

Definition A: see [1]The integrals in the group Z_n define by

$\int_o^k \bar{x} d Z_n = \bar{x} Z_n|_0^k = k\bar{x} \forall n \geq 2$ k= 0, I, 2, …….., n-1 $\int_o^k \bar{x} dZ_n = \{0\bar{x} , 1\bar{x} , \ldots\ldots (n-1)\bar{x}\}$

we can give general definition of the integrals by the following:

$$\int \bar{x} d Zn = \begin{cases} Z_n & \times = p \text{ and } \quad n\backslash p \\ < \bar{x} > & \times = p \text{ and } n\backslash p \text{ or } x \neq p \end{cases}$$

Example 1 :Take $(Z_8 , +_8)$, so

$\int_o^k \bar{3}\, dZ_8 = \int_o^7 \bar{3}\, dZ_8 = Z_8$

$\int_o^k \bar{2}\, dZ_8 = \int_o^7 \bar{4}\, dZ_8 = < \bar{2} >$

$\int_o^k \bar{5}\, dZ_8 = Z_8$

Now, we are ready to give the new definition of the derivative of cyclic elements in Z_n .

Definition B : Let G = Z_n be a group mod n , then the derivative of any generated (cyclic) element of Z_n , define as the following:

d(< $\bar{0}$ >) =$\bar{0}$

d(< $\bar{p}$ >) = p ∀p , p is aprime number

$$d(< Zn >) = \begin{cases} 1 & or \\ p & \quad n \times p \end{cases} \quad , n \in Z^+ \quad n \neq p$$

d(Zp)= all the non – Zero elements of Z p .

d(< $\bar{n}$ >)=$\bar{n}$ n≥2

we can show that by the following example

Example 2:- Take $(Z_6 , +_6)$

so d (Z_6) = $\begin{cases} 1 & \\ & p \quad\quad n \times p \end{cases}$

Thus , d (Z_6) = $\{1 \quad \bar{5} \quad 6 \times \bar{5}$

d($< \bar{2} >$)$\bar{2}$, d($< \bar{3} >$) = $\bar{3}$

and d ($< \bar{0} >$) =$\bar{o}$

Example 3:- Take $(Z_4, +_4)$

d(Z_4) =$\{1 \quad \bar{3} \quad 4 \times \bar{3}$

d($< \bar{2} >$) = $\bar{2}$ and d ($< \bar{0} >$) =$\bar{0}$

clearly we have only $< \bar{0} >$, $< \bar{2} >$are two cyclic subgroups of Z_4 .

Definition C: The derivative element of the subgroup H of G = Z_n is said to be called (neat – derivative)element ,∀ prime number p ,∀ h ≠ 0∈ H

If d (h) = p^x , for some x ∈G ,then d (h) = ph_o = h for some h_o∈H.we shall say a subgroup H of Z_n is neat – derivative , if (∀p) p is prime number and∀h∈ H , x∈Z_n,if d (h) = p^x in G then p (h) = h_o , h_o∈H.

Example 4 : Take G= Z_6 , and H= $< \bar{3} >$, H= $\{\bar{0} , \bar{3}\}$.

We can show that , the all elements of H (we have only $\bar{3}$ ∈ H) is neat derivativeTake p=2

d($\bar{3}$) = 2$\bar{x}$ = $\bar{3}\bar{x}$ ∉Z_6

so Take p=3

d($\bar{3}$) = 3$\bar{x}$ = 3$\bar{1}$ in Z_6

To show d ($\bar{3}$) = ph_o = 3h_o h_o∈ H

d ($\bar{3}$) = 3 . $\bar{3}$ ∈ H

Take p= 5

clearly(d($\bar{3}$) = 5X in G)

d($\bar{3}$) = 5.$\bar{3}$ = $\bar{3}$ in H

thus ,∀p , p >2 ∀h∈ H

If d(h) = p^x in G, then d(h) = ph_o = h in H.Therefore H is derivative – neat in G

We shall denotedby H, the p- derivative neat subgroup of G.

Example 5:- Take G= Z_{12} and H_0=$< \bar{2} >$

Take p=2 and $\bar{2}$ ∈$< \bar{2} >$

sod($\bar{2}$) = 2.$\bar{1}$ = $\bar{2}$ in G

But then is no element h_o∈H ∋ d ($\bar{2}$) ≠ 2 h_oTherefore, H=$< \bar{2} >$ is not p – derivative neat in G .

Definition E: A subgroup H of the Z_n is said to be p_o – derivative neat in G if for some prime number p_o , and ∀h∈ H , d(h) = p_ox in G then d(h) = $p_o h_o$ in H for some h_o∈ H.

Example 6 : Take G= Z_8 , H= $< \bar{2} >$

ifp_o= 2 h=$\bar{2}$ ∈ H , d($\bar{2}$) = 2.$\bar{1}$in G

But There is no element ($h_o \in H$) $\ni d(\bar{2}) = \bar{2}h_o = \bar{2}$Now , take p=3 , test $h_o = \bar{2} \in H$

Clearly $d(\bar{2}) = 3.\bar{6} = \bar{2}$ in G and $h_0 = \bar{6} \in < \bar{2} >$, hence $d(\bar{2}) = 3.\bar{6}$ in H Now , test $\bar{4} \in H$ $d(\bar{4}) = 3.\bar{4} = \bar{4}$ in G and $\bar{4}$ G H ,so $d(\bar{4}) = 3.\bar{4}$in H

Test $\bar{6} \in H$

($d(\bar{6}) = 3\,\bar{x}$) in G $\bar{x} = \bar{2} \in G$ and $d(\bar{6}) = 3.\bar{2} = 6$ in H ($\bar{2} \in H$)

clearly $d(\bar{h}) = p_0 X$ in G and $d(\bar{h}) = 3\, h_o$ in H.

Hence H is 3- neat derivative in G .

Note : For any p- neat derivative in G , is p_o – neat derivative in G .

We are ready to show some new results of the p- neat derivative of element of the cyclic subgroups.

I. **New Results** .I

Theorem 1 : Let A and B are two p- neat derivative in G then

1) $A \cap B$ is a p- neat derivative in G

2) A+B is a p- neat derivative in G

Proof (1):Let h be any element in $A \cap B$, and ($\forall p$) p is prime number

suppose , $d(h) = p\bar{x}$ in G ($x \in G$)

since $h \in A \cap B$, so $\exists$an element$g \in G \ni d(h) = p\,g \in A \cap B$

$d(h) = pg$ in A But A is p- neat derivative so $d(h) = pa_o$

for some $a_o \in A$ and $d(h) = pg$ in B . B is p- neat derivative.

Thus , $d(h) = ph_o$ for some $b_o \in B$ so $d(h) = p\,a_o = pb_o$

we get $p(a_o - b_o) = o$ and thus $a_o = b_o \in A \cap B$

Therefore,$d(h) = p(a_o) = p(b_o)$ $a_o, b_o \in A \cap B$

That mean , $A \cap B$ is p- neat derivative in G .

(2) To prove, A+B is p- neat derivative in G

Let X be any element in A+B and suppose that $d(x) = pg$ in G

since $x \in A+B$, $x = a+b$ for some $a \in A$ and some $b \in B$

we have $d(x) = d(a+b) = pg \in G$ so $da + db = pg$, $da \in A$ and $db \in B$ But A and B are p- neat derivative in G.

Thus $da = a_o$ for some $a_o \in A$$db = b_o$ for some $b_o \in B$

we obtain $d(a+b) = da + db = a_o + b_o \in A+B$

consequently A+B is p- neat derivation.

Theorem 2: Let A be a neat – derivative subgroup of B in G – then

i) A is a p- neat – derivative in G

ii) If B is p- neat derivative in G , then B/A is p- neat derivative in G/A

Proof: i) For all a∈A and for all p is prime number p we have d(a) = pg for some g∈G But A ⊆B and A is p- neat derivative in B so , d(a) = pb for some b∈B

and∃a_o∈ A ∋ d (a) = pa_o in A

which means that A is p- neat derivative in G .

(ii) Let b+A∈B/A for some b∈B

(∀p) p is prime number , suppose thatd(b+A) = p(g+A) in G/A (g∈G) therefore d(b) = pg in G

But we have B is p- neat derivative in G so , d(b) = pb_o for some b_o∈B

we get , d(b+A) = p(g+A) = b_3 = pb_o+pA = p(b_o+A) .

Thus d(b+A)= p(g+A) in G/A d(b+A)= p(b_o+A) in B/A

which means that B/A is p- neat derivative in G/A .

References

[1] H. M. A. Abdullah ,Atefa J. S. Abdullah and Rawaa E. E.,Some new results of the integrals in the group mod-n, Journal of progressive Research in Mathematics , V 12 . is sue 2 June 13 , 2017
[2] H. M. A. Abdullah , pure (1-2) and 3 subgroup in Abelian groups . pU. M. A ser A Vd 3 no 3-1 pp 135-139 Italy, 1992.
[3] L. Fvchs., Abelian group , USA ,1998
[4] TaToKimiko ,on abelian group every subgroup of which is neat . comment .M .Uniu .st.pauli 1980