Number Set Theory and Collatz Conjecture

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Abstract: *Set theory is widely use in the mathematics to study sets. The number is grain of mathematics, which categories in the set of integers (Z), the set of natural number (positive integers) (N), the set of rational numbers (Q), and set of real number (R). Here is study of the natural number by using the arithmetic series distribution and by the set theory. So, study of number in sequential series and by set distribution is called as 'number set theory'. Here we use the set theory is in slightly different way and in different notation. The theory study the arithmetic series problem, like Collatz conjecture / (3n+1) problem. Collatz conjecture is most famous (3n+1) arithmetic series problem. (3n+1) is one of the set which, we represent in number set theory is S{3n+1}. This set is divided into many sub-set for the deep understanding of the (3n+1) problem. Here is study of the collatz conjecture using the number set theory.*

Keywords — *set theory, collatz conjecture, Arithmetic series.*

I. INTRODUCTION

For the understanding of number system, we must have to understand number pattern by using the arithmetic series and set theory [1]. Generally numbers are categories in four parts, set of integers (Z), the set of natural number (positive integers) (N), the set of rational numbers (Q), and set of real number (R). The set of natural number is also sub-divided into even number and odd number. Is this classification is proper classification of natural number? What is base of classification in even and odd number? '2' is first base element of in the prime numbers and multiplication of natural number (n), give a probability of all natural number in multiple of 2 is 50 percentages. So, it appears in alternating way and it called as even number. Left remaining number is called as odd numbers. But odd number is not simply taken as in one single set. But base element for set is prime numbers. So, it has infinite sets with different probability percentages. For example, for base element prime number 3, its probability percentage is approximately 16.7. For bigger base element prime number, probability percentage is smaller.

The Collatz conjecture is named after Lothal Collatz. It is also known as the (3n+1) conjecture, and by many other name [2]. Collatz conjecture state that any positive integer number n multiply with 3 and add 1, will give either odd or even. If it even then divide with 2 and get odd and again multiply with 3 and add 1, and if it even then divide with 2 get odd. This cycle is always be eventually reach at 1, and get trivial cycle (4, 2, 1). Mathematical statement is –

$$C(n) = \begin{cases} (3n+1) & , \quad n\ odd \\ \dfrac{n}{2} & , \quad n\ even \end{cases}$$

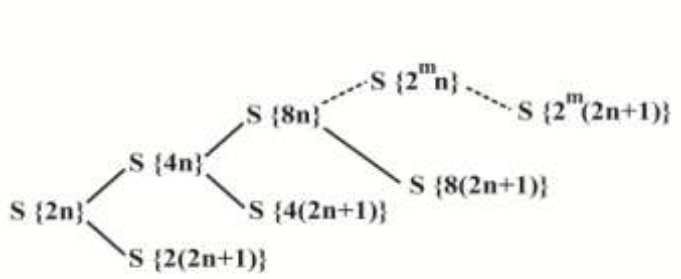
Where, n = 1, 2, 3,……..

II. SET OF ARITHMETIC SERIES FOR NATURAL NUMBER

Set of arithmetic series representation for natural numbers is –

$$S\{n\} = \{1, 2, 3,\ldots,.\}$$

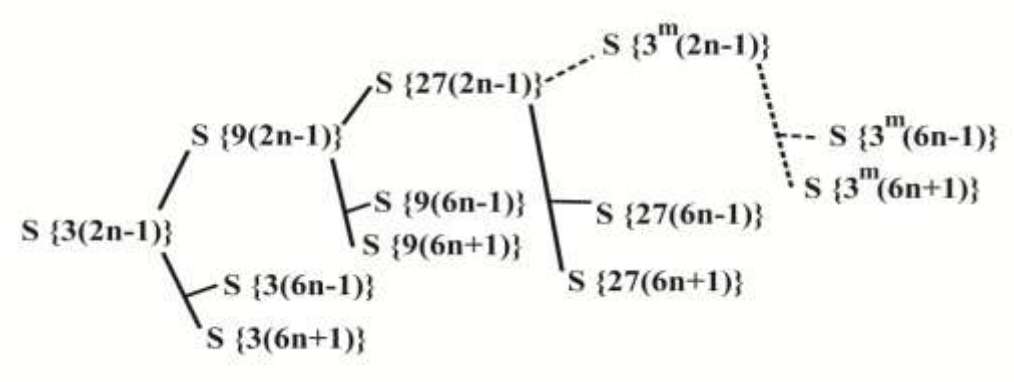
Similarly, for even is S{2n} and for odd is S{2n+1}. Set of the even series is based on the '2' base prime element. So, series is sub-divided into $S\{2^m\}$ and $S\{2^m(2n+1)\}$. Where, m = 1, 2, 3,…,. and n = 1, 2, 3,....,. Set Distribution in even number is –



And the total for first prime number element '2' series is represented as –

$$S\{2n\} = S\{2^m\} \cup S\{2^m(2n+1)\}.$$

Similarly for the prime number element 3 is –



And total series for prime number element 3 is –

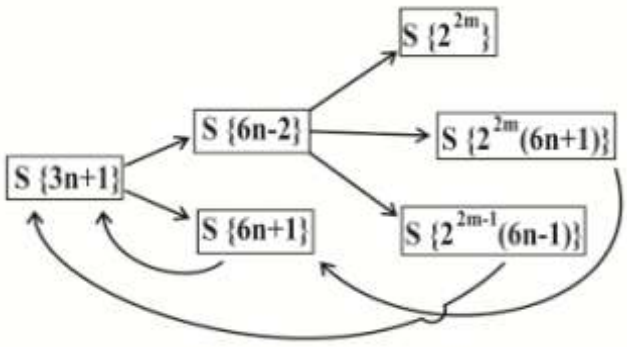
$$S\{3^m(2n-1)\} = S\{3^m\} \cup [S\{3^m(6n-1)\} \cup S\{3^m(6n+1)\}]$$

For all prime number elements, the set of number distribution is different for different-different and also unique. For collatz like conjecture, it is useful to separate to set with equal number distribution. For example, S{3n+1} is sub divided into S{6n-1} and S{6n+1}.

III. Brief Description of Conjecture

A. *Flow Chart of Collatz Conjecture*

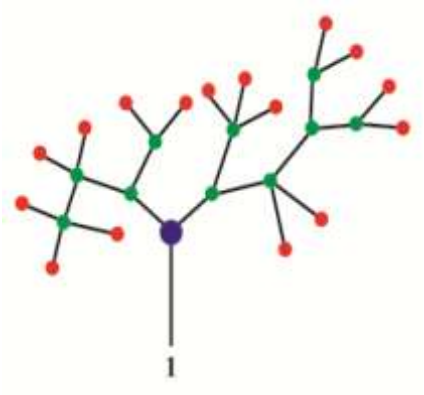
By using number set theory, S{3n+1} is subdivided into two new set S{6n-2} even set, and S{6n+1} odd set. Even set S{6n-2} is also subdivided into three new set, $S\{2^{2m}\}$, $S\{2^{2m}(6n+1)\}$, and $S\{2^{2m}(6n-1)\}$. So, flowchart for Collatz conjecture is –



B. *Data Tree for Collatz Conjecture*

All numbers in S{3n+1} is connected in a such a way that, it create data tree which is shortly described as –

1) All odd elements are use in chain called as 'nodes'.
2) All the branches of the data tree start up with the 'root nodes' and converge to 1.
3) Every node have infinite branches except 'root nodes' or all nodes are 'branch nodes' except 'root nodes'.



Short description of the figure is –

1) All red dots represent the 'root nodes' of the data tree.
2) All green dots represent the 'branch nodes' of the data tree.
3) Black lines represent branches.
4) Dark blue dot represent 'last nodes' of the data tree.

IV. Set distribution in Collatz conjecture

A. 'root nodes' for the S{3n+1} is –

$$S\{3(2n-1)\} = \{3, 9, 15, \dots\dots.\}$$

B. Set of 'last nodes' is –

$$S\left\{\sum_{n=0}^{n} 2^{2n}\right\} = \{1, 5, 21, 85, \dots\dots.\}$$

C. 'Last nodes' are subdivided into three sets –

First set is, subset of the S{3(2n+1)},which is –

$$S\left\{\sum_{\phi=1}^{3n} 2^{6n-2\phi}\right\} = \{21, 1365, 87381, \dots\dots.\}$$

Due to subset of the series of the S{3(2n+1)}. So, this is 'root nodes' and it iterate only one time and directly give 1. Thereto, this set is also we called as 'exterior nodes'.

Second and third set is subset of the –

$$S\left\{\sum_{\phi=1}^{\psi(n)} 2^{2(\psi(n)-\phi)}\right\} = \{5, 858, 341, \dots\dots.\}$$

Where,

$$\psi(n) = \left[\frac{3n + 2^{\left(\frac{1}{2}+\frac{1}{2}(-1)^n\right)}}{2}\right]$$

These subsets are also a subset of (6n-1) and (6n+1).

Second set is subset of the (6n-1).

$$S\left\{\sum_{\phi=1}^{\psi_1(n)} 2^{2(\psi_1(n)-\phi)}\right\} = \{5, 341, 21845, \dots\dots.\}$$

Where, $\psi_1(n)$ = (3n-1).

Third set is subset of the (6n+1).

$$S\left\{\sum_{\phi=1}^{\psi_2(n)} 2^{2(\psi_2(n)-\phi)}\right\} = \{85, 5461, \dots\dots.\}$$

Where, $\psi_2(n)$ = (3n+1).

V. CONCLUSION

Conclusion of the paper is study of number, in sequential series and set distribution, in the arithmetic series problem. Here, we study collatz conjecture ((3n+1) problem) using the set disribution in S{3n+1}.

REFERENCES

[1] Norman T. Hamilton and Joseph Landin, Set theory: the structure of Arithmetic, Allyn and Bacon, inc. Boston, 1961.
[2] Jeffrey C. Lagarias, Editor, The Ultimate Challenge: The 3x+1 Problem, American Mathematical Society, 2010.