Study of Normalized Analytical Function in the View of Group Theory

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Abstract: *In this paper we introduced class N of normalized analytical functions on unit disc U*

given by f (z) = z +$\sum_{k=2}^{\infty} a_k z^k$, $a_k \in C$.We define operation '+' on N. We examine class N for group with respect to given operation. We find subgroup and factor group for (N, +)

1 INTRODUCTION AND PRELIMINARIES.

LetN denotes subclass of normalized analytical function in open unit disc U= {z: $|z| < 1$}

given by f (z) = z +$\sum_{k=2}^{\infty} a_k z^k$. (1.1)

[1] has introduced following definitions.

Definition 1.1. A group < G, *> is a set G, closed under binary operation *, such that following axioms are satisfied

G_1: For all a, b, c ∈ G, we have

(a*b) *c= a*(b*c) associativity of *.

G_2: There is an element e in G such that for all x ∈ G,

e*x = x*e=x identity element e for *

G_3: corresponding to each a ∈ G, there is an element a' in G such that

a*a' = a'*a=e inverse a' of a.

Operation 1.2. '+'

For f_1(z), f_2(z),in N we have define following operations

f_1(z)= z +$\sum_{k=2}^{\infty} t_k z^k$, f_2(z) = z +$\sum_{k=2}^{\infty} b_k z^k$.

f_1(z) +f_2(z) = z +$\sum_{k=2}^{\infty}(t_k + b_k) z^k$. (1.2)

Definition1.3 If subset H of group G is closed under binary operation of G and If H with induced

operation from G is itself a group, then H is a **subgroup** of G.

Example1.4. Show that <N, +>is a group.

Answer: f_1(z) = z +$\sum_{k=2}^{\infty} t_k z^k$, f_2(z) = z +$\sum_{k=2}^{\infty} b_k z^k$, f_3(z) = z +$\sum_{k=2}^{\infty} c_k z^k$ in N,

G_1: (f_1(z) +f_2(z)) + f_3(z) = [(z + $\sum_{k=2}^{\infty} t_k z^k$) +(z +$\sum_{k=2}^{\infty} b_k z^k$)]+ (z +$\sum_{k=2}^{\infty} c_k z^k$)

= (z +$\sum_{k=2}^{\infty}(t_k + b_k) z^k$) + (z +$\sum_{k=2}^{\infty} c_k z^k$)

= (z +$\sum_{k=2}^{\infty}(t_k + b_k + c_k) z^k$.

=L.H.S

f_1(z) + (f_2(z) +f_3(z)) = z +$\sum_{k=2}^{\infty} t_k z^k$+ [(z +$\sum_{k=2}^{\infty} b_k z^k$)+ (z +$\sum_{k=2}^{\infty} c_k z^k$)]

= (z +$\sum_{k=2}^{\infty} t_k z^k$)+ [z +$\sum_{k=2}^{\infty}(b_k + c_k) z^k$]

= z +$\sum_{k=2}^{\infty}(t_k + b_k + c_k) z^k$

= R.H.S

$Therefore$(f_1(z) +f_2(z))+f_3(z) = f_1(z) + (f_2(z) +f_3(z)).

G_2: Let I (z) = z in N such that for f(z)in N

I (z)+f (z) = (z +$\sum_{k=2}^{\infty} 0 z^k$)+ (z +$\sum_{k=2}^{\infty} a_k z^k$)

= z +$\sum_{k=2}^{\infty} a_k z^k$ = f (z)

Similarly we can show that f (z)+I (z) = f (z)

G_3: f_1(z) = z +$\sum_{k=2}^{\infty} t_k z^k$ in N their exit f_2(z) = z +$\sum_{k=2}^{\infty} -t_k z^k$ in N such that

f_1(z)+ f_2(z) = (z +$\sum_{k=2}^{\infty} t_k z^k$)+ (z +$\sum_{k=2}^{\infty} -t_k z^k$)

= (z +$\sum_{k=2}^{\infty}(t_k - t_k) z^k$)

= z =I (z)

Similarly we can show that f_2(z)+f_1(z) = I (z).

Hence <N,+> is a group.

Theorem1.5. [1] has given following result

A subset H of group G is a subgroup of G if and only if

1. H is closed under binary operation of G.
2. Identity element e of G is in H.
3. For all a∈ H, a' ∈ H.

Example1.6. Let H = {z +$\sum_{k=2}^{\infty} a_k z^k$ / a_k is integer}.

Show that H is subgroup of G.

Answer: 1.f_1(z)= z +$\sum_{k=2}^{\infty} t_k z^k$, f_2(z) = z +$\sum_{k=2}^{\infty} b_k z^k$in

f_1(z) +f_2(z) = z +$\sum_{k=2}^{\infty}(t_k + b_k) z^k$.

as t_k ,b_k ∈ Z, t_k +b_k ∈ Z. Hence f_1(z) +f_2(z) ∈H.

2. Let I (Z) =Z is identity element of (N, +).

I (Z) = z +$\sum_{k=2}^{\infty} a_k z^k$, where a_k=0.

Then I (Z) ∈H.

3 For f_1(z) = z +$\sum_{k=2}^{\infty} t_k z^k$ their exist f_2(z) = z +$\sum_{k=2}^{\infty}(-t_k) z^k$ such that

f_1(z) + f_2(z) = (z +$\sum_{k=2}^{\infty} t_k z^k$) + (z +$\sum_{k=2}^{\infty}(-t_{k)} z^k$)

=z+$\sum_{k=2}^{\infty}(t_k - t_k) z^k$

=z

=I (z).

For $t_k \in$ Z, - $t_k \in$ Z.

Hence f_2(z) ∈ H.

Therefore H is subgroup of N

Definition1.7.A subgroup H of group G is normal if gH=Hg ∀ g∈ H.

Example1.8. Show that H is Normal subgroup of N.

Answer:g.H={g.h/h ∈ H}

For f(z) = z +$\sum_{k=2}^{\infty} t_k z^k$.

f (z).H = {f(z)+h(z)/h(z) ∈ H}.

H.f (z) = {h (z) +f (z)/ h (z) ∈ H}.

But f(z)+h(z)=(z +$\sum_{k=2}^{\infty} t_k z^k$)+(z +$\sum_{k=2}^{\infty} h_k z^k$).

= z +$\sum_{k=2}^{\infty}(t_k + h_k) z^k$.

= z +$\sum_{k=2}^{\infty}(h_k + t_k) z^k$.

= (z +$\sum_{k=2}^{\infty} b_k z^k$) + (z +$\sum_{k=2}^{\infty} t_k z^k$).

=h (z) +f (z).

Hence f (z).H= H.f (z).

Therefore H is normal subgroup of G.

Example1.9. Show that set of cosets of H form group N/H.

Answer:1. **(**f (z).H)*(g (z).H) = (f (z) +g (z)).H

(f (z).H)*[(g (z).H)*(h (z).H] = (f (z).H)*[(g (z) +h (z))].H

= (f (z) + (g (z) +h (z)).H

= (f (z) +g (z) +h (z)).H

Similarly we can show that

[(f (z).H)* (g (z).H)]*(h (z).H) =(f (z) +g (z) +h (z)).H

Hence G_1 axiom satisfied.

2. TakeI (z) =z

(f (z).H)*(I (z).H) = (f (z) +I (z)).H

=f (z).H

Similarly we can show that (I(z).H)*(f(z).H) =f(z).H

Hence G_2 axiom satisfied

3. If f(z) = z +$\sum_{k=2}^{\infty} t_k z^k$, g(z) = z +$\sum_{k=2}^{\infty}(-t_k)z^k$.

Then f (z) +g (z) =I (z)

(f (z).H)*(g (z).H) = (f (z) +g (z)).H

=I (z).H

Similarly we can show that (g(z).H)*(f(z).H)=I(z).H

For f (z).H∈ N/H there exist g (z).H ∈ N/H such that

(f (z).H)*(g (z).H) = (g (z).H)*(f (z).H) =I (z).H

Hence G_3 is satisfied.

Therefore (N/H,*) is a group.

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