Continuous Acceptance Sampling Plans for Truncated Lomax Distribution Based on CUSUM Schemes

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ABSTRACT

This paper study "Continuous Acceptance Sampling plans for Truncated Lomax distribution based on CUSUM Schemes" by Gauss-Chebyshev integration method. Assuming that the life time of an item produced is distributed according to Lomax distribution. Generally life tests experiments are carried out to determine an optimal truncated point. Truncated distributions are employed many practical situations where there is a constraint a lower and upper limits of the variable understudy. Based on these understanding we optimize CASP-CUSUM Schemes through the truncated Lomax distribution by using Gauss-Chebyshev integration method. At various parameter values of the underlying distribution, we determine probability of acceptance.

Keywords: CASP-CUSUM Schemes, Optimal Truncated point, Truncated Lomax Distribution.

I. INTRODUCTION

Customer satisfaction determines the success of a new product and only products at high value meet needs clients who expect them to perform correctly in their whole life cycle. In order to fulfill such requirements the minimum of variation of parameters should be assured within the manufacturing processes and the product itself.

Quality is relating to one or more desirable characteristics that a product or service should possess. Quality has become one of the most important consumer decision factors in the selection among competing products and services. Quality improvement methods can be applied to any area within a company or organization, including manufacturing process, development, engineering design, finance and accounting, marketing, distribution and logistics, customer service, and field service of products.

In order to tackle the development of advanced technologies, the reliability of products has become a significant matter of concern. It regards with respect to failure avoidance rather than probability of failure. Product failure occurs when the product is not able to perform its objective function and does not meet its requirements. Thus truncation of a product is capability to fulfill intended tasks for a specified performance period.

Acceptance sampling plan is an essential tool in the Statistical Quality Control. In most of the statistical quality control experiment, it is not possible to perform hundred percent inspections, due to various reasons. The acceptance sampling plan was the first applied in the US military for testing the bullets during World War II. For instance, if every bullet tested in advance, no bullets are available for shipment, and on the other hand if no bullets are tested, then disaster may occur in the battle field at the crucial time. Acceptance sampling plan is a middle path between 100% inspection and no inspection.

A classical field of quality control is acceptance sampling; it deals mainly with the following problem namely product control. Product control deals with inspection of all the items in the lot with respect to certain qualitative characteristics.

Truncation of a product can be defined as "the time period over which a product meets the standards of quality for the period of expected use". The objective of the truncation is to study, characterization, and measurement, analysis of failures and repairs and consequences to improving system operational time.

Truncated distribution can be used to simplify the asymptotic theory of robust estimators of location and regression. These are useful when the underlying distribution is exponential, double exponential, normal, Cauchy and also examine the sample median, trimmed means and two stage trimmed means behavior at these distribution.

The items which are conforming the quality specifications required by consumer are referred as quality items. Quality of an item is subjected to the reliability; one should adopt certain measures such as life testing through various probability models, preventing measures, sampling inspection CUSUM Schemes etc. In the process of improving the quality of products it should be examined whether, the items produced performing their intended duties or not. The items are available up to the warranty time, and how best they satisfy the consumer needs.

Life tests experiments are carried out in order to obtain the life time of an item (i.e. time to its failure or the stops working satisfactory). Sometimes, it may be time consuming process as we have to wait until all the products fail in a life test, if the life times of products are high. One can use the truncated life test for saving time and money, because 100% inspection involves more time, more money, man power, material, machinery etc. Even the sample finite 100% inspection practically not feasible in case of explosive type materials like crackers, bombs, batteries, bulbs etc. The test can be performed without waiting until all the products fail, and then testing time can be reduced significantly. For the purpose of reduces test time and cost, obliviously truncated life models.

Hawkins, D. M. [4] proposed a fast accurate approximation for ARL's of a CUSUM Control Charts. This approximation can be used to evaluate the ARL's for Specific parameter values and the out of control ARL's location and scale CUSUM Charts.

Kakoty. S., Chakravaborthy A.B. [6] determined CASP-CUSUM charts under the assumption that the variable under study follows a Truncated Normal Distribution. Generally truncated distributions are employed in many practical phenomena where there is a constraint on the lower and upper limits of the variable under study. For example, in the production engineering items, the sorting procedure eliminates items above or bellows designated tolerance limits. It is worthwhile to note that any continuous variable be first approximated as an exponential variable.

Vardeman.S, Di-ou Ray [10] introduced CUSUM control charts under the restriction that the values are regard to quality is exponentially distributed. Further the phenomena under study is the occurrence of rate of rare events and the inter arrival times for a homogenous poison process are identically independently distributed exponential random variables.

Lonnie. C. Vance [7], considered Average Run Length of cumulative Sum Control Charts for controlling normal means and to determine the parameters of a CUSUM Chart. To determine the parameters of CUSUM Chart the acceptable and rejectable quality levels along with the desired respective ARL's are consider.

Muhammed Riaz, Nasir Abbas[8] and Ronald J.M.M Does proposed two Runs rule schemes for the CUSUM Charts. The performance of the CUSUM and EWMA Charts are compared with the usual CUSUM and weighted CUSUM, the first initial response CUSUM compared with usual EWMA Schemes. This comparison stated that the proposed schemes perform better for small and moderate shifts.

Mohammed Akhtar. P and Sarma K.L.A.P [1] analyzed and Optimization of CASP-CUSUM Schemes based on truncated two parametric Gamma distribution and evaluate L (0), L' (O) and probability of Acceptance and also Optimized CASP-CUSUM Schemes based numerical results.

Narayana Murthy,B.R. and Mohammed Akhtar.P[11] proposed an Optimization of CASP CUSUM Schemes based on Truncated Log-logistic distribution and evaluate the probability of acceptance for different paremter values.

Sainath.B and Mohammed Akhtar .P [13] studied an Optimization of CASP-CUSUM Schemes based on truncated Burr distribution and the results were analyzed at different values of the parameters.

Venkatesulu.G and Mohammed Akhtar.P[14] determined Truncated Gompertz Distribution and its Optimization of CASP-CUSUM Schemes by changing the values of the parameters and finally critical comparisons are drawn based on the obtained numerical results.

In the present paper it is determined Type-C OC curves of CASP-CUSUM schemes when the variable under study follows truncated Lomax Distribution. Thus it is more worthwhile to study some interesting characteristics of Type-C OC Curves based on this distribution.

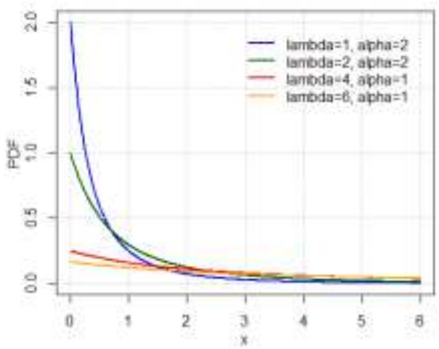
LOMAX DISTRIBUTION

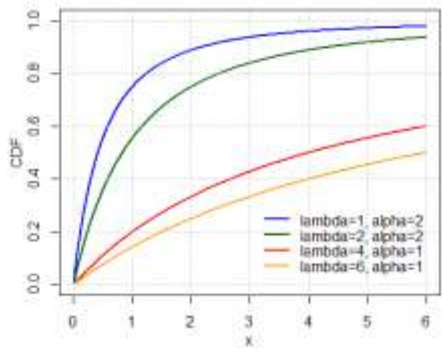
The Lomax distribution, also called "Pareto type II" distribution is a particular case of the generalized Pareto distribution. The Lomax distribution has been used in the literature in a number of ways. It has been used as an alternative to the exponential distribution when the data are heavy tailed. The Lomax distribution has applications in economics, actuarial modeling, queuing problems and biological sciences.

Definition: A continuous random variable X assuming non-negative values is said to have Lomax Distribution with parameters α, λ> 0, and its probability density function is given by:

$$f(x) = \frac{\alpha}{\lambda}\left[1+\frac{x}{\lambda}\right]^{-(\alpha+1)} \qquad \text{…. (1.1)}$$

Where $\lambda > 0$ is the scale parameter and $\alpha > 0$ is the shape parameter of the Lomax distribution.





Truncated Lomax Distribution

It is the ratio of probability density function of the Lomax distribution to their cumulative distribution function at the point B.

The random variable X is said to follow a truncated Lomax Distribution as:

$$f_B(x) = \frac{\frac{\alpha}{\lambda}\left[1+\frac{x}{\lambda}\right]^{-(\alpha+1)}}{1-\left[1+\frac{B}{\lambda}\right]^{-\alpha}}, \quad x \geq 0,\ \alpha > 0 \qquad \text{…. (1.2)}$$

Where' **B'** is the truncated point of the Lomax Distribution.

A new distribution with two extra parameters named a Truncated Lomax distribution is proposed which is more flexible than many well-known heavily tailed distributions. The importance of Truncated Lomax distribution is illustrated by means of the two real data sets. The results indicate that the new distribution can provide better fits than Exponential, Weibull, Gamma, Log-normal, Log-logistic and generalized extreme value distribution in insurance. Therefore, Truncated Lomax distribution can be alternative to modeling the catastrophe loss in insurance applications.

2. DESCRIPTION OF THE PLAN AND TYPE- C OC CURVE

Beattie [2] has suggested the method for constructing the continuous acceptance sampling plans. The procedure, suggested by him consists of a chosen decision interval namely, "Return interval" with the length h', above the decision line is taken. We plot on the chart the sum $S_m = \sum (X_i - k_1) X_i's(i = 1,2,3........)$ is distributed independently and k_1 is the reference value. If the sum lies in the area of normal chart, the product is accepted and if it lies of the return chart, then the product is rejected, subject to the following assumptions.

1. When the recently plotted point on the chart touches the decision line, then the next point to be plotted at the maximum, i.e., h+h'
2. When the decision line is reached or crossed from above, the next point on the chart is to be plotted from the baseline.

When the CUSUM falls in the return chart, network or a change of specification may be employed rather than outright rejection.

The procedure in brief is given below.

1. Start plotting the CUSUM at 0.
2. The product is accepted when $S_m = \sum (X_i - k) < h$; when $S_m < 0$, return cumulative to 0.
3. When h $< S_m <$ h+h' the product is rejected: when S_m crossed h, i.e., when $S_m >$h+h' and continuously rejecting product until $S_m >$h+h' return cumulative to h+h'

The type-C, OC function, which is defined as the probability of acceptance of an item as function of incoming quality, when sampling rate is the same in acceptance and rejection regions. Then the probability of acceptance P (A) is given by

$$P(A) = \frac{L(0)}{L(0) + L'(0)} \qquad \text{...... (2.1)}$$

Where L (0) = Average Run Length in acceptance zone and

L' (0) = Average Run Length in rejection zone.

Page E.S. [8] has introduced the formulae for L (0) and L' (0) as

$$L(0) = \frac{N(0)}{1 - P(0)} \qquad \text{...... (2.2)}$$

$$L'(0) = \frac{N'(0)}{1 - P'(0)} \qquad \text{...... (2.3)}$$

Where P (0) =Probability for the test starting from zero on the normal chart,

N (0) = ASN for the test starting from zero on the normal chart,

P' (0) = Probability for the test on the return chart and

N' (0) = ASN for the test on the return chart

He further obtained integral equations for the quantities

P (0), N (0), P' (0), N' (0) as follows:

$$P(z) = F(k_1 - z) + \int_0^h P(y) f(y + k_1 - z) dy \ , \qquad \text{...... (2.4)}$$

$$N(z) = 1 + \int_0^h N(y) f(y + k_1 - z) dy \ , \qquad \text{...... (2.5)}$$

$$P'(z) = \int_{k_1 + z}^{B} f(y) dy + \int_0^h P'(y) f(-y + k_1 + z) dy \qquad \text{...... (2.6)}$$

$$N'(z) = 1 + \int_0^h N'(y) f(-y + k_1 + z) dy \ , \qquad \text{...... (2.7)}$$

$$F(x) = 1 + \int_A^h f(x)dx \; :$$

$$F(k_1 - z) = 1 + \int_A^{k_1 - z} f(y)dy$$

and z is the distance of the starting of the test in the normal chart from zero.

3. METHOD OF SOLUTION

We first express the integral equation (2.4) in the form

$$F(X) = Q(X) + \int_c^d R(x,t)F(t)dt \qquad \text{...... (3.1)}$$

Where

$F(X) = P(z),$

$Q(X) = F(k - z),$

$R(X,t) = f(y + k - z)$

Let the integral $I = \int_c^d f(x)dx$ be transformed to

...... (3.2)

$$I = \frac{d-c}{2}\int_c^d f(y)dy = \frac{d-c}{2}\sum a_i f(t_i)$$

Where $y = \frac{2x - (c-d)}{d-c}$ where a_i's and t_i's respectively the weight factor and abscissa for the Gauss-Chebyshev polynomial, given in Jain M.K. and et al [4] using (3.1) and (3.2),(2.4) can be written as

$$F(X) = Q(X)\frac{d-c}{2}\sum a_i R(x,t_i)F(t_i) \qquad \text{...... (3.3)}$$

Since equation (3.3) should be valid for all values of x in the interval (c, d), it must be true for $x=t_i$, i = 0 (1) n then obtain.

...... (3.4)

$$F(t_i) = Q(t_i) + \frac{d-c}{2}\sum a_i R(t_j,t_i)F(t_i) \qquad j = 0(1)n$$

Substituting

$F(t_i) = F_i, Q(t_i) = Q_i, i = 0(1)n, in \quad (3.4),$ we get

$$F_0 = Q_0 + \frac{d-c}{2}[a_0 R(t_0,t_0)F_0 + a_1 R(t_0,t_1)F_1 + \text{..........} .a_n R(t_0,t_n)F_n)]$$

$$F_1 = Q_1 + \frac{d-c}{2}[a_0 R(t_1,t_0)F_0 + a_1 R(t_1,t_1)F_1 + \text{..........} .a_n R(t_1,t_n)F_n)]$$

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$$F_n = Q_n + \frac{d-c}{2}[a_0 R(t_n,t_0)F_0 + a_1 R(t_n,t_1)F_1 + \text{..........} .a_n R(t_n,t_n)F_n)] \qquad \text{...... (3.5)}$$

In the system of equations except F_i , i= 0,1,2……………n are known and hence can be solved for F_i, we solved the system of equations by the method of Iteration. For this we write the system (3.5) as

$$[1 - Ta_0 R(t_0,t_0)]F_0 = Q_0 + T[a_0 R(t_0,t_0)F_0 + a_1 R(t_0,t_1)F_1 + \text{.........} \; a_n R(t_0,t_n)F_n)]$$

$$[1 - Ta_1 R(t_1, t_1)]F_1 = Q_1 + T[a_0 R(t_1, t_0)F_0 + a_1 R(t_1, t_1)F_1 + \ldots\ldots\ldots \ a_n R(t_1, t_n)F_n)]$$

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$$[1 - Ta_n R(t_n, t_n)]F_n = Q_n + T[a_0 R(t_n, t_0)F_0 + a_1 R(t_n, t_1)F_1 + \ldots\ldots\ldots \ a_n R(t_n, t_n)F_n)] \qquad \ldots..(3.6)$$

Where $T = \dfrac{d - c}{2}$

To start the Iteration process, let us put $F_1 = F_2 = \ldots. = F_n = 0$ in the first equation of (3.6), we then obtain a rough value of F_0. Putting this value of F_0 and $F_1 = F_2 = \ldots. = F_n = 0$ on the second equation, we get the rough value F_1 and so on. This gives the first set of values F_i i= 0,1,2,...,n which are just the refined values of F_i i= 0,1,2,…,n. The process is continued until two consecutive sets of values are obtained up to a certain degree of accuracy. In the similar way solutions P' (0), N (0), N' (0) can be obtained.

4. COMPUTATION OF ARL's AND P (A)

We developed computer programs to solve the equations (2.4), (2.5), (2.6) and (2.7) and we got the following results given in the Tables (4.1) to (4.24).

TABLE-4.1
Values of ARL's AND TYPE-C OC CURVES when
α=2, λ =4, k=1.5, h=0.10, h'=0.10

B	L(0)	L'(0)	P(A)
2.1	11.55481	1.9672639	0.8545145988
2.0	14.23293	2.0276411	0.8753032088
1.9	19.00994	2.0996311	0.9005365372
1.8	29.92328	2.1868854	0.9318943024
1.7	79.84055	2.2947578	0.9720612764

TABLE-4.2
Values of ARL's and TYPE-C OC CURVES when
α=2, λ =4, k=2, h=0.10, h'=0.10

B	L(0)	L'(0)	P(A)
2.6	16.12509	1.7699907	0.9010906816
2.5	20.04141	1.7997228	0.9175993800
2.4	27.09236	1.8333746	0.9366178513
2.3	43.53524	1.8717581	0.9587782025
2.2	125.37740	1.9159253	0.9849487543

TABLE-4.3
Values of ARL's AND TYPE-C OC CURVES when
α=2, λ =4, k=2.5, h=0.10, h'=0.10

B	L(0)	L'(0)	P(A)
3.1	22.14269	1.6617905	0.9301900268
3.0	27.90147	1.6792951	0.9432353377
2.9	38.56524	1.6985635	0.9578141570
2.8	64.97897	1.7198679	0.9742144346
2.7	240.49442	1.7435414	0.9928023815

TABLE-4.4
Values of ARL's and TYPE-C OC CURVES When
α=2, λ =4, k=3, h=0.10, h'=0.10

B	L(0)	L'(0)	P(A)
3.6	30.05311	1.5939482	0.9496335983
3.5	38.57830	1.6053915	0.9600486755
3.4	55.06450	1.6177710	0.9714589715
3.3	100.40211	1.6312029	0.9840130210
3.2	778.61249	1.6458230	0.9978906512

TABLE-4.5
Values of ARL's AND TYPE-C OC CURVES when
α=2, λ =4, k=1.5, h=0.12, h'=0.12

B	L(0)	L'(0)	P(A)
2.1	14.12652	2.3804674	0.8557903767
2.0	18.02655	2.4845493	0.8788680434
1.9	25.66819	2.6120420	0.9076371789
1.8	47.33793	2.7717428	0.9446864724
1.7	542.31726	2.9774745	0.9945396781

TABLE-4.6
Values of ARL's and TYPE-C OC CURVES when
α=2, λ =4, k=1.5, h=0.18, h'=0.18

B	L(0)	L'(0)	P(A)
2.3	20.23001	4.6633110	0.8126682043
2.2	26.13974	5.1343265	0.8358279467
2.1	37.86080	5.7782068	0.8675907850
2.0	72.10518	6.7109442	0.9148531556
1.9	1670.68457	8.1821671	0.9951263666

TABLE-4.7
Values of ARL's AND TYPE-C OC CURVES when
α=2, λ =4.5, k=1.5, h=0.20, h'=0.20

B	L(0)	L'(0)	P(A)
2.3	14.97682	3.1709139	0.8252722025
2.2	18.57917	3.3654804	0.8466377854
2.1	24.97241	3.6097531	0.8737061024
1.9	39.41126	3.9254141	0.9094204903
1.8	102.50545	4.3488874	0.9593007565

Table-4.8
Values of ARL's And Type-C OC Curves when
α=2, λ =4.5, k=2, h=0.20, h'=0.20

B	L(0)	L'(0)	P(A)
2.8	19.63176	2.5927384	0.8833386898
2.7	24.30739	2.6747277	0.9008703828
2.6	32.48593	2.7696147	0.9214417338
2.5	50.43513	2.8806651	0.9459697604
2.4	121.44909	3.0123408	0.9757969975

TABLE-4.9
Values of ARL's AND TYPE-C OC CURVES when
α=2, λ =4.5, k=2.5, h=0.20, h'=0.20

B	L(0)	L'(0)	P(A)
3.3	26.19493	2.3080285	0.9190249443
3.2	32.81628	2.3527057	0.9331028461
3.1	44.69670	2.4024832	0.9489910007
2.9	72.20257	2.4582708	0.9670741558
2.8	205.11890	2.5212071	0.9878578186

TABLE-4.10
Values of ARL's and TYPE-C OC CURVES when
α=2, λ =4.5, k=3, h=0.20, h'=0.20

B	L(0)	L'(0)	P(A)
3.8	35.37514	2.1395493	0.9429677129
3.7	45.32010	2.1674635	0.9543572068
3.6	64.22472	2.1978874	0.9669105411
3.5	114.08570	2.2311678	0.9808182120
3.4	613.21057	2.2677162	0.9963155389

TABLE-4.11
Values of ARL's AND TYPE-C OC CURVES when
α=2, λ =4, k=3, h=0.10, h'=0.10

B	L(0)	L'(0)	P(A)
4.3	13.24844	1.5331647	0.8962788582
4.2	14.22590	1.5402158	0.9023084641
4.1	15.41829	1.5477262	0.9087749124
4.0	16.90480	1.5557401	0.9157261848
3.9	18.80901	1.5643080	0.9232178330

TABLE-4.12
Values of ARL's and TYPE-C OC CURVES when
α=2, λ =4, k=3, h=0.12, h'=0.12

B	L(0)	L'(0)	P(A)
4.3	14.99346	1.7004458	0.8981397152
4.2	16.18979	1.7106211	0.9044367671
4.1	17.66564	1.7214884	0.9112045765
4.0	19.53133	1.7331173	0.9184969664
3.9	21.96409	1.7455885	0.9263765216

TABLE-4.13
Values of ARL's AND TYPE-C OC CURVES when
α=2, λ =4, k=3, h=0.15, h'=0.15

B	L(0)	L'(0)	P(A)
4.3	18.55009	2.0204971	0.9017773867
4.2	20.26078	2.0378706	0.9086101651
4.1	22.42188	2.0565202	0.9159862995
4.0	25.23726	2.0765877	0.9239730835
3.9	29.05568	2.0982358	0.9326493740

TABLE-4.14
Values of ARL's and TYPE-C OC CURVES when
α=2, λ =4, k=3, h=0.18, h'=0.18

B	L(0)	L'(0)	P(A)
4.3	24.05692	2.4671352	0.9069849849
4.2	26.75110	2.4972489	0.9146191478
4.1	30.28933	2.5298080	0.9229167104
4.0	35.13998	2.5651145	0.9319689870
3.9	42.19631	2.6035221	0.9418854713

TABLE-4.15
Values of ARL's AND TYPE-C OC CURVES when
α=2, λ =4, k=3, h=0.20, h'=0.20

B	L(0)	L'(0)	P(A)
4.3	29.77987	2.8778050	0.911879658
4.2	33.74896	2.9224274	0.9203076959
4.1	39.18913	2.9709945	0.9295306802
4.0	47.10046	3.0240400	0.9396694303
3.9	59.65480	3.0821996	0.9508711100

TABLE-4.16
Values of ARL's and TYPE-C OC CURVES when
α=2, λ =4, k=3, h=0.25, h'=0.25

B	L(0)	L'(0)	P(A)
4.3	69.43465	4.7808723	0.9355812669
4.2	90.70218	4.9284010	0.9484641552
4.1	132.92952	5.0940590	0.9630928636
4.0	257.06653	5.2813592	0.9798688889
3.9	8215.00098	5.4947886	**0.9993315935**

TABLE-4.17
Values of ARL's AND TYPE-C OC CURVES when
α=2, λ =4, k=3, h=0.15, h'=0.15

B	L(0)	L'(0)	P(A)
3.8	34.52708	2.1216524	0.9421084523
3.7	43.01743	2.1470571	0.9524613619
3.6	57.96990	2.1747067	0.9638420343
3.5	91.27293	2.2049038	0.9764125347
3.4	230.20418	2.2380085	0.9903717637

TABLE-4.18
Values of ARL's and TYPE-C OC CURVES when
α=2, λ =4, k=2, h=0.18, h'=0.18

B	L(0)	L'(0)	P(A)
2.8	26.93155	3.4454691	0.8865764737
2.7	34.74672	3.6035326	0.9060362577
2.6	50.06136	3.7920883	0.9295850396
2.5	93.52734	4.0208073	0.9587813020
2.4	1106.95068	4.3039303	0.9961269498

TABLE-4.19
Values of ARL's AND TYPE-C OC CURVES when
α=2, λ =4, k=2, h=0.20, h'=0.20

B	L(0)	L'(0)	P(A)
2.9	29.713214	4.2461734	0.8749629259
2.8	38.15330	4.4890919	0.8947269917
2.7	54.37271	4.7848639	0.9191166162
2.6	98.21504	5.1526895	0.9501518607
2.5	63950604	5.6223497	0.9912849069

TABLE-4.20
Values of ARL's and TYPE-C OC CURVES when
α=2, λ =4, k=3, h=0.20, h'=0.20

B	L(0)	L'(0)	P(A)
4.0	47.10046	3.0240400	0.9396694303
3.9	59.65480	3.0821996	0.9508711100
3.8	82.62214	3.1462328	0.9633170962
3.7	138.09215	3.2170589	0.9772338867
3.6	461.43591	3.2957976	0.9929081798

TABLE-4.21
Values of ARL's AND TYPE-C OC CURVES when
α=1, λ =4, k=2, h=0.12, h'=0.12

B	L(0)	L'(0)	P(A)
.5	11.69545	1.3853960	0.8940897584
2.4	14.99139	1.4013257	0.9145153165
2.3	21.49567	1.4191756	0.9380673766
2.2	40.32782	1.4393133	0.9655395746
2.1	738.26984	1.4622059	0.9980233312

TABLE-4.22
Values of ARL's and TYPE-C OC CURVES when
α=2, λ =4.5, k=3, h=0.10, h'=0.10

B	L(0)	L'(0)	P(A)
3.6	23.01853	1.4042108	0.9425039887
3.5	28.67420	1.4116585	0.9530787985
3.4	38.72478	1.4196817	0.9646356702
3.3	61.53769	1.4283473	0.9773156047
3.2	163.61160	1.4377333	0.9912890792

TABLE-4.23
Values of ARL's AND TYPE-C OC CURVES when
α=2, λ =4.5, k=2.5, h=0.12, h'=0.12

B	L(0)	L'(0)	P(A)
3.1	19.95287	1.5794438	0.9266477227
3.0	25.12374	1.5950038	0.9403039217
2.9	34.66338	1.6120968	0.9555596113
2.8	58.14226	1.6309550	0.9727143049
2.7	208.72723	1.6518599	0.9921481609

TABLE-4.24
Values of ARL's and TYPE-C OC CURVES when
α=2, λ =4.5, k=3, h=0.12, h'=0.12

B	L(0)	L'(0)	P(A)
3.6	26.56414	1.5188400	0.9459159970
3.5	33.96438	1.5290967	0.9569189548
3.4	48.09967	1.5401767	0.9689729810
3.3	85.82152	1.5521799	0.9822351336
3.2	502.63605	1.5652231	0.9968956113

5. NUMERICAL RESULTS AND CONCLUSIONS

At the hypothetical values of the parameters α, λ, k, h and h' are given at the top of each table, we determined optimum truncated point B at which P (A) the probability of accepting an item is maximum and also obtained ARL's values which represent the acceptance zone L(0) and rejection zone L'(0) values. The values of truncated point B of random variable X, L(0), L'(0) and the values for Type-C Curve, i.e. P (A) are given in columns I, II, III, and IV respectively.

From the above tables 4.1 to 4.24 we made the following conclusions

1. From the Table 4.1 to 4.24, it is observed that the values of P(A) is increased as the value of truncated point decreases. Thus, the truncated point of the random variable and the various parameters for CASP-CUSUM are related.

2. From the Table 4.1 to 4.24, we observed that it could be maximized the truncated point B by increasing value of *k.*

3. From Table 4.1 to 4.24, it is observed that at the maximum level of probability of acceptance P (A) the truncated point B from 5.0 to 1.2 as the value of h changes from 0.10 to 0.25.

4. From the Table 4.1 to 4.24, it was observed that the value of L (0) and P (A) are increased as the value of truncated point decreases thus the truncated point of the random variable and the various parameters for CASP-CUSUM are related.

5. From the Table 4.1 to 4.24, it was observed that the truncated point B changes from 3.0 to1 .1 and P (A) is as $h \to 0.25$ maximum **i.e. 0.999331.** Thus truncated point B and h are inversely related and h and P (A) are positively related.

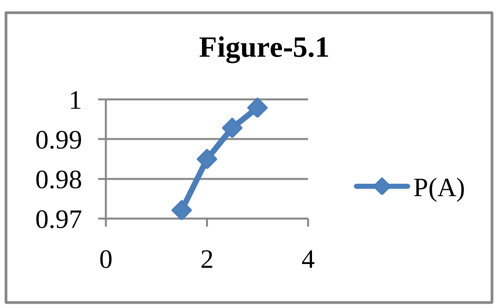
6. From Table 4.1 to 4.24 it is observed that the optimal truncated point changes from 1.7 to 3.9 as $h \to 0.25$.

7. It is observed that the Table -5.1 values of Maximum Probabilities increased as the increased values of '**k**' as shown in the Figure-5.1.

Table-5.1

α=2, λ=4, h=0.10, h'=0.10

k	P(A)
1.5	0.972061
2	0.984949
2.5	0.992802
3	0.997891

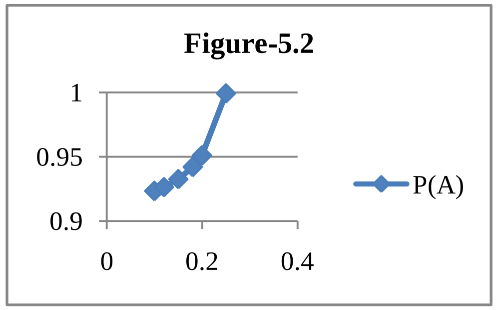


8. It is observed that the Table-5.2 values of Maximum Probabilities increased as the values of h and h' as shown in the Figure-5.2.

Table-5.2

α=2, λ=4, B=3.9, k=3

h and h'	P(A)
0.10	0.923217
0.12	0.926376
0.15	0.932649
0.18	0.941885
0.20	0.950871
0.25	0.999331



9. The various relations exhibited among the ARL's and Type-C OC curves with the parameters of the CASP-CUSUM based on the above table 4.1 to 4.24 are observed from the following Table.

Table 5.3
CONSOLIDATED TABLE

B	A	λ	h	h'	k	P(A)
1.7	2	4	0.10	0.10	1.5	0.972061
3.2	2	4	0.10	0.10	3	0.997890
1.7	2	4	0.12	0.12	1.5	0.994539
2.4	2	4	0.18	0.18	2	0.996126
3.2	2	4	0.10	0.10	3	0.997890
2.5	2	4	0.20	0.20	2	0.991284
3.6	2	4	0.20	0.20	3	0.992908
1.7	2	4.5	0.10	0.10	1.5	0.961777
3.2	2	4.5	0.10	0.10	3	0.991289
3.3	2	4.5	0.15	0.15	3	0.991325
1.9	2	4	0.18	0.18	1.5	0.995126
1.7	2	4.5	0.12	0.12	1.5	0.974898
3.9	***2***	***4***	***0.25***	***0.25***	***3***	***0.999331***
2.8	2	4	0.25	0.25	2	0.997206
2.7	2	4.5	0.12	0.12	2.5	0.992148
2.3	2	4.5	0.18	0.18	2	0.992314
2.2	2	4	0.12	0.12	2	0.998616

By observing the Table- 5(c), we can conclude that the optimum CASP-CUSUM schemes which have the values of ARL and P (A) reach their maximum i.e., 8215, 0.999331 respectively, is

$$\begin{bmatrix} B = 3.9 \\ \alpha = 2 \\ \lambda = 4 \\ k = 3 \\ h = 0.25 \\ h' = 0.25 \end{bmatrix}$$

On similar lines we can obtain CASP-CUSUM schemes when a particular parameter is fixed at a point, for example, if we fix the value of $k = 2$, in that case only the maximum value of probability of acceptance P(A)= 0.998616, is

$$\begin{bmatrix} B = 2.2 \\ \alpha = 2 \\ \lambda = 4 \\ k = 2 \\ h = 0.12 \\ h' = 0.12 \end{bmatrix}$$

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