Certain integrals involving generalized multivariable Aleph function and Wright's function

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Abstract
In this paper we define the generalized multivariable Aleph-function in terms of multiple integrals contour. Further we establish certains integrals involving product of the generalized multivariable Aleph-function with exponential function and Fox-Wright's generalized hypergeometric function. Being unified and general nature, these integrals yield a number known and new results as special cases. At the end, we study several corollaries.

Keywords: Generalized multivariable Aleph-function, Mellin-barnes integrals contour, generalized multivariable I-function, Aleph-function, Fox-Wright's generalized hypergeometric function.

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1.Introduction and preliminaries.

Throughout this paper, let $\mathbb{C},\mathbb{R}$ and $\mathbb{N}$ be set of complex numbers, real numbers, and positive integers respectively. Also $\mathbb{N}_0=\mathbb{N}\cup\{0\}$.

The generalized Aleph-function of r-variables is a general higher transcendental function. It will be defined and represented as follows :

$$\aleph(z_1,\cdots,z_r)=\aleph^{m,\mathfrak{n}:m_1,n_1,\cdots,m_r,n_r}_{p_i,q_i,\tau_i;R:p_{i^{(1)}},q_{i^{(1)}},\tau_{i^{(1)}};R^{(1)};\cdots;p_{i^{(r)}},q_{i^{(r)}};\tau_{i^{(r)}};R^{(r)}}\left(\begin{array}{c|c} \mathrm{z}_1 & [(\mathrm{a}_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)})_{1,\mathfrak{n}}], \\ \vdots & \cdot \\ \vdots & [(\mathrm{b}_j;\beta_j^{(1)},\cdots,\beta_j^{(r)})_{1,m}], \\ \mathrm{z}_r & \end{array}\right.$$

$$[\tau_i(a_{ji};\alpha_{ji}^{(1)},\cdots,\alpha_{ji}^{(r)})_{\mathfrak{n}+1,p_i}]:[(\mathrm{c}_j^{(1)};\gamma_j^{(1)})_{1,n_1}],[\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)};\gamma_{ji^{(1)}}^{(1)})_{n_1+1,p_i^{(1)}}];\cdots;$$
$$[\tau_i(b_{ji};\beta_{ji}^{(1)},\cdots,\beta_{ji}^{(r)})_{m+1,q_i}]:[(\mathrm{d}_j^{(1)};\delta_j^{(1)})_{1,m_1}],[\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)};\delta_{ji^{(1)}}^{(1)})_{m_1+1,q_i^{(1)}}];\cdots;$$

$$\left.\begin{array}{c}[(\mathrm{c}_j^{(r)};\gamma_j^{(r)})_{1,n_r}],[\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)};\gamma_{ji^{(r)}}^{(r)})_{n_r+1,p_i^{(r)}}] \\ \cdot \\ [(\mathrm{d}_j^{(r)};\delta_j^{(r)})_{1,m_r}],[\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)};\delta_{ji^{(r)}}^{(r)})_{m_r+1,q_i^{(r)}}]\end{array}\right)=\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^{r}\theta_k(s_k)z_k^{s_k}\,\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{1.1}$$

with $\omega=\sqrt{-1}$

$$\psi(s_1,\cdots,s_r)\frac{\prod_{j=1}^{m}\Gamma(b_j-\sum_{k=1}^{r}\beta_j^{(k)}s_k)\prod_{j=1}^{\mathfrak{n}}\Gamma(1-a_j+\sum_{k=1}^{r}\alpha_j^{(k)}s_k)}{\sum_{i=1}^{R}[\tau_i\prod_{j=\mathfrak{n}+1}^{p_i}\Gamma(a_{ji}-\sum_{k=1}^{r}\alpha_{ji}^{(k)}s_k)\prod_{j=m+1}^{q_i}\Gamma(1-b_{ji}+\sum_{k=1}^{r}\beta_{ji}^{(k)}s_k)]} \tag{1.2}$$

and

$$\theta_k(s_k)=\frac{\prod_{j=1}^{m_k}\Gamma(d_j^{(k)}-\delta_j^{(k)}s_k)\prod_{j=1}^{n_k}\Gamma(1-c_j^{(k)}+\gamma_j^{(k)}s_k)}{\sum_{i^{(k)}=1}^{R^{(k)}}[\tau_{i^{(k)}}\prod_{j=m_k+1}^{q_{i^{(k)}}}\Gamma(1-d_{ji^{(k)}}^{(k)}+\delta_{ji^{(k)}}^{(k)}s_k)\prod_{j=n_k+1}^{p_{i^{(k)}}}\Gamma(c_{ji^{(k)}}^{(k)}-\gamma_{ji^{(k)}}^{(k)}s_k)]},k=1,\cdots,r \tag{1.3}$$

1) $[(c_j^{(1)};\gamma_j^{(1)})]_{1,n_1}$ stands for $(c_1^{(1)};\gamma_1^{(1)}),\cdots,(c_{n_1}^{(1)};\gamma_{n_1}^{(1)})$.

2) $p_i, q_i, R, p_{i^{(1)}}, q_{i^{(1)}}, R^{(1)}; \cdots ; p_{i^{(r)}}, q_{i^{(r)}}, R^{(r)}, m, \mathfrak{n} : m_1, n_1, \cdots , m_r, n_r \in \mathbb{N}^+$ and verify:

$0 \leqslant \mathfrak{n} \leqslant p_i, i = 1, \cdots , R \,; 0 \leqslant m \leqslant q_i, i = 1, \cdots , R \,; 0 \leqslant n_k \leqslant p_{i^{(k)}}, i = 1, \cdots , R^{(k)}; k = 1, \cdots , r$

$0 \leqslant m_k \leqslant q_{i^{(k)}}, i = 1, \cdots , R^{(k)}; k = 1, \cdots , r.$

3) $\alpha_j^{(k)} \in \mathbb{R}^+; (j = 1, \cdots , \mathfrak{n}); (k = 1, \cdots , r).\ \beta_j^{(k)} \in \mathbb{R}^+; (j = 1, \cdots , m); (k = 1, \cdots , r).$

$\alpha_{ji}^{(k)} \in \mathbb{R}^+; (i = 1, \cdots , R); (j = \mathfrak{n} + 1, \cdots , p_i); (k = 1, \cdots , r).$

$\beta_{ji}^{(k)} \in \mathbb{R}^+; (i = 1, \cdots , R); (j = m + 1, \cdots , q_i); (k = 1, \cdots , r).$

$\delta_j^{(k)} \in \mathbb{R}^+; (j = 1, \cdots , m_k); (k = 1, \cdots , r).\ \gamma_j^{(k)} \in \mathbb{R}^+; (j = 1, \cdots , n_k); (k = 1, \cdots , r).$

$\delta_{ji^{(k)}}^{(k)} \in \mathbb{R}^+; (i = 1, \cdots , R^{(k)}); (j = m_k + 1, \cdots , q_{i^{(k)}}); (k = 1, \cdots , r).$

$\gamma_{ji^{(k)}}^{(k)} \in \mathbb{R}^+; (i = 1, \cdots , R^{(k)}); (j = n_k + 1, \cdots , p_{i^{(k)}}); (k = 1, \cdots , r).$

$\tau_i \in \mathbb{R}^+; (i = 1, \cdots , R).\ \tau_{i^{(k)}} \in \mathbb{R}^+; (i = 1, \cdots , R^{(k)}), (k = 1, \cdots , r).$

4) $b_j \in \mathbb{C}, (j = 1, \cdots , m).\ a_j \in \mathbb{C}, (j = 1, \cdots , \mathfrak{n}).a_{ji} \in \mathbb{C}; (i = 1, \cdots , R), (j = \mathfrak{n} + 1, \cdots , p_i).$

$b_{ji} \in \mathbb{C}; (i = 1, \cdots , R), (j = m + 1, \cdots , q_i).\ d_j^{(k)} \in \mathbb{C}; (k = 1, \cdots , r), (j = 1, \cdots , m_k).$

$c_j^{(k)} \in \mathbb{C}; (k = 1, \cdots , r), (j = 1, \cdots , n_k).\ d_{ji^{(k)}}^{(k)} \in \mathbb{C}; (k = 1, \cdots , r), (i = 1, \cdots , R^{(k)}), (j = m_k + 1, \cdots , q_k).$

$c_{ji^{(k)}}^{(k)} \in \mathbb{C}; (k = 1, \cdots , r), (i = 1, \cdots , R^{(k)}), (j = n_k + 1, \cdots , p_k).\ z_k \neq 0; (k = 1, \cdots , r).$

The contour L_k is in the $s_k (k = 1, \cdots , r)$ plane and run from $\sigma - i\infty$ to $\sigma + i\infty$ where σ is a real number with loop , if necessary to ensure that the poles of $\Gamma(1 - a_j + \sum_{i=1}^{r} \alpha_j^{(k)} s_k) (j = 1, \cdots , m)$;$\Gamma(d_j^{(k)} - \delta_j^{(k)} s_k)\ (j = 1, \cdots , m_k)$ lie to the right of the contour L_k and the poles of $\Gamma(1 - a_j + \sum_{i=1}^{r} \alpha_j^{(k)} s_k)\ (j = 1, \cdots , \mathfrak{n})$; $\Gamma(1 - c_j^{(k)} + \gamma_j^{(k)} s_k)$ $(j = 1, \cdots , n_k)$ lie to the left of the contour L_k . The condition for absolute convergence of multiple Mellin-Barnes type contour (1.1) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$|arg z_k| < \frac{1}{2} A_i^{(k)} \pi$, where $A_i^{(k)} = \sum_{j=1}^{\mathfrak{n}} \alpha_j^{(k)} + \sum_{j=1}^{m} \beta_j^{(k)} - \tau_i \sum_{j=\mathfrak{n}+1}^{p_i} \alpha_{ji}^{(k)} - \tau_i \sum_{j=m+1}^{q_i} \beta_{ji}^{(k)} + \sum_{j=1}^{n_k} \gamma_j^{(k)} -$

$$\tau_{i^{(k)}} \sum_{j=n_k+1}^{p_{i^{(k)}}} \gamma_{ji^{(k)}}^{(k)} + \sum_{j=1}^{m_k} \delta_j^{(k)} - \tau_{i^{(k)}} \sum_{j=m_k+1}^{q_{i^{(k)}}} \delta_{ji^{(k)}}^{(k)} > 0 \tag{1.4}$$

The generalized Aleph-function of r-variables is analytic if

$$\tau_i \sum_{j=\mathfrak{n}}^{p_i} \alpha_{ji}^{(k)} - \tau_i \sum_{j=1}^{q_i} \beta_{ji}^{(k)} + \tau_{i^{(k)}} \sum_{j=1}^{p_{i^{(k)}}} \gamma_{ji^{(k)}}^{(k)} - \tau_{i^{(k)}} \sum_{j=1}^{q_{i^{(k)}}} \delta_{ji^{(k)}}^{(k)} < 0 \tag{1.5}$$

with $k = 1, \cdots , r, i = 1, \cdots , R$, $i^{(k)} = 1, \cdots , R^{(k)}$.

Following the lines of Braaksma ([2], p. 278), it can be shown that

$\aleph(z_1, \cdots , z_r) = 0(|z_1|^{\alpha_1}, \cdots , |z_r|^{\alpha_r})$, $max(|z_1|, \cdots , |z_r|) \rightarrow 0$

$\aleph(z_1, \cdots , z_r) = 0(|z_1|^{\beta_1}, \cdots ,|z_r|^{\beta_r}), max(|z_1|, \cdots , |z_r|) \rightarrow \infty$

where $k=1,\cdots,r:\alpha_k=min[Re(d_j^{(k)}/\delta_j^{(k)})], j=1,\cdots,m_k$ and $\beta_k=max[Re((c_j^{(k)}-1)/\gamma_j^{(k)})], j=1,\cdots,n_k$.

For convenience, we shall use the following notations in this paper.

$$V=m_1,n_1;\cdots;m_r,n_r \;;\; \mathrm{W}= p_{i^{(1)}},q_{i^{(1)}},\tau_{i^{(1)}};R^{(1)},\cdots,p_{i^{(r)}},q_{i^{(r)}},\tau_{i^{(r)}};R^{(r)} \tag{1.6}$$

$$A=\{(a_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)})_{1,\mathfrak{n}}\},\{\tau_i(a_{ji};\alpha_{ji}^{(1)},\cdots,\alpha_{ji}^{(r)})_{\mathfrak{n}+1,p_i}\}:\{(c_j^{(1)};\gamma_j^{(1)})_{1,n_1}\},$$

$$\{\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)};\gamma_{ji^{(1)}}^{(1)})_{n_1+1,p_{i^{(1)}}}\};\cdots;\{(c_j^{(r)};\gamma_j^{(r)})_{1,n_r}\},\{\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)};\gamma_{ji^{(r)}}^{(r)})_{n_r+1,p_{i^{(r)}}}\} \tag{1.7}$$

$$B=\{(b_j;\beta_j^{(1)},\cdots,\beta_j^{(r)})_{1,m}\},\{\tau_i(b_{ji};\beta_{ji}^{(1)},\cdots,\beta_{ji}^{(r)})_{m+1,q_i}\}:\{(d_j^{(1)};\delta_j^{(1)})_{1,m_1}\},\{\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)};\delta_{ji^{(1)}}^{(1)})_{m_1+1,q_{i^{(1)}}}\}$$

$$;\cdots;\{(d_j^{(r)};\delta_j^{(r)})_{1,m_r}\},\{\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)};\delta_{ji^{(r)}}^{(r)})_{m_r+1,q_{i^{(r)}}}\} \tag{1.8}$$

Remark 1 :
If $m=0$, the generalized Aleph-function of r-variables defined above reduces to Aleph-function of r-variables defined by Ayant [1].

Remark 2 :
If $\tau_i,\tau_{i^{(1)}},\cdots,\tau_{i^{(r)}}\rightarrow 1$, we get the generalized I-function of r-variables. This function generalizes the multivariable I-function defined by Sharma and Ahmad [13]. It's defined as follows :

$$\mathbf{I}(z_1,\cdots,z_r)=\mathbf{I}^{m,\mathfrak{n}:m_1,n_1,\cdots,m_r,n_r}_{p_i,q_i;R:p_{i^{(1)}},q_{i^{(1)}};R^{(1)};\cdots;p_{i^{(r)}},q_{i^{(r)}};R^{(r)}}\left(\begin{array}{c|c} \mathrm{z}_1 & [(\mathrm{a}_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)})_{1,\mathfrak{n}}], \\ \vdots & \cdot \\ \vdots & [(\mathrm{b}_j;\beta_j^{(1)},\cdots,\beta_j^{(r)})_{1,m}], \\ \mathrm{z}_r & \end{array}\right.$$

$$[(\mathrm{a}_{ji};\alpha_{ji}^{(1)},\cdots,\alpha_{ji}^{(r)})_{\mathfrak{n}+1,p_i}]:[(\mathrm{c}_j^{(1)};\gamma_j^{(1)})_{1,n_1}],[(c_{ji^{(1)}}^{(1)};\gamma_{ji^{(1)}}^{(1)})_{n_1+1,p_i^{(1)}}];\cdots;$$
$$[(\mathrm{b}_{ji};\beta_{ji}^{(1)},\cdots,\beta_{ji}^{(r)})_{m+1,q_i}]:[(\mathrm{d}_j^{(1)};\delta_j^{(1)})_{1,m_1}],[(d_{ji^{(1)}}^{(1)};\delta_{ji^{(1)}}^{(1)})_{m_1+1,q_i^{(1)}}];\cdots;$$

$$\left.\begin{array}{c}[(\mathrm{c}_j^{(r)};\gamma_j^{(r)})_{1,n_r}],[(c_{ji^{(r)}}^{(r)};\gamma_{ji^{(r)}}^{(r)})_{n_r+1,p_i^{(r)}}]\\ \cdot \\ [(\mathrm{d}_j^{(r)};\delta_j^{(r)})_{1,m_r}],[(d_{ji^{(r)}}^{(r)};\delta_{ji^{(r)}}^{(r)})_{m_r+1,q_i^{(r)}}]\end{array}\right)=\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^{r}\theta_k(s_k)z_k^{s_k}\,\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{1.9}$$

with $\omega=\sqrt{-1}$

$$\psi(s_1,\cdots,s_r)\frac{\prod_{j=1}^{m}\Gamma(b_j-\sum_{k=1}^{r}\beta_j^{(k)}s_k)\prod_{j=1}^{\mathfrak{n}}\Gamma(1-a_j+\sum_{k=1}^{r}\alpha_j^{(k)}s_k)}{\sum_{i=1}^{R}[\prod_{j=\mathfrak{n}+1}^{p_i}\Gamma(a_{ji}-\sum_{k=1}^{r}\alpha_{ji}^{(k)}s_k)\prod_{j=m+1}^{q_i}\Gamma(1-b_{ji}+\sum_{k=1}^{r}\beta_{ji}^{(k)}s_k)]} \tag{1.10}$$

and

$$\theta_k(s_k)=\frac{\prod_{j=1}^{m_k}\Gamma(d_j^{(k)}-\delta_j^{(k)}s_k)\prod_{j=1}^{n_k}\Gamma(1-c_j^{(k)}+\gamma_j^{(k)}s_k)}{\sum_{i^{(k)}=1}^{R^{(k)}}[\prod_{j=m_k+1}^{q_{i^{(k)}}}\Gamma(1-d_{ji^{(k)}}^{(k)}+\delta_{ji^{(k)}}^{(k)}s_k)\prod_{j=n_k+1}^{p_{i^{(k)}}}\Gamma(c_{ji^{(k)}}^{(k)}-\gamma_{ji^{(k)}}^{(k)}s_k)]},k=1,\cdots,r \tag{1.11}$$

under the same existence conditions that generalized Aleph-function of r-variables with $\tau_i,\tau_{i^{(1)}},\cdots,\tau_{i^{(r)}}\rightarrow 1$.

Remark 3 :
If $\tau_i,\tau_{i^{(1)}},\cdots,\tau_{i^{(r)}}\rightarrow 1$ and $R=R^{(1)}=\cdots=R^{(r)}=1$, the generalized multivariable Aleph-function reduces in generalized multivariable H-function. This function is an extension of the multivariable H-function defined by

Srivastava and Panda [16,17]. It's is defined as follows.

$$\mathbf{H}(z_1,\cdots,z_r)=\mathbf{H}^{m,\mathfrak{n}:m_1,n_1,\cdots,m_r,n_r}_{p,q:R:p_1,q_1;\cdots;p_r,q_r}\left(\begin{array}{c|c} \mathrm{z}_1 & [(\mathrm{a}_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)})]_{1,p}:\ [(\mathrm{c}_j^{(1)};\gamma_j^{(1)})_{1,p_1}];\cdots;[(\mathrm{c}_j^{(r)};\gamma_j^{(r)})_{1,p_r}] \\ \vdots & \cdot \\ \vdots & \\ \mathrm{z}_r & [(\mathrm{b}_j;\beta_j^{(1)},\cdots,\beta_j^{(r)})]_{1,q}:\ [(\mathrm{d}_j^{(1)};\delta_j^{(1)})_{1,q_1}];\cdots;[(\mathrm{d}_j^{(r)};\delta_j^{(r)})_{1,q_r}] \end{array}\right)$$

$$=\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^{r}\theta_k(s_k)z_k^{s_k}\,\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{1.12}$$

with $\omega=\sqrt{-1}$

$$\psi(s_1,\cdots,s_r)\frac{\prod_{j=1}^{m}\Gamma(b_j-\sum_{k=1}^{r}\beta_j^{(k)}s_k)\prod_{j=1}^{\mathfrak{n}}\Gamma(1-a_j+\sum_{k=1}^{r}\alpha_j^{(k)}s_k)}{[\prod_{j=\mathfrak{n}+1}^{p}\Gamma(a_j-\sum_{k=1}^{r}\alpha_j^{(k)}s_k)\prod_{j=m+1}^{q}\Gamma(1-b_j+\sum_{k=1}^{r}\beta_j^{(k)}s_k)]} \tag{1.13}$$

and

$$\theta_k(s_k)=\frac{\prod_{j=1}^{m_k}\Gamma(d_j^{(k)}-\delta_j^{(k)}s_k)\prod_{j=1}^{n_k}\Gamma(1-c_j^{(k)}+\gamma_j^{(k)}s_k)}{\prod_{j=m_k+1}^{q_k}\Gamma(1-d_j^{(k)}+\delta_j^{(k)}s_k)\prod_{j=n_k+1}^{p_k}\Gamma(c_j^{(k)}-\gamma_j^{(k)}s_k)]},k=1,\cdots,r \tag{1.14}$$

under the same existence conditions that generalized Aleph-function of r-variables with $\tau_i,\tau_{i^{(1)}},\cdots,\tau_{i^{(r)}}\rightarrow 1$and $R=R^{(1)}=\cdots=R^{(r)}=1$,

The Wright's generalized hypergeometric function ${}_{p'}\psi_{q'}(.)$ given by Wright [21] defined as :

$${}_{p'}\psi_{q'}(z)={}_{p'}\psi_{q'}\left[\begin{array}{c|c} (\mathrm{e}_j,E_j)_{1,p'} & \\ \cdot & \mathrm{z} \\ (\mathrm{f}_j,F_j)_{1,q'} & \end{array}\right]=\sum_{h=0}^{\infty}\frac{\prod_{j=1}^{p'}\Gamma(e_j+E_jh)}{\prod_{j=1}^{q'}\Gamma(f_j+F_jh)}\frac{z^h}{h!} \tag{1.15}$$

Where $E_j(j=1,\cdots,p')$ and $F_j(j=1,\cdots,q')$ are real and positive numbers and

$$1+\sum_{j=1}^{q'}F_j-\sum_{j=1}^{p'}E_j>0 \tag{1.16}$$

We shall use this notation in this paper.

$$A_h=\frac{\prod_{j=1}^{p'}\Gamma(e_j+E_jh)}{\prod_{j=1}^{q'}\Gamma(f_j+F_jh)} \tag{1.17}$$

2. Required result.

We have the following relation, see Rainville [10].

Lemma.

$$\sum_{h,u=0}^{\infty}A(h,u)=\sum_{u=0}^{\infty}\sum_{h=0}^{u}A(h,u-h) \tag{2.1}$$

3. Main integrals.

In this section, we have etablished four integrals involving product of generalized Aleph-function of r-variables and Wright's generalized hypergeometric function.

Theorem 1.

$$\int_0^t x^{\rho-1}(t-x)^{\sigma-1}e^{-zx}{}_p\psi_{q'}(ax^\zeta(t-x)^\eta)\ \aleph_{p_i,q_i,\tau_i;R:W}^{m,\mathfrak{n}:V}\left(\begin{array}{c}\mathrm{z}_1x^{\mu_1}(t-x)^{\upsilon_1}\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z}_rx^{\mu_r}(t-x)^{\upsilon_r}\end{array}\right)\mathrm{d}x= e^{-zt}t^{\rho+\sigma-1}\sum_{u=0}^{\infty}\sum_{h=0}^{u}A_h$$

$$\frac{z^{u-h}}{(u-h)!}t^{(\zeta+\eta-1)h+u}\ \aleph_{p_i+2,q_i+1,\tau_i;R:W}^{m,\mathfrak{n}+2:V}\left(\begin{array}{c|c}\mathrm{z}_1t^{\mu_1+\upsilon_1} & (1\text{-}\rho-\zeta h;\mu_1,\cdots,\mu_r),(1-\sigma-(\eta-1)h-u;\upsilon_1,\cdots,\upsilon_r),A\\ \cdot & \cdot\\ \cdot & \cdot\\ \cdot & \cdot\\ \mathrm{z}_rt^{\mu_r+\upsilon_r} & (1\text{-}\rho-\sigma-(\zeta+\eta-1)h-u;\mu_1+\upsilon_1,\cdots,\mu_r+\upsilon_r),B\end{array}\right) \quad (3.1)$$

A_h is defined by (1.18).

provided

1) $0\leqslant\mu_k, 0\leqslant\upsilon_k$ (not both zero simultaneously) such that $\upsilon_k-\mu_k>0.(k=1,\cdots,r)$.

2) ζ and η are non-negative integers such that $1\leqslant\zeta+\eta$.

3) $|arg z_k(x^{\mu_k}(t-x)^{\upsilon_k})|\ <\frac{1}{2}A_i^{(k)}\pi$, where $A_i^{(k)}$ is defined by (1.4).

4)$Re\,(\rho+\zeta h)+\sum_{k=1}^{r}\mu_k\min_{\substack{0\leqslant H\leqslant m\\0\leqslant l\leqslant m_k}}Re\left(\sum_{H'=1}^{r}\frac{b_H}{\beta_H^{(H')}}+\frac{d_l^{(k)}}{\delta_l^{(k)}}\right)>0$ and

$$Re\,(\sigma+\eta h)+\sum_{k=1}^{r}\upsilon_k\min_{\substack{0\leqslant H\leqslant m\\0\leqslant l\leqslant m_k}}Re\left(\sum_{H'=1}^{r}\frac{b_H}{B_H^{(H')}}+\frac{d_l^{(k)}}{\delta_l^{(k)}}\right)>0.$$

5)$1+\sum_{j=1}^{q'}F_j-\sum_{j=1}^{p'}E_j>0$

Proof

We note I this integral, we have

$$I=e^{-zt}\int_0^t x^{\rho-1}(t-x)^{\sigma-1}e^{-z(t-x)}{}_p\psi_{q'}(ax^\zeta(t-x)^\eta)\ \aleph_{p_i,q_i,\tau_i;R:W}^{m,\mathfrak{n}:V}\left(\begin{array}{c}\mathrm{z}_1x^{\mu_1}(t-x)^{\upsilon_1}\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z}_rx^{\mu_r}(t-x)^{\upsilon_r}\end{array}\right)\mathrm{d}x \quad (3.2)$$

Now, we replace $e^{(t-x)z}$ by $\sum_{u=0}^{\infty}\frac{(t-x)^uz^u}{u!}$ and express the Wright's generalized hypergeometric function and

generalized Aleph-function of r-variables with the help of (1.16) and (1.1) respectively, we get

$$I=e^{-zt}\int_0^t x^{\rho-1}(t-x)^{\sigma-1}\sum_{u=0}^{\infty}\frac{(t-x)^uz^u}{u!}\sum_{h=0}^{\infty}A_h\frac{a^hx^{\zeta h}(t-x)^{\eta h}}{h!}\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)$$

$$\prod_{k=1}^{r}\theta_k(s_k)z_k^{s_k}x^{\mu_ks_k}(t-x)^{\upsilon_ks_k}\,\mathrm{d}s_1\cdots\mathrm{d}s_r\mathrm{d}x$$

$$=e^{-zt}\int_0^t x^{\rho-1}(t-x)^{\sigma-1}\sum_{u=0}^{\infty}\sum_{h=0}^{\infty}A_h\frac{a^hx^{\zeta h}(t-x)^{\eta h+u}z^u}{h:u!}\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)$$

$$\prod_{k=1}^{r}\theta_k(s_k)z_k^{s_k}x^{\mu_k s_k}(t-x)^{\upsilon_k s_k}\,\mathrm{d}s_1\cdots\mathrm{d}s_r\mathrm{d}x$$

Now, we use the lemma, we use

$$I=e^{-zt}\int_0^t x^{\rho-1}(t-x)^{\sigma-1}\sum_{u=0}^{\infty}\sum_{h=0}^{u}A_h\frac{a^h x^{\zeta h}(t-x)^{(\eta-1)h+u}z^{u-h}}{h!(u-h)!}\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)$$

$$\prod_{k=1}^{r}\theta_k(s_k)z_k^{s_k}x^{\mu_k s_k}(t-x)^{\upsilon_k s_k}\,\mathrm{d}s_1\cdots\mathrm{d}s_r\mathrm{d}x$$

We interchange the order of (u,h)-summations and $(s_1,\cdots,s_r)$-integrals (which is permissible under the stated conditions), we obtain

$$I=e^{-zt}\sum_{u=0}^{\infty}\sum_{h=0}^{u}A_h\frac{a^h z^{u-h}}{h!(u-h)!}\int_0^t x^{\rho-1}(t-x)^{\sigma-1}\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)$$

$$\prod_{k=1}^{r}\theta_k(s_k)z_k^{s_k}x^{\mu_k s_k}(t-x)^{\upsilon_k s_k}\,\mathrm{d}s_1\cdots\mathrm{d}s_r\mathrm{d}x$$

We interchange the order x-integral and $(s_1,\cdots,s_r)$-integrals (which is permissible under the stated conditions), we obtain

$$I=e^{-zt}\sum_{u=0}^{\infty}\sum_{h=0}^{u}A_h\frac{a^h z^{u-h}}{h!(u-h)!}\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^{r}\theta_k(s_k)z_k^{s_k}$$

$$\left[\int_0^t x^{\rho+\zeta h+\sum_{k=1}^{r}\mu_k s_k-1}(t-x)^{\sigma+(\eta-1)h+u+\sum_{k=1}^{r}\upsilon_k s_k-1}\mathrm{d}x\right]\mathrm{d}s_1\cdots\mathrm{d}s_r$$

Take $x=tv$ in the inner integral, the above expression reduces to

$$I=e^{-zt}t^{\rho+\sigma-1}\sum_{h=0}^{u}A_h\frac{a^h z^{u-h}}{h!(u-h)!}\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^{r}\theta_k(s_k)z_k^{s_k}\,t^{(\mu_k+\upsilon_k)s_k}$$

$$\left[\int_0^t s^{\rho+\zeta h+\sum_{k=1}^{r}\mu_k s_k-1}(t-x)^{\sigma+(\eta-1)h+u+\sum_{k=1}^{r}\upsilon_k s_k-1}\mathrm{d}v\right]\mathrm{d}s_1\cdots\mathrm{d}s_r$$

By using the definition of the Beta function, we get

$$I=e^{-zt}t^{\rho+\sigma-1}\sum_{u=0}^{\infty}\sum_{h=0}^{u}A_h\frac{a^h z^{u-h}}{h!(u-h)!}\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^{r}\theta_k(s_k)z_k^{s_k}\,t^{(\mu_k+\upsilon_k)s_k}$$

$$\frac{\Gamma(\rho+\zeta h+\sum_{k=1}^{r}\mu_k s_k)\Gamma(\sigma+(\eta-1)h+u+\sum_{k=1}^{r}\upsilon_k s_k)}{\Gamma(\rho+\sigma+(\zeta+\eta-1)h+\sum_{k=1}^{r}(\mu_k+\upsilon)s_k)}\mathrm{d}s_1\cdots\mathrm{d}s_r$$

now interpreting the above multiple Mellin-Barnes integrals contour in generalized Aleph-function of r-variables, we obtain the desired result (3.1) after algebric manipulations.

Theorem 2.

$$\int_0^t x^{\rho-1}(t-x)^{\sigma-1}e^{-zx}{}_p\psi_{q'}(ax^{\zeta}(t-x)^{\eta})\,\aleph^{m,\mathfrak{n}:V}_{p_i,q_i,\tau_i;R:W}\left(\begin{array}{c}\mathrm{z}_1x^{-\mu_1}(t-x)^{-\upsilon_1}\\ \cdot\\ \cdot\\ \cdot\\ \mathrm{z}_r x^{-\mu_r}(t-x)^{-\upsilon_r}\end{array}\right)\mathrm{d}x=e^{-zt}t^{\rho+\sigma-1}\sum_{u=0}^{\infty}\sum_{h=0}^{u}A_h$$

$$\frac{z^{u-h}}{(u-h)!}t^{(\zeta+\eta-1)h+u}\aleph^{m+2,\mathfrak{n}:V}_{p_i+1,q_i+2,\tau_i;R:W}\left(\begin{array}{c|c} \mathrm{z}_1 t^{-\mu_1-\upsilon_1} & (\rho+\sigma+(\zeta+\eta-1)h-u;\mu_1+\upsilon_1,\cdots,\mu_r+\upsilon_r),A \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \mathrm{z}_r t^{-\mu_r-\upsilon_r} & (\rho+\zeta h;\mu_1,\cdots,\mu_r),(\sigma+(\eta-1)h+u;\upsilon_1,\cdots,\upsilon_r),B \end{array}\right) \quad (3.3)$$

under the same existence conditions 1), 2) and 5) that (3.1) and

3) $\left|arg z_k(x^{-\mu_k}(t-x)^{-\upsilon_k})\right| < \frac{1}{2}A_i^{(k)}\pi$, where $A_i^{(k)}$ is defined by (1.4).

4) $Re\,(\rho+\zeta h)-\sum_{k=1}^{r}\mu_k \max_{\substack{0\leqslant H\leqslant n\\0\leqslant l\leqslant n_k}} Re\left(\sum_{H'=1}^{r}\frac{a_H-1}{\alpha_H^{(H')}}+\frac{c_l^{(k)}-1}{\gamma_l^{(k)}}\right)<0$ and

$.Re\,(\sigma+\eta h)-\sum_{k=1}^{r}\upsilon_k \max_{\substack{0\leqslant H\leqslant n\\0\leqslant l\leqslant n_k}} Re\left(\sum_{H'=1}^{r}\frac{a_H-1}{\alpha_H^{(H')}}+\frac{c_l^{(k)}-1}{\gamma_l^{(k)}}\right)<0$

Theorem 3.

$$\int_0^t x^{\rho-1}(t-x)^{\sigma-1}e^{-zx}{}_p\psi_{q'}(ax^\zeta(t-x)^\eta)\ \aleph^{m,\mathfrak{n}:V}_{p_i,q_i,\tau_i;R:W}\left(\begin{array}{c} \mathrm{z}_1 x^{\mu_1}(t-x)^{-\upsilon_1} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_r x^{\mu_r}(t-x)^{-\upsilon_r} \end{array}\right)\mathrm{d}x = e^{-zt}t^{\rho+\sigma-1}\sum_{u=0}^{\infty}\sum_{h=0}^{u}A_h\frac{z^{u-h}}{(u-h)!}$$

$$t^{(\zeta+\eta-1)h+u}\aleph^{m+1,\mathfrak{n}+1:V}_{p_i+1,q_i+2,\tau_i;R:W}\left(\begin{array}{c|c} \mathrm{z}_1 t^{\mu_1-\upsilon_1} & (1\text{-}\rho-\zeta h;\mu_1,\cdots,\mu_r),A \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \mathrm{z}_r t^{\mu_r-\upsilon_r} & (\sigma+u-(\eta-1)h;\upsilon_1,\cdots,\upsilon_r),(1-\rho-\sigma-(\zeta+\eta-1)h-u;\mu_1-\upsilon_1,\cdots,\mu_r-\upsilon_r),B \end{array}\right) \quad (3.4)$$

under the same existence conditions 1), 2) and 5) that (3.1) and

3) $\left|arg z_k(x^{\mu_k}(t-x)^{-\upsilon_k})\right| < \frac{1}{2}A_i^{(k)}\pi$, where $A_i^{(k)}$ is defined by (1.4).

4) $Re\,(\rho+\zeta h)+\sum_{k=1}^{r}\mu_k \min_{\substack{0\leqslant H\leqslant m\\0\leqslant l\leqslant m_k}} Re\left(\sum_{H'=1}^{r}\frac{b_H}{\beta_H^{(H')}}+\frac{d_l^{(k)}}{\delta_l^{(k)}}\right)>0$ and

$.Re\,(\sigma+\eta h)-\sum_{k=1}^{r}\upsilon_k \max_{\substack{0\leqslant H\leqslant n\\0\leqslant l\leqslant n_k}} Re\left(\sum_{H'=1}^{r}\frac{a_H-1}{\alpha_H^{(H')}}+\frac{c_l^{(k)}-1}{\gamma_l^{(k)}}\right)<0$

where $0<\mu_k, 0\leqslant\upsilon_k$ and $0\leqslant\mu_k-\upsilon_k(k=1,\cdots,r)$.

Theorem 4.

$$\int_0^t x^{\rho-1}(t-x)^{\sigma-1}e^{-zx}{}_p\psi_{q'}(ax^\zeta(t-x)^\eta)\ \aleph^{m,\mathfrak{n}:V}_{p_i,q_i,\tau_i;R:W}\left(\begin{array}{c} \mathrm{z}_1 x^{-\mu_1}(t-x)^{\upsilon_1} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_r x^{-\mu_r}(t-x)^{\upsilon_r} \end{array}\right)\mathrm{d}x = e^{-zt}t^{\rho+\sigma-1}\sum_{u=0}^{\infty}\sum_{h=0}^{u}A_h$$

$$\frac{z^{u-h}}{(u-h)!}t^{(\zeta+\eta-1)h+u}\aleph^{m+1,\mathfrak{n}+1:V}_{p_i+1,q_i+2,\tau_i;R:W}\left(\begin{array}{c|c} z_1t^{-\mu_1+\upsilon_1} & \text{(1-u-}\sigma+(\eta-1)h-u;\upsilon_1,\cdots,\upsilon_r),A \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ z_rt^{-\mu_r+\upsilon_r} & (\rho+\zeta h;\mu_1,\cdots,\mu_r),(1-\rho-\sigma-u-(\zeta+\eta-1)h;\upsilon_1-\mu_1,\cdots,\upsilon_r-\mu_r),B \end{array}\right) \quad (3.5)$$

under the same existence conditions 1), 2) and 5) that (3.1), and

3) $|argz_k(x^{-\mu_k}(t-x)^{\upsilon_k})| < \frac{1}{2}A_i^{(k)}\pi$, where $A_i^{(k)}$ is defined by (1.4).

4) $Re\,(\rho+\zeta h)-\sum_{k=1}^{r}\mu_k\max_{\substack{0\leqslant H\leqslant\mathfrak{n}\\0\leqslant l\leqslant n_k}}Re\left(\sum_{H'=1}^{r}\frac{a_H-1}{\alpha_H^{(H')}}+\frac{c_l^{(k)}-1}{\gamma_l^{(k)}}\right)<0$ and

$Re\,(\sigma+\eta h)+\sum_{k=1}^{r}\upsilon_k\min_{\substack{0\leqslant H\leqslant m\\0\leqslant l\leqslant m_k}}Re\left(\sum_{H'=1}^{r}\frac{b_H}{B_H^{(H')}}+\frac{d_l^{(k)}}{\delta_l^{(k)}}\right)>0$.

where $0<\upsilon_k, 0\leqslant\mu_k$ and $0\leqslant\upsilon_k-\mu_k(k=1,\cdots,r)$.

The theorems 2 to 4 can be proved by the similar methods that theorem 1.

In the following section, we have seen several corollaries.

4. Corollaries.

The generalized Aleph-function of r-variables reduces in generalized **H**-function of r-variables concerning the two following corollaries. We consider the theorem 1 and we obtain.

Corollary 1.

$$\int_0^t x^{\rho-1}(t-x)^{\sigma-1}e^{-zx}{}_p\psi_{q'}(ax^{\zeta}(t-x)^{\eta})\,\mathbf{H}\left(\begin{array}{c} z_1x^{\mu_1}(t-x)^{\upsilon_1} \\ \cdot \\ \cdot \\ \cdot \\ z_rx^{\mu_r}(t-x)^{\upsilon_r} \end{array}\right)\mathrm{d}x=e^{-zt}t^{\rho+\sigma-1}\sum_{u=0}^{\infty}\sum_{h=0}^{u}A_h$$

$$\frac{z^{u-h}}{(u-h)!}t^{(\zeta+\eta-1)h+u}\,\mathbf{H}^{m,\mathfrak{n}+2:V}_{p_i+2,q_i+1,\tau_i;R:W}\left(\begin{array}{c|c} z_1t^{\mu_1+\upsilon_1} & \text{(1-}\rho-\zeta h;\mu_1,\cdots,\mu_r),(1-\sigma-(\eta-1)h-u;\upsilon_1,\cdots,\upsilon_r),A_1 \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ z_rt^{\mu_r+\upsilon_r} & \text{(1-}\rho-\sigma-(\zeta+\eta-1)h-u;\mu_1+\upsilon_1,\cdots,\mu_r+\upsilon_r),B_1 \end{array}\right) \quad (4.1)$$

where

$$A_1=[(a_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)})]_{1,p}:[(\mathrm{c}_j^{(1)};\gamma_j^{(1)})_{1,p_1}];\cdots;[(c_j^{(r)};\gamma_j^{(r)})_{1,p_r}] \quad (4.2)$$

$$B_1=[(b_j;\beta_j^{(1)},\cdots,\beta_j^{(r)})]_{1,q}:[(d_j^{(1)};\delta_j^{(1)})_{1,q_1}];\cdots;[(d_j^{(r)};\delta_j^{(r)})_{1,q_r}] \quad (4.3)$$

under the same existence conditions 1), 2), 4) and 5) that (3.1) and

3) $|argz_k(x^{\mu_k}(t-x)^{\upsilon_k})| < \frac{1}{2}B_j^{(k)}\pi$, where

$$B_j^{(k)}=\sum_{j=1}^{\mathfrak{n}}\alpha_j^{(k)}+\sum_{j=1}^{m}\beta_j^{(k)}-\sum_{j=\mathfrak{n}+1}^{p}\alpha_j^{(k)}-\sum_{j=m+1}^{q}\beta_j^{(k)}+\sum_{j=1}^{n_k}\gamma_j^{(k)}-\sum_{j=n_k+1}^{p_k}\gamma_j^{(k)}+\sum_{j=1}^{m_k}\delta_j^{(k)}-\sum_{j=m_k+1}^{q_k}\delta_j^{(k)}>0$$

Now, we consider the theorem 2, we get.

Corollary 2.

$$\int_0^t x^{\rho-1}(t-x)^{\sigma-1}e^{-zx}{}_p\psi_{q'}(ax^\zeta(t-x)^\eta)\,\mathbf{H}\begin{pmatrix} z_1x^{-\mu_1}(t-x)^{-\upsilon_1} \\ \cdot \\ \cdot \\ \cdot \\ z_rx^{-\mu_r}(t-x)^{-\upsilon_r}\end{pmatrix}\mathrm{d}x = e^{-zt}t^{\rho+\sigma-1}\sum_{u=0}^{\infty}\sum_{h=0}^{u}A_h$$

$$\frac{z^{u-h}}{(u-h)!}t^{(\zeta+\eta-1)h+u}\mathbf{H}^{m+2,\mathfrak{n}:V}_{p_i+1,q_i+2,\tau_i;R:W}\left(\begin{array}{c|c} z_1t^{-\mu_1-\upsilon_1} & (\rho+\sigma+(\zeta+\eta-1)h-u;\mu_1+\upsilon_1,\cdots,\mu_r+\upsilon_r),A_1 \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ z_rt^{-\mu_r-\upsilon_r} & (\rho+\zeta h;\mu_1,\cdots,\mu_r),(\sigma+(\eta-1)h+u;\upsilon_1,\cdots,\upsilon_r),B_1\end{array}\right) \quad (4.4)$$

where A_1 and B_1 are defined respectively by (4.2) and (4.3).

under the same existence conditions 1), 2), 4) and 5) that (3.3) and

3) $\left|arg z_k(x^{-\mu_k}(t-x)^{-\upsilon_k})\right| < \frac{1}{2}B_j^{(k)}\pi$, where

$$B_j^{(k)} = \sum_{j=1}^{\mathfrak{n}}\alpha_j^{(k)} + \sum_{j=1}^{m}\beta_j^{(k)} - \sum_{j=\mathfrak{n}+1}^{p}\alpha_j^{(k)} - \sum_{j=m+1}^{q}\beta_j^{(k)} + \sum_{j=1}^{n_k}\gamma_j^{(k)} - \sum_{j=n_k+1}^{p_k}\gamma_j^{(k)} + \sum_{j=1}^{m_k}\delta_j^{(k)} - \sum_{j=m_k+1}^{q_k}\delta_j^{(k)} > 0.$$

The generalized Aleph-function of r-variables reduces in Aleph-function of one variable defined by Sudland et al. [18,19] concerning the two following corollaries, see Suthar et al. [20] for more details. We consider the theorem 3 and we obtain

Corollary 3.

$$\int_0^t x^{\rho-1}(t-x)^{\sigma-1}e^{-zx}{}_p\psi_{q'}(ax^\zeta(t-x)^\eta)\,\aleph(\ \mathrm{Z}\,\mathrm{x}^{\mu}(t-x)^{-\upsilon}\ )\mathrm{d}x = e^{-zt}t^{\rho+\sigma-1}\sum_{u=0}^{\infty}\sum_{h=0}^{u}A_h\frac{z^{u-h}}{(u-h)!}$$

$$t^{(\zeta+\eta-1)h+u}\aleph^{m+1,\mathfrak{n}+1}_{p_i+1,q_i+2,\tau_i;R}\left(\begin{array}{c|c} & (1\text{-}\rho-\zeta h;\mu),A_2 \\ & \cdot \\ \mathrm{Z}t^{\mu-\upsilon} & \cdot \\ & \cdot \\ & (\sigma+u-(\eta-1)h;\upsilon),(1-\rho-\sigma-(\zeta+\eta-1)h-u;\mu-\upsilon),B_2\end{array}\right) \quad (4.5)$$

where

$$A_2 = (a_j,\alpha_j)_{1,n},[\tau_i(a_{ji},\alpha_{ji})]_{n+1,p_i;r}\ ;\ B_2 = (b_j,\beta_j)_{1,m},[c_i(b_{ji},\beta_{ji})]_{m+1,q_i;r} \quad (4.6)$$

under the following existence conditions

1) $0<\mu, 0\leqslant\upsilon$ and $0\leqslant\mu-\upsilon$.

2) ζ and η are non-negative integers such that $1\leqslant\zeta+\eta$.

3) $\left|arg Z(x^{\mu}(t-x)^{-\upsilon})\right| < \frac{1}{2}C^{(i)}\pi$, where $C^{(i)} = \sum_{j=1}^{n}\alpha_j - \tau_i\sum_{j=n+1}^{p_i}\alpha_{ji} + \sum_{j=1}^{m}\beta_j - \tau_i\sum_{j=m+1}^{q_i}\beta_{ji}$

4) $Re\,(\rho+\zeta h)+\mu\min\limits_{1\leqslant l\leqslant m}Re\left(\frac{b_l}{\beta_l}\right)>0$ and $Re\,(\sigma+\eta h)-\upsilon\max\limits_{1\leqslant l\leqslant n}Re\left(\frac{a_l-1}{\alpha_l}\right)<0.$

5) $1+\sum_{j=1}^{q'}F_j-\sum_{j=1}^{p'}E_j>0.$

Corollary 4.

$$\int_0^t x^{\rho-1}(t-x)^{\sigma-1}e^{-zx}{}_{p'}\psi_{q'}(ax^{\zeta}(t-x)^{\eta})\,\aleph(\ \mathrm{Zx}^{-\mu}(t-x)^{\upsilon}\ )\mathrm{d}x= e^{-zt}t^{\rho+\sigma-1}\sum_{u=0}^{\infty}\sum_{h=0}^{u}A_h$$

$$\frac{z^{u-h}}{(u-h)!}t^{(\zeta+\eta-1)h+u}\aleph^{m+1,\mathrm{n}+1}_{p_i+1,q_i+2,\tau_i;R}\left(\mathrm{Zt}^{-\mu+\upsilon}\ \middle|\ \begin{array}{c}(1\text{-u-}\sigma+(\eta-1)h-u;\upsilon),A_2\\ \cdot\\ \cdot\\ \cdot\\ (\rho+\zeta h;\mu),(1-\rho-\sigma-u-(\zeta+\eta-1)h;\upsilon-\mu),B_2\end{array}\right) \tag{4.7}$$

A_2 and B_2 are defined by (4.6).

under the same existence conditions 2), 3) and 5) that (4.6) and

$Re\,(\rho+\zeta h)-\mu\max_{1\leqslant l\leqslant n}Re\left(\frac{a_l-1}{\alpha_l}\right)<0$ and $Re\,(\sigma+\eta h)+\upsilon\min_{1\leqslant l\leqslant m}Re\left(\frac{b_l}{\beta_l}\right)>0.$

where $0<\upsilon, 0\leqslant\mu$ and $0\leqslant\upsilon-\mu$.

Remarks :
By using the similar methods, we can obtain the sames integrals about the multivariable A-function [6], the A-function [5], the modified multivariable H-function [9].
By using the corollaries studied, we can obtain the similar integrals concerning the I-function [11], the H-function ([3], [8]).

5. Conclusion.

The generalized multivariable Aleph-function presented in this paper is quite basic in nature. Therefore, on specializing the parameters of this function, we may obtain various other special functions of one and several variables such as Aleph-function of two variables ([7],[12]), multivariable I-function [13], I-function of two variables [14], H-function of two variables [15], Aleph-function, I-function, H-function, G-function and other. As special cases, and therefore, various unified integral presentations can be obtained as special cases of our results.

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