Sum of Domination and Independence Numbers of Tetra Bipartite Graphs

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***Abstract* —** *The study of independence and domination numbers of graphs is that the quickest growing space in graph theory. Domination number and independence numbers area unit utilized in such fields as algorithmic styles, social sciences, communication networks etc. A set D of vertices is the dominating set of graph G if each vertex of V to D is adjacent to different vertex of D. The domination number of graph is that the minimum cardinality of a dominating set of G, it's denoted by gamma(G). Independence number is that the highest cardinality of associate degree independent set of vertices of a graph Domination range is that the cardinality of a minimum dominating set of a graph. During this paper we tend to are presenting results on domination and independence numbers of tetra bipartite graphs.*

Keywords — *Independent numbers, Domination numbers, Graph, Tetra Bipartite graph.*

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I. Introduction

The graph theory has been a biggest growth attributable to its interaction with within the last 5 decades and application in many areas like Operation analysis , Life Sciences, Engineering, Physical Sciences etc. its several and varied applications in such fields as communications, networks, recursive styles, social sciences, etc. The conception of dominating set happens in an exceedingly form of issues. Many of issues are intended by communication network issues. Mathematical study of domination of a graph started around 1960 and it had been additional developed in late 1950's and 1960's. The Domination number was firstly established by Claude Berge. Independent and Dominating sets are among the foremost well-studied graph sets. Domination are often a great tool for determining business network and creating selections. Let vertex set V of any Graph G is denoted by V(G). For a positive integer n, a group of vertices D in an exceedingly graph G is alleged to be a n-dominating set if every vertex of G not contained in D has a minimum of 'n' neighbours in D. The order of a smallest n-dominating set of G is termed the n-domination variety, and it's denoted by $\gamma_n(G)$.

By definition a dominating set coincides with a 1- dominating set and $\gamma_1(G)$ is that the domination number $\gamma(G)$ of G. The idea of domination was given formal Mathematical definition proposed by C. Berge and O. Ore [1]. Ore started the world domination in his famous book [1] this idea lived in hibernation till 1975 once a paper [3] published in 1977.

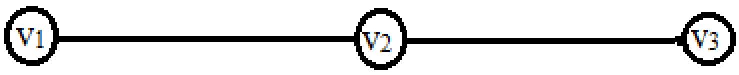
This paper brought to light interesting concepts and potentiality of being applied in several kind of areas. The analysis in domination theory has been broadly speaking introduced in [9], [10].Independent sets were introduced into the subject area on noisy channels [4]. In 1982, Xuong and Payan [2] and severally in 1985, Kinch and Roberts fink, Jacobson [5] characterised the graphs achieving equality in Ore's bound.

II. DOMINATION SET AND INDEPENDENT SET DOMINATING

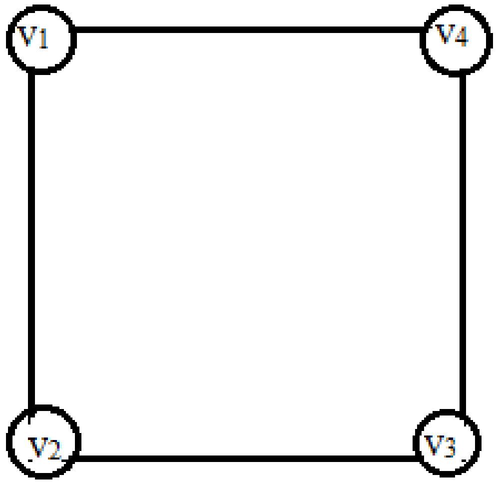
A set of vertices of a graph, such that each vertex in that graph is either in that set or adjacent to some other vertex in that set

The number of vertices in a very minimum dominating set of a given graph usually denoted as \gamma=\gamma (G).

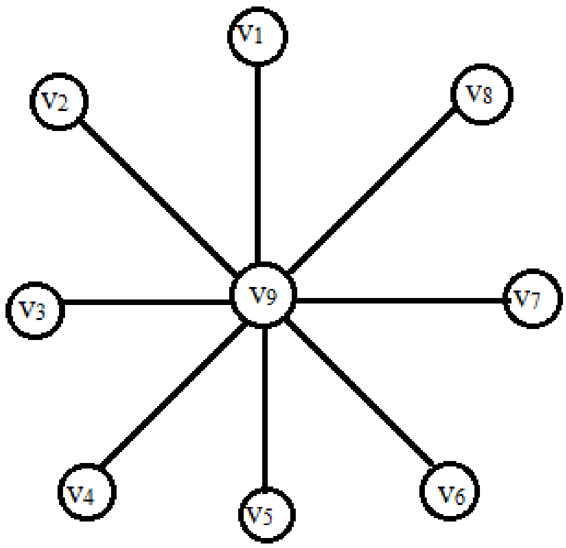
The number γ (G) represent the cardinality of a minimum dominating set of G.



Dominating set $\{v_1\}$



Dominating set $\{v_1, v_4\}$



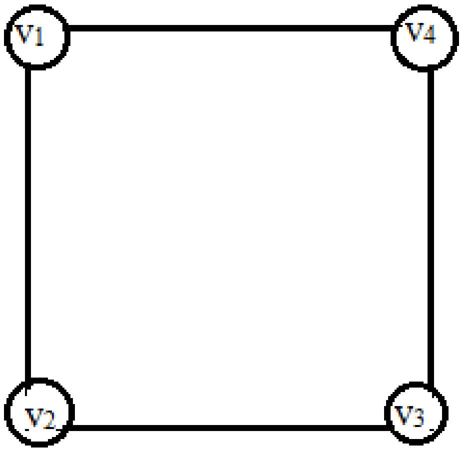
Dominating set $\{v_9\}$

III. INDEPENDENT DOMINATION SET

A set of vertices of a graph, such no pair of them are adjacent to every other, in different words, a group of vertices that are all "independent'' of each other.

In a graph the number of vertices in a maximum independent set is usually denoted as \alpha=\alpha (G).

The number α_i (G) represent the cardinality of a maximum independent set of G



Independent dominating set { v_1, v_3}

Independent dominating set{v_1, v_3}

IV. DOMINATION NUMBER AND INDEPENDENCE NUMBER OF SIMPLE GRAPHS

A well-known upper bound for the domination range of a graph was given by Ore [1] in 1962.

Theorem 3.1 ([9]) If G could be a graph while not isolated vertices, then $\gamma(G) \leq |G|/2$. In 1982, Payan and Xuong [2] and severally, in 1985, Fink, Jacobson, Kinch and Roberts [5] defined the graphs achieving equality in Ore's positive.

Theorem 3.2 ([2, 5]) Let G be a connected graph then $\gamma(G) = |G|/2$ if and on condition that G is that the corona graph of any connected graph J or G is isomorphic to the cycle C4. In [5], Chellali, Favaron and Blidia studied the link between the 2-domination number and so the independence number of a tree particularly, they established that the quantitative relation $\gamma_2(T)/\ \gamma_i(T)$ for a treeT is contained in an exceedingly small interval.

Theorem 3.3 [6] For any tree, $\gamma_i(T) \leq \gamma_2\ (T) \leq 3/2\ \gamma_i\ (T)$.

Theorem 3.4 [7] If a simple graph G with n vertices has a vertex with degree n – 1 , then the domination number $\gamma(G)$ is one.

Theorem 3.5 [7] An independent set of a graph G is dominating if and only if it is maximal.

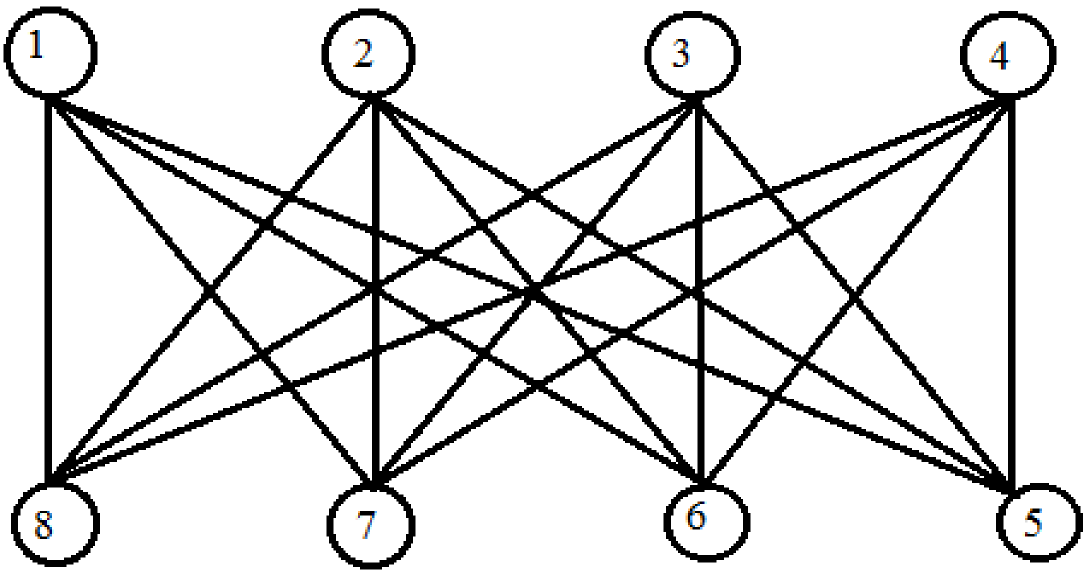
Theorem 3.6 [7] If G is a complete graph, then $\gamma(G) = \gamma_i(G)$

V. RESULTS AND CONCLUSION

DOMINATION AND INDEPENDENCE OF TETRABIPARTITE GRAPHS

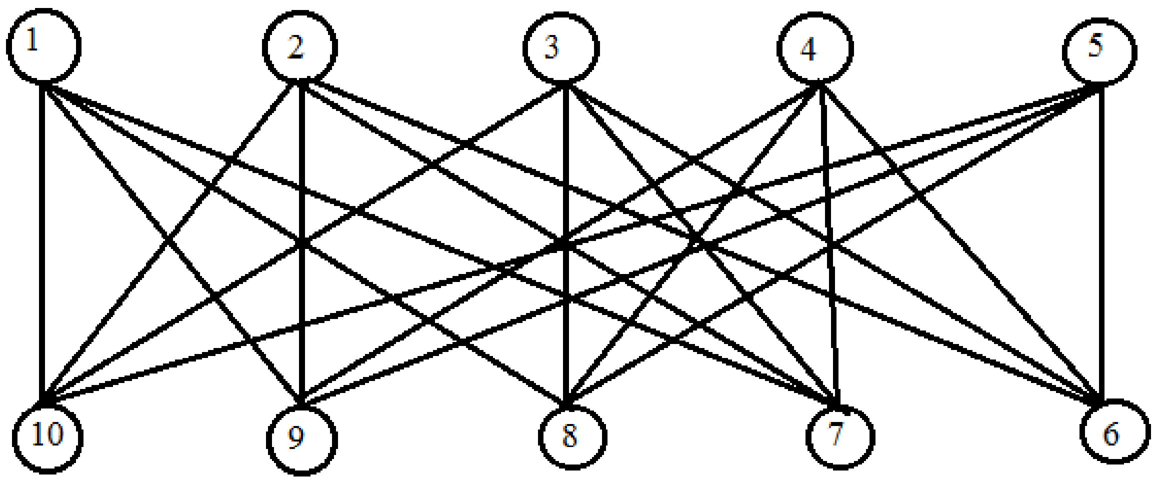
Let G be a Tetra bipartite graph and n be the number of its vertices. We know that the minimum number of vertices that a tetra bipartite graph will have is 8.

1. For n = 8, we have the tetra bipartite graph as given below.



Here { 1, 5} is a dominating set. Then $\gamma(G) = 2 = 8/4 = n/4$.
The largest independent set is {1,2,3,4}.Then $\alpha_i(G) = 4 = n/2$.

2. For n = 10, we have the tetra bipartite graph as given below.



Here {1, 6} is a dominating set. Then γ (G) = 2 < 10/4 = n/4.
The largest independent set is { 1,2,3,4,5 }.Then α_i(G) = 5 = n /2.

Theorem 4.1 For a tetra bipartite graph the domination number γ is less than or equal to 1/4 of the number of vertices.

Proof. Suppose a tetra bipartite graph with n vertices. Then the set of vertices is partitioned off into 2 sets with each set consisting of n/2 vertices. conjointly since the graph is tetra , every vertex is of degree four. Therefore every vertex in each of the sets are going to be adjacent to four vertices only. That is, every vertex dominates four vertices. Therefore at the most n/4 vertices are enough to dominate all the other vertices. Thus the domination number of a graph are less than or equal to 1/4 of the number of vertices.

Theorem 4.2 For a Tetra bipartite graph, the independence number γ_i is equal to 1/2 of the number of vertices.
Proof. Consider a tetra bipartite graph with 'n' vertices. The vertex set of the given graph can be sepreated into two independent sets consisting of n/2 vertices each. So that the greatest independent set will consist of n /2 vertices.Thus the independence number of graph is n /2.

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