N-fractional calculus and multivariable I-function and generalized multivariable polynomials

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Abstract
By the application of a result given by Nishimoto ([13], (2006),p. 35-44), we investigate the differintegrals of multivariable I-function and class of multivariable polynomials containing general power functions in its argument $\prod_{j=1}^{r}((z_j-A_j)^{\tau_j}-f_j)^{\sigma_j}$. The results derived are of most general character includes, among others, the results for differintegrals of power functions given by Nishimoto [10,11,12,13], Saxena and Nishimoto [20], Romero et al. [18,19], Gupta et al. [6] and Jaimini and Nishimoto [7] and others. At the end, we shall see several corollaries and particular cases.

Keywords: General class of multivariable polynomials, fractional calculus, Mellin-barnes integrals contour, multivariable I-function, H-function.

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1.Introduction and preliminaries.

Recently, Saxena and Nishimoto [20] have studied the N-fractional calculus and multivariable H-function with general arguments. In our paper, we evaluate the r-dimensional N-fractional calculus concerning a class of multivariable polynomials defined by Srivastava [16] and the multivariable I-function defined by Prasad [25] with general arguments.

Following Nishimoto [11], we define the r-dimensional N-fractional diffeintegral of a function of r-variables in the following form :

Let $D_j=\left\{\underset{-}{D_j},\underset{+}{D_j}\right\}, C_j=\left\{\underset{-}{C_j},\underset{+}{C_j}\right\}$

$\underset{-}{C_j}$ be a curve along the cut joining two points z_j and $-\infty+\omega Im(z_j)$,

$\underset{+}{C_j}$ be a curve along the cut joining two points z_j and $\infty+\omega Im(z_j)$,

$\underset{-}{D_j}$ be a domain surrounded by $\underset{-}{C_j}$,

$\underset{+}{D_j}$ be a domain surrounded by $\underset{+}{C_j}$.

Further, let $f=f(z_1,\cdots,z_r)$ be an analytic function of r-variables in a domain $D=D_1\times D_2\times\cdots\times D_r$ where each D_j is surrounded by C_j then the fractional differintegral of an arbitrary order υ_j for $z_j(\upsilon_j\in\mathbb{R},z_j\in\mathbb{C},j=1,\cdots,r)$of the function $f(z_1,\cdots,z_r)$, if$|(f)_{\upsilon_1,\cdots,\upsilon_r}|$ exists, is defined by

$$f_{\upsilon_1,\cdots,\cdots,\upsilon_r}=f_{\upsilon_1,\cdots,\cdots,\upsilon_r}(z_1,\cdots,z_r)={}_{C_1,\cdots,C_r}f_{\upsilon_1,\cdots,\cdots,\upsilon_r}(z_1,\cdots,z_r)$$

$$=\frac{\prod_{j=1}^{r}\Gamma(\upsilon_j+1)}{(2\pi\omega)^r}\int_{C_1}\cdots\int_{C_r}\frac{f(\zeta_1,\cdots,\zeta_r)}{\prod_{j=1}^{r}(\zeta_j-z_j)^{\upsilon_j+1}}\mathrm{d}\zeta_1\cdots\mathrm{d}\zeta_r \tag{1.1}$$

$$(f)_{-m_1,\cdots,-m_r}=\lim_{\upsilon_j\to -m_j}f_{\upsilon_1,\cdots,\upsilon_r}(m_j\in\mathbb{Z}^+,j=1,\cdots,r)$$

where

$$-\pi\leqslant arg(\zeta_j-z_j)\leqslant\pi \text{ for } C_j=\underline{C_j} \tag{1.2}$$

The generalized polynomials of multivariables defined by Srivastava [25], is given in the following manner :

$$S_{N_1,\cdots,N_r}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_r}[y_1,\cdots,y_r] = \sum_{K_1=0}^{[N_1/\mathfrak{M}_1]} \cdots \sum_{K_r=0}^{[N_r/\mathfrak{M}_r]} \frac{(-N_1)_{\mathfrak{M}_1 K_1}}{K_1!} \cdots \frac{(-N_r)_{\mathfrak{M}_r K_r}}{K_r!} A[N_1,K_1;\cdots;N_r,K_r] y_1^{K_1} \cdots y_r^{K_r} \quad (1.3)$$

where $\mathfrak{M}_1,\cdots,\mathfrak{M}_r$ are arbitrary positive integers and the coefficients $A[N_1,K_1;\cdots;N_r,K_r]$ are arbitrary constants, real or complex.

We shall note

$$a_r = \frac{(-N_1)_{\mathfrak{M}_1 K_1}}{K_1!} \cdots \frac{(-N_r)_{\mathfrak{M}_r K_r}}{K_r!} A[N_1,K_1;\cdots;N_r,K_r] \quad (1.4)$$

The multivariable I-function defined by Prasad [15] generalizes the multivariable H-function studied by Srivastava and Panda [26,27]. This function of r-variables is defined in term of multiple Mellin-Barnes type integral :

$$I(z_1,z_2,\cdots,z_r) = I_{p_2,q_2,p_3,q_3;\cdots;p_r,q_r:p^{(1)},q^{(1)};\cdots;p^{(r)},q^{(r)}}^{0,n_2;0,n_3;\cdots;0,n_r:m^{(1)},n^{(1)};\cdots;m^{(r)},n^{(r)}} \left(\begin{array}{c|l} \mathrm{z}_1 & (\mathrm{a}_{2j};\alpha'_{2j},\alpha''_{2j})_{1,p_2};\cdots; \\ \cdot & \\ \cdot & \\ \cdot & \\ \mathrm{z}_r & (\mathrm{b}_{2j};\beta'_{2j},\beta''_{2j})_{1,q_2};\cdots; \end{array} \right.$$

$$\left. \begin{array}{l} (\mathrm{a}_{rj};\alpha_{rj}^{(1)},\cdots,\alpha_{rj}^{(r)})_{1,p_r} : (a_j^{(1)},\alpha_j^{(1)})_{1,p^{(1)}};\cdots;(a_j^{(r)},\alpha_j^{(r)})_{1,p^{(r)}} \\ \\ (\mathrm{b}_{rj};\beta_{rj}^{(1)},\cdots,\beta_{rj}^{(r)})_{1,q_r} : (b_j^{(1)},\beta_j^{(1)})_{1,q^{(1)}};\cdots;(b_j^{(r)},\beta_j^{(r)})_{1,q^{(r)}} \end{array} \right)$$

$$= \frac{1}{(2\pi\omega)^r} \int_{L_1} \cdots \int_{L_r} \phi(s_1,\cdots,s_r) \prod_{i=1}^{r} \phi_i(s_i) z_i^{s_i} \mathrm{d}s_1 \cdots \mathrm{d}s_r \quad (1.5)$$

where

$$\phi_i(s_i) = \frac{\prod_{j=1}^{m^{(i)}} \Gamma(b_j^{(i)} - \beta_j^{(i)} s_i) \prod_{j=1}^{n^{(i)}} \Gamma(1 - a_j^{(i)} + \alpha_j^{(i)} s_i)}{\prod_{j=m^{(i)}+1}^{q^{(i)}} \Gamma(1 - b_j^{(i)} + \beta_j^{(i)} s_i) \prod_{j=n^{(i)}+1}^{p^{(i)}} \Gamma(a_j^{(i)} - \alpha_j^{(i)} s_i)} \ , i = 1,\cdots,r \quad (1.6)$$

and

$$\phi(s_1,\cdots,s_r) = \frac{\prod_{j=1}^{n_2} \Gamma(1 - a_{2j} + \sum_{i=1}^{2} \alpha_{2j}^{(i)} s_i) \prod_{j=1}^{n_3} \Gamma(1 - a_{3j} + \sum_{i=1}^{3} \alpha_{3j}^{(i)} s_i) \cdots}{\prod_{j=n_2+1}^{p_2} \Gamma(a_{2j} - \sum_{i=1}^{2} \alpha_{2j}^{(i)} s_i) \prod_{j=n_3+1}^{p_3} \Gamma(a_{3j} - \sum_{i=1}^{3} \alpha_{3j}^{(i)} s_i) \cdots}$$

$$\frac{\cdots \prod_{j=1}^{n_r} \Gamma(1 - a_{rj} + \sum_{i=1}^{r} \alpha_{rj}^{(i)} s_i)}{\cdots \prod_{j=n_r+1}^{p_r} \Gamma(a_{rj} - \sum_{i=1}^{r} \alpha_{rj}^{(i)} s_i) \prod_{j=1}^{q_2} \Gamma(1 - b_{2j} - \sum_{i=1}^{2} \beta_{2j}^{(i)} s_i)}$$

$$\times \frac{1}{\prod_{j=1}^{q_3} \Gamma(1 - b_{3j} + \sum_{i=1}^{3} \beta_{3j}^{(i)} s_i) \cdots \prod_{j=1}^{q_r} \Gamma(1 - b_{rj} - \sum_{i=1}^{r} \beta_{rj}^{(i)} s_i)} \quad (1.7)$$

The defined integral of the above function, the existence and convergence conditions, see Y.N. Prasad [15]. Throughout the present document, we assume that the existence and convergence conditions of the multivariable I-function. The condition for absolute convergence of multiple Mellin-Barnes type contour (1.7) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$|argz_i| < \frac{1}{2}\Omega_i \pi$, where

$$\Omega_i=\sum_{k=1}^{n^{(i)}}\alpha_k^{(i)}-\sum_{k=n^{(i)}+1}^{p^{(i)}}\alpha_k^{(i)}+\sum_{k=1}^{m^{(i)}}\beta_k^{(i)}-\sum_{k=m^{(i)}+1}^{q^{(i)}}\beta_k^{(i)}+\left(\sum_{k=1}^{n_2}\alpha_{2k}^{(i)}-\sum_{k=n_2+1}^{p_2}\alpha_{2k}^{(i)}\right)+\cdots+$$

$$\left(\sum_{k=1}^{n_s}\alpha_{sk}^{(i)}-\sum_{k=n_s+1}^{p_s}\alpha_{sk}^{(i)}\right)-\left(\sum_{k=1}^{q_2}\beta_{2k}^{(i)}+\sum_{k=1}^{q_3}\beta_{3k}^{(i)}+\cdots+\sum_{k=1}^{q_s}\beta_{sk}^{(i)}\right) \tag{1.8}$$

where $i=1,\cdots,r$. The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable I-function.We may establish the the asymptotic expansion in the following convenient form :

$$I(z_1,\cdots,z_r)=0(\,|z_1|^{\alpha_1},\cdots,|z_r|^{\alpha_r}\,)\,,max(\,|z_1|,\cdots,|z_r|\,)\rightarrow 0$$

$$I(z_1,\cdots,z_r)\;=0(\,|z_1|^{\beta_1},\cdots,|z_r|^{\beta_r}\,)\,,min(\,|z_1|,\cdots,|z_r|\,)\rightarrow\infty$$

where $k=1,\cdots,r:\alpha_k'=min[Re(b_j^{(k)}/\beta_j^{(k)})],j=1,\cdots,m^{(k)}$ and

$$\beta_k'=max[Re((a_j^{(k)}-1)/\alpha_j^{(k)})],j=1,\cdots,n^{(k)}$$

In this paper, we shall note

$$U=p_2,q_2;p_3,q_3;\cdots;p_{r-1},q_{r-1} \tag{1.9}$$

$$V=0,n_2;0,n_3;\cdots;0,n_{r-1} \tag{1.10}$$

$$\mathbf{A}=(a_{2k};\alpha_{2k}^{(1)},\alpha_{2k}^{(2)})_{1,p_2};\cdots;(a_{(r-1)k};\alpha_{(r-1)k}^{(1)},\alpha_{(r-1)k}^{(2)},\cdots,\alpha_{(r-1)k}^{(r-1)})_{1,p_{r-1}}:\;(a_{rk};\alpha_{rk}^{(1)},\alpha_{rk}^{(2)},\cdots,\alpha_{rk}^{(r)})_{1,p_r} \tag{1.11}$$

$$\mathbf{B}=(b_{2k};\beta_{2k}^{(1)},\beta_{2k}^{(2)})_{1,q_2};\cdots;(b_{(r-1)k};\beta_{(r-1)k}^{(1)},\beta_{(r-1)k}^{(2)},\cdots,\beta_{(r-1)k}^{(r-1)})_{1,q_{r-1}}:(b_{rk};\beta_{rk}^{(1)},\beta_{rk}^{(2)},\cdots,\beta_{rk}^{(r)})_{1,q_r} \tag{1.12}$$

2. Required results.

Lemma 1.

Let $f(z_1,\cdots,z_r)=\prod_{j=1}^{r}((z_j-A)^{\tau_j}-f_j)^{\sigma_j}$

be an analytic function having no branch points inside or on C_j, then r-dimensional N-fractional differintegral is given by

$$f_{\upsilon_1,\cdots,\upsilon_r}(z_1,\cdots,z_r)=\left\{\prod_{j=1}^{r}[((z_j-A)^{\tau_j}-f_j)^{\sigma_j}]\right\}_{\upsilon_1,\cdots,\upsilon_r}=\prod_{j=1}^{r}[((z_j-A)^{\tau_j}-f_j)^{\sigma_j}]_{\upsilon_j} \tag{2.1}$$

see Saxena and Nishimoto [20] about the proof.

Lemma 2.

By using the lemma 1 and Nishimoto's result [13], we obtain

$$f_{\upsilon_1,\cdots,\upsilon_r}(z_1,\cdots,z_r)=e^{-\omega\pi\sum_{j=1}^{r}\upsilon_j}\prod_{j=1}^{r}\left[\frac{(z_j-A)^{\sigma_j\tau_j-\upsilon_j}}{\Gamma(-\sigma_j)}\sum_{l_j=0}^{\infty}\frac{\Gamma(l_j-\sigma_j)\Gamma(\upsilon_j+l_j\tau_j-\sigma_j\tau_j)}{\Gamma(l_j\tau_j-\sigma_j\tau_j)}\left(\frac{f_j}{(z_j-A)^{\tau_j}}\right)^{l_j}\right] \tag{2.2}$$

Provided that

$\left|\frac{\Gamma(\upsilon_j+l_j\tau_j-\sigma_j\tau_j)}{\Gamma(l_j\tau_j-\sigma_j\tau_j)}\right|<\infty, z_j\neq A, (z_j-A)^{\tau_j}\neq f_j, \sigma_j\in\mathbb{C}, \tau_j\in\mathbb{R}^+, \upsilon_j\in\mathbb{R}$ and $\left|\frac{f_j}{(z_j-A)^{\tau_j}}\right|<1$ for $j=1,\cdots,r$

It is interesting to note that for $\tau_1 = \cdots = \tau_r = 1$, (2.2) reduces to a result given by Garg et al. ([3],p. 191, eq. (2.1)).

Lemma 3.

$$[((z-A)^\tau - f)((z-A)^\tau - g)]_\upsilon = e^{-\omega\pi\upsilon} \sum_{u=0}^{\infty} \frac{(-\rho)_u}{u!}(g-f)^u \sum_{l=0}^{\infty} \frac{(z-A)^{(\rho+\sigma-u)\tau-\upsilon}}{\Gamma(u-\sigma-\rho)}$$

$$\frac{\Gamma(l+u-\sigma-\rho)\Gamma(\upsilon+(u+l-\sigma-\rho)\tau)}{\Gamma((l+u-\sigma-\rho)\tau)}\left(\frac{f}{(z-A)^\tau}\right)^l \tag{2.3}$$

where $\left|\frac{\Gamma(\upsilon+(u+l-\sigma-\rho)\tau)}{\Gamma((l+u-\sigma-\rho)\tau)}\right| < \infty$, $z^\tau \neq f, , \sigma \in \mathbb{C}, \tau \in \mathbb{R}^+, \upsilon \in \mathbb{R}$ and $\left|\frac{f}{(z-A)^\tau}\right| < 1$.

Proof

$$[((z-A)^\tau - f)((z-A)^\tau - g)]_\upsilon = \sum_{u=0}^{\infty} \frac{(-\rho)_u}{u!}(g-f)^u \Big[((z-A)^\tau - f)_{\sigma+\rho-u}\Big]_\upsilon = e^{-\omega\pi\upsilon}$$

$$\sum_{u=0}^{\infty} \frac{(-\rho)_u}{u!}(g-f)^u \sum_{l=0}^{\infty} \frac{(z-A)^{(\rho+\sigma-u)\tau-\upsilon}}{\Gamma(u-\sigma-\rho)} \frac{\Gamma(l+u-\sigma-\rho)\Gamma(\upsilon+\tau(l+u-\sigma-\rho)\tau_j)}{\Gamma(\tau(l+u-\sigma-\rho)\tau_j)}\left(\frac{f}{(z-A)^\tau}\right)^l$$

Lemma 4.

$$[((z_j-A)^{\tau_j} - f_j)^{\sigma_j}((z_j-A)^{\tau_j} - g_j)^{\rho_j}]_{\upsilon_1,\cdots,\upsilon_r} = \prod_{j=1}^{r} e^{-\omega\pi\upsilon_j} \sum_{u_j=0}^{\infty} \frac{(-\rho_j)_{u_j}}{u_j!}(g_j-f_j)^{u_j} \sum_{l_j=0}^{\infty} \frac{(z_j-A)^{(\rho_j+\sigma_j-u_j)\tau_j-\upsilon_j}}{\Gamma(u_j-\sigma_j-\rho_j)}$$

$$\frac{\Gamma(l_j+u_j-\sigma_j-\rho_j)\Gamma(\upsilon_j+(u_j+l_j-\sigma_j-\rho_j)\tau_j)}{\Gamma((u_j+l_j-\sigma_j-\rho_j)\tau_j)}\left(\frac{f_j}{(z_j-A)^{\tau_j}}\right)^l \tag{2.4}$$

where $\left|\frac{\Gamma(\upsilon_j+(u_j+l_j-\sigma_j-\rho_j)\tau_j)}{\Gamma((u_j+l_j-\sigma_j-\rho_j)\tau_j)}\right| < \infty, z_j \neq A, (z_j-A)^{\tau_j} \neq f_j, \sigma_j \in \mathbb{C}, \tau_j \in \mathbb{R}^+, \upsilon_j \in \mathbb{R}$ and $\left|\frac{f_j}{(z_j-A)^{\tau_j}}\right| < 1$

for $j = 1, \cdots, r$.

Proof
By using the lemma 1 and lemma 3, we obtain the lemma 4.

3. Main result.

We have the following result.

Theorem.

$$[\prod_{j=1}^{r}[((z_j-A)^{\tau_j}-f_j)^{\sigma_j}][((z_j-A)^{\tau_j}-g_j)^{\rho_j}]S_{N_1,\cdots,N_r}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_r}(y_1[(z_1-A)^{\tau_1}-f_1]^{\alpha_1}[(z_1-A)^{\tau_1}-g_1]^{\beta_1},\cdots,$$

$$y_r[(z_r-A)^{\tau_r}-f_r]^{\alpha_r}[(z_r-A)^{\tau_r}-g_r]^{\beta_r})\,I(\lambda_1[(z_1-A)^{\tau_1}-f_1]^{\mu_1}[(z_1-A)^{\tau_1}-g_1]^{\eta_1},\cdots,$$

$$\lambda_r[(z_r-A)^{\tau_r}-f_r]^{\mu_r}[(z_r-A)^{\tau_r}-g_r]^{\eta_r})]_{\upsilon_1,\cdots,\upsilon_r}$$

$$= e^{-\omega\pi\sum_{j=1}^{r}\upsilon_j} \sum_{u_1,\cdots,u_r=0}^{\infty} \prod_{j=1}^{r}\left[\frac{(g_j-f_j)^{u_j}}{u_j!}(z_j-A)^{\tau_j(\sigma_j+\rho_j-u_j)-\upsilon_j}\right] \sum_{l_1,\cdots,l_r=0}^{\infty} \prod_{j=1}^{r}\left[\left(\frac{f_j}{(z_j-A)^{\tau_j}}\right)^{l_j}\frac{1}{l_j!}\right]$$

$$\sum_{K_1=0}^{[N_1/\mathfrak{M}_1]} \cdots \sum_{K_r=0}^{[N_r/\mathfrak{M}_r]} a_r \prod_{j=1}^{r} y_j^{K_j} (z_j - A)^{K_j \tau_j (\alpha_j + \beta_j)} I_{U:p_r,q_r;Y'}^{V;0,n_r;X'} \left(\begin{array}{c} \lambda_1 (z_1 - A)^{\tau_1(\mu_1+\eta_1)} \\ \cdot \\ \cdot \\ \cdot \\ \lambda_r (z_r - A)^{\tau_r(\mu_r+\eta_r)} \end{array} \middle| \begin{array}{c} \mathbf{A} : \mathrm{A'} \\ \cdot \\ \cdot \\ \mathbf{B} : \mathrm{B'} \end{array} \right) \tag{3.1}$$

where

$$X' = m^{(1)} + 3, n^{(1)}; \cdots ; m^{(r)} + 3, n^{(r)} \tag{3.2}$$

$$Y' = p^{(1)} + 3, q^{(1)} + 3; \cdots ; p^{(r)} + 3, q^{(r)} + 3 \tag{3.3}$$

$$A' = (a_k^{(1)}; \alpha_k^{(1)})_{1,p^{(1)}}, (-\rho_1 - K_1\beta_1; \eta_1), (u_1 - \sigma_1 - \rho_1 - K_1(\alpha_1 + \beta_1); \mu_1 + \eta_1),$$

$$(\tau_1(l_1 + u_1 - \sigma_1 - \rho_1 - K_1(\alpha_1 + \beta_1)); \tau_1(\mu_1 + \eta_1)); \cdots ;(a_k^{(r)}; \alpha_k^{(r)})_{1,p^{(r)}}, (-\rho_r - K_r\beta_r; \eta_r),$$

$$(u_r - \sigma_r - \rho_r - K_r(\alpha_r + \beta_r); \mu_r + \eta_r),(\tau_r(l_r + u_r - \sigma_r - \rho_r - K_r(\alpha_r + \beta_r)); \tau_r(\mu_r + \eta_r)) \tag{3.4}$$

$$B' = (b_k^{(1)}; \beta_k^{(1)})_{1,q^{(1)}}, (u_1 - \rho_1 - K_1\beta_1; \eta_1), (l_1 + u_1 - \sigma_1 - \rho_1 - K_1(\alpha_1 + \beta_1); \mu_1 + \eta_1),$$

$$(\upsilon_1 + \tau_1(l_1 + u_1 - \sigma_1 - \rho_1 - K_1(\alpha_1 + \beta_1)); \tau_1(\mu_1 + \eta_1)); \cdots ;(b_k^{(r)}; \beta_k^{(r)})_{1,q^{(r)}}, (u_r - \rho_r - K_r\beta_r; \eta_r),$$

$$(l_r + u_r - \sigma_r - \rho_r - K_r(\alpha_r + \beta_r); \mu_r + \eta_r),(\upsilon_r + \tau_r(l_r + u_r - \sigma_r - \rho_r - K_r(\alpha_r + \beta_r)); \tau_r(\mu_r + \eta_r)) \tag{3.5}$$

Provided that

$z_j^{\tau_j} \neq g_j, f_j; \alpha_j, \beta_j, \mu_j, \eta_j, \tau_j \in \mathbb{R}^+, z_j \neq A$, $\upsilon_j \in \mathbb{R}$ for $j = 1, \cdots , r$

$\tau_j Re(\sigma_j + \eta_j + K_j(\alpha_j + \beta_j)) + \tau_j(\mu_j + \eta_j) \max_{1 \leqslant l \leqslant n^{(j)}} Re \left(\frac{a_l^{(j)} - 1}{\alpha_l^{(j)}} \right) < \upsilon_j < 0 \,; j = 1, \cdots , r$

$|arg \lambda_j ((z_j - A)^{\tau_j} - f_j)^{\mu_j} ((z_j - A)^{\tau_j} - g_j)^{\eta_j}| < \frac{1}{2} \Omega_j \pi,$ where Ω_j is defined by (1.8) and the multiple series in the left-hand side of (3.1) is absolutely and uniformely convergent.

Proof

To establish (3.1), we first express the class of multivariable polynomials $S_{N_1,\cdots,N_r}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_r}[.]$ in series forms with the help of (1.3), we employ the definition of the N-fractional calculus given by Nishimoto (1.1) on the left-hand side of equation (3.1), we express the multivariable I-function in terms of its equivalent multiple Mellin-Barnes integrals contour with the help of (1.5), we interchange the order of $(K_1, \cdots , K_r)$-finite multiple summations and $(s_1, \cdots , s_r)$-integrals (which is permissible under the stated conditions), we obtain (say I),

$$I = \sum_{K_1=0}^{[N_1/\mathfrak{M}_1]} \cdots \sum_{K_r=0}^{[N_r/\mathfrak{M}_r]} a_r \prod_{j=1}^{r} y_j^{K_j} \frac{1}{(2\pi\omega)^r} \int_{L_1} \cdots \int_{L_r} \phi(s_1, \cdots , s_r) \prod_{i=1}^{r} \phi_i(s_i) \lambda_i^{s_i}$$

$$\left[((z_i - A)^{\tau_i} - f_i)^{\sigma_i + \alpha_i K_i + \mu_i s_i} ((z_i - A)^{\tau_i} - g_i)^{\rho_i + \beta_i K_i + \eta_i s_i} \right]_{\upsilon_1, \cdots, \upsilon_r} \mathrm{d}s_1 \cdots \mathrm{d}s_r$$

Now, we use the lemma 4, we get

$$I = \sum_{K_1=0}^{[N_1/\mathfrak{M}_1]} \cdots \sum_{K_r=0}^{[N_r/\mathfrak{M}_r]} a_r \frac{1}{(2\pi\omega)^r} \int_{L_1} \cdots \int_{L_r} \phi(s_1, \cdots , s_r) \prod_{j=1}^{r} \phi_j(s_j) \lambda_j^{s_j} e^{-\omega\pi\upsilon_j} \sum_{u_j=0}^{\infty} \frac{(-\rho_j - \beta_j K_j - \eta_j s_j)_{u_j}}{u_j!} (g_j - f_j)^{u_j}$$

$$\sum_{l_j=0}^{\infty}(z_j-A)^{(\rho_j+\sigma_j-u_j)\tau_j-\upsilon_j}\left(\frac{f_j}{(z_j-A)^{\tau_j}}\right)^{l_j}\frac{\Gamma(l_j+u_j-(\alpha_j+\beta_j)K_j-(\mu_j+\eta_j)s_j-\sigma_j-\rho_j)}{\Gamma(u_j-(\alpha_j+\beta_j)K_j-(\mu_j+\eta_j)s_j-\sigma_j-\rho_j)}$$

$$\frac{\Gamma(\upsilon_j+(u_j+l_j-\sigma_j-(\alpha_j+\beta_j)K_j-(\mu_j+\eta)s_j-\rho_j)\tau_j)}{\Gamma((u_j+l_j-\sigma_j-(\alpha_j+\beta_j)K_j-(\mu_j+\eta_j)s_j-\rho_j)\tau_j)}\mathrm{d}s_1\cdots\mathrm{d}s_r$$

we interchange the order of $(u_j,l_j)_{1\leqslant j\leqslant r}$ multiple series and $(s_1,\cdots,s_r)$-integrals (which is permissible under the stated conditions), we get

$$I=\sum_{K_1=0}^{[N_1/\mathfrak{M}_1]}\cdots\sum_{K_r=0}^{[N_r/\mathfrak{M}_r]}a_r\prod_{j=1}^{r}e^{-\omega\pi\upsilon_j}\sum_{u_j=0}^{\infty}\frac{(z_j-A)^{(\rho_j+\sigma_j-u_j)\tau_j-\upsilon_j}}{u_j!}(g_j-f_j)^{u_j}\sum_{l_j=0}^{\infty}\left(\frac{f_j}{(z_j-A)^{\tau_j}}\right)^{l_j}\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}$$

$$\prod_{j=1}^{r}\frac{\Gamma(l_j+u_j-(\alpha_j+\beta_j)K_j-(\mu_j+\eta_j)s_j-\sigma_j-\rho_j)\Gamma(\upsilon_j+(u_j+l_j-\sigma_j-(\alpha_j+\beta_j)K_j-(\mu_j+\eta)s_j-\rho_j)\tau_j)}{\Gamma(u_j-\sigma_j-(\alpha_j+\beta_j)K_j-(\mu_j+\eta_j)s_j-\rho_j)\Gamma((u_j+l_j-\sigma_j-(\alpha_j+\beta_j)K_j-(\mu_j+\eta_j)s_j-\rho_j)\tau_j)}$$

$$\frac{\Gamma(u_j-\rho_j-\beta_jK_j-\eta_js_j)}{\Gamma(-\rho_j-\beta_jK_j-\eta_js_j)}\phi(s_1,\cdots,s_r)\prod_{j=1}^{r}\phi_j(s_j)\lambda_j^{s_j}\mathrm{d}s_1\cdots\mathrm{d}s_r$$

$$=e^{-\omega\pi\sum_{j=1}^{r}\upsilon_j}\sum_{u_1,\cdots,u_r=0}^{\infty}\prod_{j=1}^{r}\left[\frac{(g_j-f_j)^{u_j}}{u_j!}(z_j-A)^{\tau_j(\sigma_j+\rho_j-u_j)-\upsilon_j}\right]\sum_{l_1,\cdots,l_r=0}^{\infty}\prod_{j=1}^{r}\left[\left(\frac{f_j}{(z_j-A)^{\tau_j}}\right)^{l_j}\frac{1}{l_j!}\right]$$

$$\sum_{K_1=0}^{[N_1/\mathfrak{M}_1]}\cdots\sum_{K_r=0}^{[N_r/\mathfrak{M}_r]}a_r\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\phi(s_1,\cdots,s_r)\prod_{j=1}^{r}\phi_j(s_j)\lambda_j^{s_j}\frac{\Gamma(u_j-\rho_j-\beta_jK_j-\eta_js_j)}{\Gamma(-\rho_j-\beta_jK_j-\eta_js_j)}$$

$$\frac{\Gamma(l_j+u_j-(\alpha_j+\beta_j)K_j-(\mu_j+\eta_j)s_j-\sigma_j-\rho_j)\Gamma(\upsilon_j+(u_j+l_j-\sigma_j-(\alpha_j+\beta_j)K_j-(\mu_j+\eta)s_j-\rho_j)\tau_j)}{\Gamma(u_j-\sigma_j-(\alpha_j+\beta_j)K_j-(\mu_j+\eta_j)s_j-\rho_j)\Gamma((u_j+l_j-\sigma_j-(\alpha_j+\beta_j)K_j-(\mu_j+\eta_j)s_j-\rho_j)\tau_j)}\mathrm{d}s_1\cdots\mathrm{d}s_r$$

Finally interpreting the multiple Mellin-Barnes integrals contour in multivariable I-function, we obtain the desired result (3.1) after algebric manipulations.

Remarks :

We note that the technique employed here may be used in extending the result (3.1) to a product of any finite number of power functions in the arguments of the multivariable I-function and multivariable polynomials instead of two. The formula (3.1) can be extended to product of any finite number of multivariable polynomials and multivariable I-functions.

4. Special cases.

If we set $A=0$ in the formula (3.1), we obtain the following result.

Corollary 1.

$$[\prod_{j=1}^{r}(z_j^{\tau_j}-f_j)^{\sigma_j}(z_j^{\tau_j}-g_j)^{\rho_j}S_{N_1,\cdots,N_r}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_r}(y_1\,(z_1^{\tau_1}-f_1)^{\alpha_1}\,(z_1^{\tau_1}-g_1)^{\beta_1},\cdots,y_r\,(z_r^{\tau_r}-f_r)^{\alpha_r}\,(z_r^{\tau_r}-g_r)^{\beta_r})$$

$$I(\lambda_1\,(z_1^{\tau_1}-f_1)^{\mu_1}\,(z_1^{\tau_1}-g_1)^{\eta_1},\cdots,(\lambda_r\,(z_r^{\tau_r}-f_r)^{\mu_r}\,(z_r^{\tau_r}-g_r)^{\eta_r})]_{\upsilon_1,\cdots,\upsilon_r}$$

$$=e^{-\omega\pi\sum_{j=1}^{r}\upsilon_j}\sum_{u_1,\cdots,u_r=0}^{\infty}\prod_{j=1}^{r}\left[\frac{(g_j-f_j)^{u_j}}{u_j!}z_j^{\tau_j(\sigma_j+\rho_j-u_j)-\upsilon_j}\right]\sum_{l_1,\cdots,l_r=0}^{\infty}\prod_{j=1}^{r}\left[\left(\frac{f_j}{z_j^{\tau_j}}\right)^{l_j}\frac{1}{l_j!}\right]$$

$$\sum_{K_1=0}^{[N_1/\mathfrak{M}_1]}\cdots\sum_{K_r=0}^{[N_r/\mathfrak{M}_r]} a_r \prod_{j=1}^{r} y_j^{K_j} z_j^{K_j\tau_j(\alpha_j+\beta_j)} I_{U:p_r,q_r;Y'}^{V;0,n_r;X'}\left(\begin{array}{c|c} \lambda_1 z_1^{\tau_1(\mu_1+\eta_1)} & \mathbf{A} : \mathrm{A'} \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \\ \lambda_r z_r^{\tau_r(\mu_r+\eta_r)} & \mathbf{B} : \mathrm{B'} \end{array}\right) \tag{4.1}$$

X', Y', A' and B' are defined respectively by (3.2), (3.3), (3.4) and (3.5).

Provided that

$z_j^{\tau_j} \neq g_j, f_j; \alpha_j, \beta_j, \mu_j, \eta_j, \tau_j \in \mathbb{R}^+, z_j \neq A$, $\upsilon_j \in \mathbb{R}$ for $j = 1, \cdots, r$

$\tau_j Re(\sigma_j + \eta_j + K_j(\alpha_j + \beta_j)) + \tau_j(\mu_j + \eta_j) \max_{1\leqslant l\leqslant n^{(j)}} Re\left(\frac{a_l^{(j)} - 1}{\alpha_l^{(j)}}\right) < \upsilon_j < 0$ for $j = 1, \cdots, r$

$|arg\lambda_j (z_j^{\tau_j} - f_j)^{\mu_j} (z_j^{\tau_j} - g_j)^{\eta_j}| < \frac{1}{2}\Omega_j\pi,$ where Ω_j is defined by (1.8) and the multiple series in the left-hand side of (4.1) is absolutely and uniformely convergent.

On the other hand if we take $A = 0$ and $\tau_1 = \cdots = \tau_r = 1$, the using the binomial formula

$$(1-z)^{-\alpha} = \sum_{k=0}^{\infty} \frac{(\alpha)_k}{k!} z^k, |z| < 1, \tag{4.2}$$

it yields the following corrected form of the result given by Garg et al. [3]

Corollary 2.

$$\prod_{j=1}^{r} [(z_j - f_j)^{\sigma_j}] [(z_j - g_j)^{\rho_j}] S_{N_1,\cdots,N_r}^{\mathfrak{M}_1,\cdots,\mathfrak{M}_r}(y_1 (z_1 - f_1)^{\alpha_1} (z_1 - g_1)^{\beta_1}, \cdots, y_r (z_r - f_r)^{\alpha_r} (z_r - g_r)^{\beta_r})$$

$$I(\lambda_1 (z_1 - f_1)^{\mu_1} (z_1 - g_1)^{\eta_1}, \cdots, \lambda_r (z_r - f_r)^{\mu_r} (z_r - g_r)^{\eta_r})]_{\upsilon_1,\cdots,\upsilon_r}$$

$$= e^{-\omega\pi \sum_{j=1}^{r} \upsilon_j} \prod_{j=1}^{r} (z_j - f_j)^{\sigma_j+\rho_j-\upsilon_j} \sum_{u_1,\cdots,u_r=0}^{\infty} \prod_{j=1}^{r} \left(\frac{(g_j - f_j)}{z_j - f_j}\right)^{u_j} \frac{1}{u_j!} \sum_{K_1=0}^{[N_1/\mathfrak{M}_1]}\cdots\sum_{K_r=0}^{[N_r/\mathfrak{M}_r]} a_r$$

$$\prod_{j=1}^{r} y_j^{K_j} (z_j - f_j)^{K_j\tau_j(\alpha_j+\beta_j)} I_{U:p_r,q_r;Y''}^{V;0,n_r;X''}\left(\begin{array}{c|c} \lambda_1 (z_1 - f_1)^{(\mu_1+\eta_1)} & \mathbf{A} : \mathrm{A''} \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \\ \lambda_r (z_r - f_r)^{(\mu_r+\eta_r)} & \mathbf{B} : \mathrm{B''} \end{array}\right) \tag{4.3}$$

where

$$X'' = m^{(1)} + 2, n^{(1)}; \cdots ; m^{(r)} + 2, n^{(r)} \tag{4.4}$$

$$Y'' = p^{(1)} + 2, q^{(1)} + 2; \cdots ; p^{(r)} + 2, q^{(r)} + 2 \tag{4.5}$$

$$A'' = (a_k^{(1)}; \alpha_k^{(1)})_{1,p^{(1)}}, (-\rho_1 - K_1\beta_1; \eta_1), (u_1 - \sigma_1 - \rho_1 - K_1(\alpha_1 + \beta_1); \mu_1 + \eta_1); \cdots ;$$

$$(a_k^{(r)}; \alpha_k^{(r)})_{1,p^{(r)}}, (-\rho_r - K_r\beta_r; \eta_r), (u_r - \sigma_r - \rho_r - K_r(\alpha_r + \beta_r); \mu_r + \eta_r) \tag{4.6}$$

$$B'' = (b_k^{(1)}; \beta_k^{(1)})_{1,q^{(1)}}, (u_1 - \rho_1 - K_1\beta_1; \eta_1), (\upsilon_1 + u_1 - \sigma_1 - \rho_1 - K_1(\alpha_1 + \beta_1); \mu_1 + \eta_1); \cdots ;$$

$$(b_k^{(r)};\beta_k^{(r)})_{1,q^{(r)}},(u_r-\rho_r-K_r\beta_r;\eta_r),(\upsilon_r+u_r-\sigma_r-\rho_r-K_r(\alpha_r+\beta_r);\mu_r+\eta_r) \tag{4.7}$$

Provided that

$z_j\neq g_j,f_j;\alpha_j,\beta_j,\mu_j,\eta_j,\tau_j\in\mathbb{R}^+,z_j\neq A$, $\upsilon_j\in\mathbb{R}$ for $j=1,\cdots,r$

for $j=1,\cdots,rRe(\sigma_j+\eta_j+K_j(\alpha_j+\beta_j))+(\mu_j+\eta_j)\max\limits_{1\leqslant l\leqslant n^{(j)}} Re\left(\dfrac{a_l^{(j)}-1}{\alpha_l^{(j)}}\right)<\upsilon_j<0$

$|arg\lambda_j(z_j-f_j)^{\mu_j}(z_j-g_j)^{\eta_j}|<\frac{1}{2}\Omega_j\pi,$ where Ω_j is defined by (1.8).

$\left|\dfrac{g_j-f_j}{z_j-f_j}\right|<1$ for $j=1,\cdots,r$ and the multiple series in the left-hand side of (4.1) is absolutely and convergent. We can use the lemma 2.

We consider the above corollary, if the multivariable I-function and class of multivariable polynomials reduce respectively to Fox's H-function [2,9] and class of polynomials of one variable [24], we obtain

Corollary 3.

$$[(z-f)^{\sigma})((z-g)^{\rho})S_N^M\left(\text{ y}(z-f)^{\alpha}(z-g)^{\beta}\right)H_{p^{(1)},q^{(1)}}^{m^{(1)},n^{(1)}}\left(\text{Z}(z-f)^{\mu}(z-g)^{\eta}\ \middle|\ \begin{matrix}(\mathrm{a}_k^{(1)};\alpha_k^{(1)})_{1,p^{(1)}}\\ .\\ (\mathrm{b}_k^{(1)};\beta_k^{(1)})_{1,q^{(1)}}\end{matrix}\right)_{\upsilon}$$

$$=e^{-\omega\pi\upsilon}(z-f)^{\sigma+\rho-\upsilon}\sum_{u=0}^{\infty}\left(\frac{g-f}{z-f}\right)^u\frac{1}{u!}\sum_{K=0}^{N/M}a_1y^Kz^{K(\alpha+\beta)}$$

$$H_{p^{(1)}+2,q^{(1)}+2}^{m^{(1)}+2,n^{(1)}}\left(\text{Z (z-f)}^{\mu+\eta}\ \middle|\ \begin{matrix}(\mathrm{a}_k^{(1)};\alpha_k^{(1)})_{1,p^{(1)}},(-\rho-K\alpha;\eta),(u-\sigma-\rho-K(\alpha+\beta);\mu+\eta)\\ .\\ (\mathrm{b}_k^{(1)};\beta_k^{(1)})_{1,q^{(1)}},(u-\rho-K\beta;\eta),(\upsilon+u-\sigma-\rho-K(\alpha+\beta);\mu+\eta)\end{matrix}\right) \tag{4.8}$$

Provided that

$z\neq g,f;\tau\in\mathbb{R}^+,g\neq f$, $\upsilon_1\in\mathbb{R}$,

$Re(\sigma+\eta)+(\mu+\eta)\max\limits_{1\leqslant l\leqslant n^{(1)}} Re\left(\dfrac{a_l^{(1)}-1}{\alpha_l^{(1)}}\right)<\upsilon<0$

$|argZ(z-f)^{\mu}(z-g)^{\eta}|<\frac{1}{2}\Omega_1\pi,$ where $\Omega_1=\sum\limits_{k=1}^{n^{(1)}}\alpha_k^{(1)}-\sum\limits_{k=n^{(1)}+1}^{p^{(1)}}\alpha_k^{(1)}+\sum\limits_{k=1}^{m^{(1)}}\beta_k^{(1)}-\sum\limits_{k=m^{(1)}+1}^{q^{(1)}}\beta_k^{(1)}$

$\left|\dfrac{g-f}{z-f}\right|<1$ and the multiple series in the left-hand side of (4.1) is absolutely and uniformely.

Consider the above corollary , by applying our result given in (4.4) to the case the Laguerre polynomials ([31], page 101, eq.(15.1.6)) and ([28], page 159) and by setting

$$S_N^1(x)\rightarrow L_N^{\alpha'}(x)$$

In which case $M=1,A_{N,K}=\binom{N+\alpha'}{N}\dfrac{1}{(\alpha'+1)_K}$ we have the following interesting consequencies of the main result.

Corollary 4.

$$[(z-f)^{\sigma})((z-g)^{\rho})\,L_N^{\alpha'}\left(\text{ y}(z-f)^{\alpha}(z-g)^{\beta}\right)$$

$$H_{p^{(1)},q^{(1)}}^{m^{(1)},n^{(1)}}\left(\mathrm{Z}(z-f)^{\mu}(z-g)^{\eta} \;\middle|\; \begin{array}{c} (\mathrm{a}_k^{(1)};\alpha_k^{(1)})_{1,p^{(1)}} \\ \cdot \\ (\mathrm{b}_k^{(1)};\beta_k^{(1)})_{1,q^{(1)}} \end{array} \right)_{\upsilon}$$

$$= e^{-\omega\pi\upsilon}(z-f)^{\sigma+\rho-\upsilon}\sum_{u=0}^{\infty}\left(\frac{g-f}{z-f}\right)^{u}\frac{1}{u!}\sum_{K=0}^{N}\binom{N+\alpha'}{N-K}a_1(-y)^K z^{K(\alpha+\beta)}\frac{(-N)_K}{K!}$$

$$H_{p^{(1)}+2,q^{(1)}+2}^{m^{(1)}+2,n^{(1)}}\left(\mathrm{Z}\ (\mathrm{z\text{-}f})^{\mu+\eta} \;\middle|\; \begin{array}{c} (\mathrm{a}_k^{(1)};\alpha_k^{(1)})_{1,p^{(1)}},(-\rho-K\alpha;\eta),(u-\sigma-\rho-K(\alpha+\beta);\mu+\eta) \\ \cdot \\ (\mathrm{b}_k^{(1)};\beta_k^{(1)})_{1,q^{(1)}},(u-\rho-K\beta;\eta),(\upsilon+u-\sigma-\rho-K(\alpha+\beta);\mu+\eta) \end{array} \right) \quad (4.9)$$

under the same conditions that (4.8).

Remark :
By the similar methods, we obtain the analog relations with the Aleph-function of several variables [1], Aleph-function of two variables [22] and one variable [29,30], the I-function of two variables ([8],[23]), the multivariable I-function [17], the I-function of one variable [21], the multivariable A-function [5], the A-function [4] and the modified multivariable H-function [16].

5. Conclusion.

Finally, it is interesting to observe that due to fairly general character of the multivariable I-function and class of multivariable polynomials, numerous interesting special cases of the main result (3.1) associated with potentially useful a variety special functions of one and several variables, orthogonal polynomials, multivariable H-function, H-function, G-function and Generalized Lauricella functions etc.

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