The Average Sum Method for the Unbalanced Assignment Problems

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Abstract- *Optimally assigning of jobs to the available resources is the fundamental requirement in the assignment problems to achieve the desired results. Several methods of assignment are described in the literature. In this research paper we have also inspiring from the Hungarian assignment method discovered an assignment method which is presented here in simple steps. Although the results are exactly equal to the results from the Hungarian assignment method but the Hungarian method takes more space, time and steps to solve numerical examples, also its step changes for different type of problems while our proposed method is simple, takes less time and does not alters the steps during the applications in the numerical examples. The presented algorithm is implemented on several sets of numerical examples and found that the obtained results are correct. The results achieved from the method are graphically represented here with a comparison.*

Keywords- *Unbalanced Assignment Problem, Hungarian Method, Optimization, Average Sum Method, Optimal Costs*.

I. INTRODUCTION

Several important methods for solving assignment problems (APs) have been so far presented in the history of assignment problems. Assignment problems (APs) are of great importance due to the reason that the proper assignment of tasks/jobs to the available processors/workers provides the optimal solutions to the given assignment problems. The meaning of all of these are one that is the efficient assignments of given jobs to the available resources/workers for providing the optimal solution to the given assignment problems. There exist many approaches which have been developed for finding optimal solutions to the assignment problems. Some important methods of assignment in the literature are Hungarian method by Kuhn and Munkres [1, 2]. Subject matter published in [3, 4] traces the history of these methods. For APs, there are alternate methods available [5, 6, 7]. Various methods have been so far presented for assignment problems in which the Hungarian method is more convenient method among them. This iterative method is based on the addition and subtraction of a constant value to each element of a row or column of the cost matrix, after that trying to achieve a complete assignment in terms of zeros. To avoid the complicacy of the Hungarian assignment method, another method of assignment is developed here which provides an efficient method of finding the optimal solution. It works on the principle of reduction of the given cost matrix. The optimal assignments can be achieved by reducing the given cost matrix having at least one zero in each row and column. The method is more important than the Hungarian assignment method because it is applicable for both balanced as well as for unbalanced assignment problems of minimization or maximization.

II. MATHEMATICAL REPRESENTATION OF AN ASSIGNMENT PROBLEM

Let c_{ij} be the cost if the j^{th} processor/person is assigned the i^{th} tasks/jobs, the problems is to find an assignment (which task should be assigned to which processor) so that the total cost for performing all tasks is minimum. This can be stated in the form of **n**x**n** cost matrix $[c_{ij}]$ or real numbers.

Tasks/Jobs

		1	2	----------- j -------		n
Processors/	1	C_{11}	C_{12}	…………… C_{ij}	…………...	C_{1n}
Persons	2	C_{21}	C_{22}	…………… C_{2j}	……..…....	C_{2n}
	¦	….	…..	…..		…..
	¦	….	…..	…..		…..
	i	C_{i1}	C_{i2}	……………. C_{ij}	....	C_{in}
	¦	¦	¦	¦		¦
	n	C_{n1}	C_{n2}	……………. C_{nj}	...……….	C_{nn}

Mathematically an assignment problem can be stated as follows:

Minimize the total cost

$$Z = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij} \quad \ldots (1)$$

where, $x_{ij} = \begin{cases} 1, & \text{if } j^{th} \text{ processor is assigned } i^{th} \text{ task} \\ 0, & \text{otherwise} \end{cases}$

Subject to the constraints

$$\sum_{i=1}^{n} x_{ij} = 1, \ (j = 1, 2, 3, \ldots\ldots\ldots, n) \quad \ldots (2)$$

and $$\sum_{j=1}^{n} x_{ij} = 1, \ (i = 1, 2, 3, \ldots\ldots\ldots, n) \quad \ldots (3)$$

The constraints equation (2) means that j^{th} processor can be assigned only one task and equation (3) means i^{th} task can be assigned to only one processor. Since x_{ij} = 0 or 1 for all i and j, it follows that exactly one variable in each of the constraints (1) and (3) is 1 and the other variables are 0.

III. THE AVERAGE SUM ASSIGNMENT METHOD (ASAM)

The average sum assignment method is applicable for all type of assignment problems (minimization, maximization, balanced and unbalanced assignment problems) to obtain the optimal cost/time and maximum throughput of the system. The method allocates in the zeros the average of the sum of perpendicular costs.

(A) ASAM FOR MINIMIZATION TYPE ASSIGNMENT PROBLEM

The computational procedure of new developed assignment method (minimization case) can be summarized in the following steps:

Step 1: *Construct the given cost matrix from the given problem*

(i) If the number of rows is equal to the no. of columns, go to step 3.
(ii) If the number of rows is not equal to the no. of columns, go to step 2.

Step 2: Add dummy row(s) or column(s) by taking each of the cost entries equal to zero.

Step 3: *Find at least one zero cost in each row and column*

(i) Identify the smallest cost in each row and subtract it from every cost of that row.
(ii) Identify the smallest cost in each column and subtract it from every cost of that column.
(iii) Thus, we can get modified cost matrix having at least one zero cost in each row and column.

Step 4: *In modified cost matrix obtained in step 3, search for an optimal assignment as*

(i) Start from the first zero cost in the modified cost matrix, allocate the average sum of each cost of row and column perpendicular to it by the following formula, and allocate the obtained value in a small bracket ( ).

$$Average\ Sum(A.S.) = \frac{Sum\ of\ costs\ perpendicular\ to\ zero}{Number\ of\ costs\ added}$$

(ii) Continue this assignment of average sum for all zero cost entries in the modified matrix till all the zero costs get allocated.
(iii) Observe the largest allocated average sum in the matrix. Get this allocation as optimal assignment.
(iv) If the equal largest average sum is observed at more than one zero cost entries, then make the optimal assignment in the zero cost which contains the lowest cost in the given matrix

Step 5: *Now form the reduced cost matrix for the optimal assignment and*

(i) Eliminate the corresponding row and column of the achieved optimal assignment and construct reduced cost matrix of rows and columns.
(ii) Go to step 3, otherwise step 4 for the assignment.

Step 6: Go to step 5, for further optimal assignment in reduced cost matrix.

Step 7: Repeat step 3 to 6, until optimal solution is attained.

Step 8: End of algorithm.

(B) ASAM FOR MAXIMIZATION TYPE ASSIGNMENT PROBLEM

This assignment method (maximization case) can be summarized in the following steps:

Step 0: Convert the given maximization assignment problem into minimization assignment problem by subtracting from the highest cost value (c_{ij}), all values of the cost matrix.

Step 1: Now repeat all the steps of the minimization case III (A) till the optimal solution.

(C) ASAM FOR UNBALANCED TYPE ASSIGNMENT PROBLEM

An assignment problem is said to be unbalanced if the number of processors/persons is not equal to the number of tasks/jobs that is the cost matrix of an unbalanced assignment problem is not a square matrix. Therefore, we make the cost matrix square by adding row(s)/column(s) with zero cost values, then adopt the same computational procedure as for minimization case III (A) to obtain the optimal solution of the given unbalanced assignment problem.

(IV) IMPLEMENTATION OF THE AVERAGE SUM ASSIGNMENT METHOD

To test the validity and effectiveness of the proposed method, it is implemented on several sets of numerical examples given here below:

(A) IMPLEMENTATION ON BALANCED ASSIGNMENT PROBLEMS

Example 1: Janta PG College has one professor, two associate professors and two assistant professors in the department of mathematics and five tasks are to be performed on one-to-one basis at the time of NAAC inspection. The execution time (in hours) of each task is given in the matrix

Tasks→ Professors↓	t_1	t_2	t_3	t_4	t_5
P_1	10	5	13	15	16
P_2	3	9	18	13	6
P_3	10	7	2	2	2
P_4	7	11	9	7	12
P_5	7	9	10	4	12

How should be assign the tasks to professors so that the time can be minimized?

Applying the new assignment method to minimize the overall execution time.

Step 1 and Step 2 of the algorithm III (A) follows here. Go to step 3.

Step 3: *Find at least one zero cost in each row and column*

(i) Subtract minimum, 5 from each element of row P_1, 3 from row P_2, 2 from row P_3, 7 from row P_4, and 4 from row P_5.

Tasks→ Professors↓	t_1	t_2	t_3	t_4	t_5
P_1	5	0	8	10	11
P_2	0	6	15	10	3
P_3	8	5	0	0	0
P_4	0	4	2	0	5
P_5	3	5	6	0	8

(ii) Since there is at least one zero in each row and column of the above matrix, it is a modified matrix, so go to next step.

Step 4: *In modified cost matrix obtained in step 3, search for an optimal assignment as below*

(i) Start from the first zero cost of the first row and allocate there the average of sum of each cost perpendicular to this zero (row P_1 and column t_2) in modified matrix by the formula described in ASAM step 4(i).

Tasks→ Professors↓	t_1	t_2	t_3	t_4	t_5
P_1	5	0(6.7)	8	10	11
P_2	0	6	15	10	3
P_3	8	5	0	0	0
P_4	0	4	2	0	5
P_5	3	5	6	0	8

(ii) Continue the assignment in the same way we get the assigned matrix

Tasks→ Professors↓	t_1	t_2	t_3	t_4	t_5
P_1	5	0(6.7)	8	10	11
P_2	0(6.2)	6	15	10	3
P_3	8	5	0(5.5)	0(4.1)	0(5)
P_4	0(3.3)	4	2	0(3.8)	5
P_5	3	5	6	0(5.2)	8

(iii) Since the largest average sum is at (P_1, t_2), so the optimal assignment is $P_1 \rightarrow t_2$.

Step 5: *Now form the reduced cost matrix for the optimal assignment*

(i) Eliminate row P_1 and column t_2 from the optimal assigned matrix above and form the reduced cost matrix as

Tasks→ Professors↓	t_1	t_3	t_4	t_5
P_2	0	15	10	3
P_3	8	0	0	0
P_4	0	2	0	5
P_5	3	6	0	8

(ii) Repeat step 1 to 4 of the method III (A) and get the optimal assigned matrix as

Tasks→ Professors↓	t_1	t_3	t_4	t_5
P_2	0(6.5)	15	10	3
P_3	8	0(5.1)	0(3)	0(4)
P_4	0(3)	2	0(2.8)	5
P_5	3	6	0(4.5)	8

(iii) Since the largest sum is at (P_2, t_1), so the optimal assignment is $P_2 \rightarrow t_1$.

Step 6: Repeat step 5 to get reduced cost matrix.

(i) Eliminate row P_2 and column t_1 from the optimal assigned matrix above and form the reduced cost matrix as

Tasks→ Professors↓	t_3	t_4	t_5
P_3	0	0	0
P_4	2	0	5
P_5	6	0	8

(ii) Go to step 1 to 4 and get the optimal assignment as

Tasks→ Professors↓	t_3	t_4	t_5
P_3	0(2)	0(0)	0(3.2)
P_4	2	0(1.7)	5
P_5	6	0(3.5)	8

(iii) Since the largest sum is at (P_5, t_4), so the optimal assignment is $P_5 \rightarrow t_4$.

Step 7: Deleting row P_5 and column t_4 reduced cost matrix as

Tasks→ Professors↓	t_3	t_5
P_3	0	0
P_4	2	5

Go to step 1 to 4, we get the assignment as below

Tasks→ Professors↓	t_3	t_5
P_3	0(0)	0(1.5)
P_4	0(1.5)	3

Repeat step 5 to achieve optimal assignment as, $P_3 \rightarrow t_5$ and $P_4 \rightarrow t_3$.

The final optimal assignment table is

Professors	Tasks	Time (in hrs)
P_1	t_2	5
P_2	t_1	3
P_3	t_5	2
P_4	t_3	9
P_5	t_4	4
Total optimal time = 23		

Hence, the professors of the department should be assigned the tasks according to the final assignment table so as to minimize the overall execution time. The total optimal time is 23 hrs.

(B) IMPLEMENTATION ON BALANCED MAXIMIZATION ASSIGNMENT PROBLEMS

Example 2: A marketing company has recruited five salesmen S_1, S_2, S_3, S_4, and S_5 to five sales districts Auraiya(A), Etawah(E), Kanpur(K), Jhansi(J) and Lucknow(L). Considering the capabilities of salesmen and the nature of districts, the marketing manager estimates that the sales per month (in hundred rupees) for each salesman in each district would be as follows:

Districts→ Salesmen↓	A	E	K	J	L
S_1	32	38	40	28	40
S_2	40	24	28	21	36
S_3	41	27	33	30	37
S_4	22	38	41	36	36
S_5	29	33	40	35	39

Find the optimum assignment of salesmen to districts that will result in maximum sales.

To solve this assignment problem, first we convert it to the minimization assignment problem.

Step 0: Since the maximum c_{ij} is 41, subtracting all cost values from it we will have the following minimization problem.

Districts→ Salesmen↓	A	E	K	J	L
S_1	9	3	1	13	1
S_2	1	17	13	20	5
S_3	0	14	8	11	4
S_4	19	3	0	5	5
S_5	12	8	1	6	2

Step 1 and Step 2 of method III (A) follow here. Now go to Step 3.

Step 3: *Find at least one zero cost in each row and column*

Applying step 3 of the algorithm described in (A) we got the modified cost matrix as:

Districts→ Salesmen↓	A	E	K	J	L

S_1	8	0	0	7	0
S_2	0	14	12	14	4
S_3	0	12	8	6	4
S_4	19	1	0	0	5
S_5	11	5	0	0	1

Step 4: *In modified cost matrix obtained in step 3, search for an optimal assignment as below*

(i) Start from the first zero of the first row in modified cost matrix at (S_1, E) and allocate there the average of sum of each cost of row S_1 and column E. The average of sum is equal to 5.8. The modified matrix becomes

Districts→ Salesmen↓	A	E	K	J	L
S_1	8	0(5.8)	0	7	0
S_2	0	14	12	14	4
S_3	0	12	8	6	4
S_4	19	1	0	0	5
S_5	11	5	0	0	1

(ii) Continue the assignment in the same way for all zero cost entries, we got the assigned matrix as

Districts→ Salesmen↓	A	E	K	J	L
S_1	8	0(5.8)	0(4.3)	7	0(3.6)
S_2	0(10.2)	14	12	14	4
S_3	0(8.5)	12	8	6	4
S_4	19	1	0(5.6)	0(6.5)	5
S_5	11	5	0(4.6)	0(5.5)	1

(iii) Since the largest average of sum is at (S_2, A), so the optimal assignment is $S_2 \rightarrow$ A.

Step 5: *Now form the reduced cost matrix for the optimal assignment*

(i) Eliminate row S_2 and column A from the optimal assigned matrix above and form the reduced cost matrix as

Districts→ Salesmen↓	E	K	J	L
S_1	0	0	7	0
S_3	12	8	6	4
S_4	1	0	0	5
S_5	5	0	0	1

(ii) Repeat step 1 to 4 of the method III (A) and get the optimal assigned matrix as

Districts→ Salesmen↓	E	K	J	L
S_1	0(3.5)	0(1.8)	7	0(2.1)
S_3	8	4	2	0(3.3)
S_4	1	0(1.6)	0(2.5)	5
S_5	5	0(1.6)	0(2.5)	1

(iii) Since the largest average of sum is at (S_1, E), so the optimal assignment is $S_1 \rightarrow$ E.

Step 6: Repeat step 5 to get reduced cost matrix.

(i) Eliminate row S_1 and column E from the optimal assigned matrix above and form the reduced cost matrix as

Districts→	K	J	L

Salesmen↓			
S_3	4	2	0
S_4	0	0	5
S_5	0	0	1

(ii) Go to step 1 to 4 and get the optimal assignment as

Districts→ Salesmen↓	K	J	L
S_3	4	2	0(3)
S_4	0(2.2)	0(1.7)	5
S_5	0(1.2)	0(0.7)	1

(iii) Since the largest average of sum is at (S_3, L), so the optimal assignment is $S_3 \rightarrow$ L.

Step 7: Deleting row S_3 and column L reduced cost matrix as

Districts→ Salesmen↓	K	J
S_4	0(0)	0(0)
S_5	0(0)	0(0)

Since all the average of sum entries in zero costs are equal to zero. Therefore start assigning S_4 or S_5 to minimum cost in the given table and remaining S_4 or S_5 to other cost value. The optimal assignments are $S_5 \longrightarrow$ J and $S_4 \longrightarrow$ K.

The final optimal assignment table is

Salesmen	Districts	Maximum Profit (in hundred rupees)
S_1	E	38
S_2	A	40
S_3	L	37
S_4	K	41
S_5	J	35
Total optimal cost = 191		

Hence, the salesmen should be assigned the districts according to the final assignment table to maximize the overall sales. The maximum profit (in hundred rupees) corresponding to each district is presented in the final assignment table above.

(C) IMPLEMENTATION ON UNBALANCED MAXIMIZATION ASSIGNMENT PROBLEMS

Example 3: The owner of a small machine shop has four machinists M_1, M_2, M_3 and M_4 to be assign five jobs J_1, J_2, J_3, J_4 and J_5 who are capable to operate any of the machines.

Jobs→ Machinists↓	J_1	J_2	J_3	J_4	J_5
M_1	62	78	50	101	82
M_2	71	84	61	73	59
M_3	87	92	111	71	81
M_4	48	64	87	77	80

Find the assignment of jobs to machinists that will result in a maximum profit. Which job should be declined?

This problem is an unbalanced and maximization type assignment problem. For the solution of this problem we convert it to the balanced assignment problem that is in a square matrix form by entering a new row with zero costs.

Jobs→ Machinists↓	J_1	J_2	J_3	J_4	J_5

M_1	62	78	50	101	82
M_2	71	84	61	73	59
M_3	87	92	111	71	81
M_4	48	64	87	77	80
M_5	0	0	0	0	0

Now the problem is of maximization type, therefore converting this given maximization assignment problem into minimization assignment problem by subtracting from the highest cost value (c_{ij}), all values of the cost matrix.

Jobs→ Machinists↓	J_1	J_2	J_3	J_4	J_5
M_1	49	33	61	10	29
M_2	40	30	50	38	52
M_3	24	19	0	40	30
M_4	63	47	24	34	31
M_5	111	111	111	111	111

Applying the average sum method III (A) to solve this problem.

Follow step 1 and step 2 of the method III (A) than go to step 3.

Find at least one zero in each row and column

(i) Subtract minimum 10 from each element of row M_1, 30 from row M_2, 0 from row M_3, 24 from row M_4, and 111 from row M_5.

Jobs→ Machinists↓	J_1	J_2	J_3	J_4	J_5
M_1	39	23	51	0	19
M_2	10	0	20	8	22
M_3	24	19	0	40	30
M_4	39	23	0	10	7
M_5	0	0	0	0	0

(ii) Since there is at least one zero in each column of the above matrix, it is a modified matrix, so go to next step.

In modified cost matrix above, search for an optimal assignment as below

(i) Start from the first zero of the first row and allocating the average of sum of each cost perpendicular to this zero by the formula described in ASAM step 4(i)

Jobs→ Machinists↓	J_1	J_2	J_3	J_4	J_5
M_1	39	23	51	0(23.7)	19
M_2	10	0	20	8	22
M_3	24	19	0	40	30
M_4	39	23	0	10	7
M_5	0	0	0	0	0

(ii) Continue this assignment of average sum for all zero cost entries in the modified cost matrix till all the zeros get allocated.

Jobs→ Machinists↓	J_1	J_2	J_3	J_4	J_5
M_1	39	23	51	0(23.7)	19
M_2	10	0(15.6)	20	8	22
M_3	24	19	0(23)	40	30
M_4	39	23	0(18.7)	10	7

M_5	0(14)	0(8.1)	0(8.8)	0(7.2)	0(8.8)

(iii) The largest average sum 23.7 is found in the first row and forth column. Therefore optimal assignment is $M_1 \rightarrow J_4$.

Now form the reduced cost matrix for the optimal assignment

(i) Eliminate first row and forth column from the achieved optimal assigned matrix above and form the reduced cost matrix as

Jobs→ Machinists↓	J_1	J_2	J_3	J_5
M_2	10	0	20	22
M_3	24	19	0	30
M_4	39	23	0	7
M_5	0	0	0	0

(ii) Go to step 1 to step 4 of method III (A) and find optimal assignment as

Jobs→ Machinists↓	J_1	J_2	J_3	J_5
M_2	10	0(15.6)	20	22
M_3	24	19	0(15.5)	30
M_4	39	23	0(14.8)	7
M_5	0(12.1)	0(7)	0(3.3)	0(9.8)

The largest average sum 15.6 is in second row and second column. So the optimal assignment is $M_2 \rightarrow J_2$.

Deleting row M_2 and column J_2 from above matrix we get reduced cost matrix as

Jobs→ Machinists↓	J_1	J_3	J_5
M_3	24	0(13.5)	30
M_4	39	0(11.5)	7
M_5	0(15.7)	0(0)	0(9.2)

Repeat step 5 to obtain optimal assignment $M_5 \rightarrow J_1$ as maximum average sum exists in fifth row and first column

Again reduced cost matrix by eliminating M_5 and J_1 is

Jobs→ Machinists↓	J_3	J_5
M_3	0(11.5)	23
M_4	0(0)	0(11.5)

Allocating the maximum average sum by III (A) we have assignment as $M_3 \rightarrow J_3$ and $M_4 \rightarrow J_5$.

The final optimal assignment table

Jobs	Machinists	Costs
J_1	-	-
J_2	M_2	84
J_3	M_3	111
J_4	M_1	101
J_5	M_4	80
	Maximum profit =	376

Hence the optimal assignment of jobs should be according to the final optimal assignment table above so that to achieve the maximum profit. The maximum profit is 376. Here the job J_1 has not been assigned to any of the machinists, therefore, it should be declined.

(D) IMPLEMENTATION ON UNBALANCED ASSIGNMENT PROBLEMS

Example 4: A distributed computing system (DCS) has four tasks T = {t_1, t_2, t_3, t_4} to be performed by the three processors P = {P_1, P_2, P_3} of the system. The processors differ in efficiency. The estimates of time, each processor would take to perform is given as

Time→ Processors↓	t_1	t_2	t_3	t_4
P_1	9	13	35	18
P_2	26	27	20	15
P_3	15	6	15	20

How would be assign the tasks on one to one basis to minimize the total man-hours?

To solve the above unbalanced assignment problem we convert it to the balanced assignment problem that is in a square matrix form by entering a new row with zero costs.

Time→ Processors↓	t_1	t_2	t_3	t_4
P_1	9	13	35	18
P_2	26	27	20	15
P_3	15	6	15	20
P_4	0	0	0	0

Now solving it applying the average sum method used in III (A) we will have the following final optimal assignment table

Processors	Tasks	Time (in hrs)
P_1	t_1	9
P_2	t_4	15
P_3	t_2	6
P_4	t_3	0
Total optimal time = 30		

Hence the available processors of the system should be assigned according to the final optimal assignment table to minimize the total man-hours.

V. RESULTS AND DISCUSSIONS

The average sum assignment method is implemented on different types of assignment problems given in the form of numerical examples 1, 2, 3 and 4 above, the obtained results are 23, 191, 376 and 30. These results of the numerical examples are equal to that of the results by Hungarian assignment method. This new proposed method of assignment provides the optimal solutions when applied to the assignment problems of types balanced, minimization, maximization and unbalanced assignment problems. The achieved results are also represented graphically in the figure plotted below.

VI. CONCLUSIONS

The proposed average sum assignment methods III (A), (B) and (C) are applied here in several sets of different types of numerical examples and it is found that the discovered methods provide the optimal solutions exactly equal to the Hungarian method. The obtained results of the numerical examples are also demonstrated with the results of Hungarian method in the comparison graph below. The Hungarian method takes more time and space to solve the assignment problems, also its steps change for different types of problems but the simple steps in less iterations of the discovered method remains the same even for the different types of assignment problems. Thus the projected method is valid and relevant for all kinds of assignment problems. This method fulfils the requirement of efficiently and optimally assigning the jobs/tasks to the available resources.

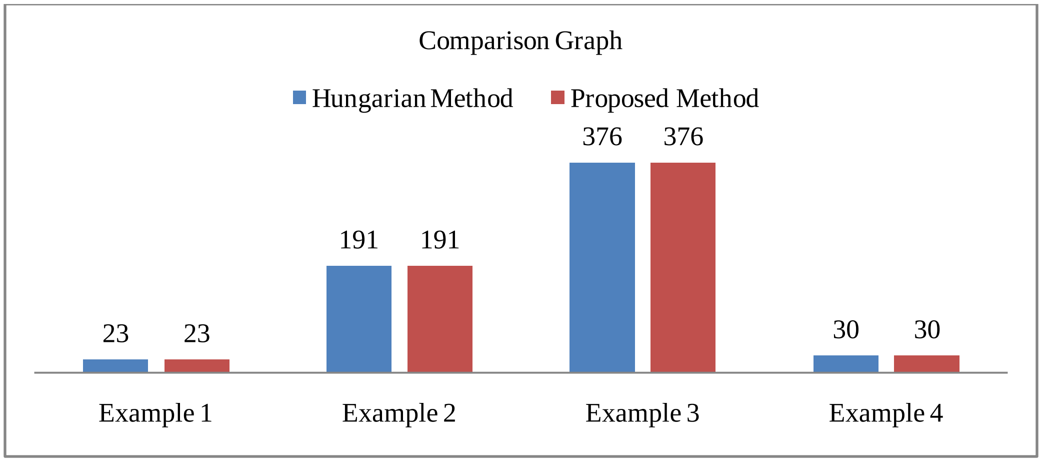

Figure 1: Comparison graph between Hungarian algorithm and proposed algorithm

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