

Buongiorno Model with Revised Boundary Conditions for Hydromagnetic Forced Convective Nanofluid Flow Past a Rotating Porous Disk

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Abstract - An analysis has been carried out to investigate the effects of thermophoresis and Brownian motion on hydromagnetic flow of a viscous, incompressible, electrically conducting nanofluid over a rotating porous disk. Two types of nanofluids such as copper-water nanofluid and silver-water nanofluid are considered. Governing equations of the problem are transformed into set of non-linear ordinary differential equations utilizing similarity transformations. The resulting non-linear differential equations are solved numerically by utilizing Nachtsheim-Swigert shooting scheme for satisfaction of asymptotic boundary conditions along with Runge - Kutta Fehlberg Method. The effects of different parameters on the velocity as well as on temperature are depicted graphically. Numerical values of radial and tangential skin friction coefficients and the non-dimensional rate of heat transfer are shown in tabular form.

Keyword – MHD, Nanofluid, Rotating Disk, Forced convection, Suction

I. INTRODUCTION

The flow due to a rotating disk which is one of the classical problems in fluid mechanics has received much attention in several industrial, geothermal, geophysical, technological and engineering processes. The reason of such a great interest can be attributed to its having a three-dimensional exact similarity solution which is significant in the study of engineering flows in rotating machinery, lubrication, rotating heat exchangers, rotating disk reactors for bio-fuels production, computer disk drives, centrifugal pumps, viscometers and some aerodynamic related problems in fluid mechanics.

The pioneering study of fluid flow due to an infinite rotating disk has been carried out by Von Karman (1921). He first formulated the problem and then the governing partial differential equations have been reduced to the ordinary differential equations by defining appropriate transformations. Cochran (1934) obtained asymptotic solutions for the steady problem formulated by von Karman and Benton (1966) and solved the unsteady state of this problem. Kuiken (1971) discussed the effects of normal blowing on the flow near the rotating disk of finite extent. Parter and Rajagopal (1984) further proved that when the disks rotate with different angular velocities about distinct axes or a common axis there is a one-parameter family of solutions.

Some interesting effects of the magnetic field on the steady hydromagnetic flow due to the rotation of a disk of infinite or finite extent was examined by El-Mistikawy and Attia (1990) and El-Mistikawy et al. (1991). The steady hydromagnetic boundary layer flow due to an infinite disk rotating with a uniform angular velocity in the presence of an axial magnetic field was investigated by Aboul-Hassan and Attia (1997). Kelson and Desseaux (2000) revised the classical von Karman rotating disk problem. They reexamined the flow of a porous heated rotating disk, motivated by the view that the problem can serve as a prototype for practical swirl flows. Maleque and Sattar (2003, 2005), Attia (2007), and Hayat et al. (2007, 2008) have carried out studies on steady and unsteady MHD flow due to a rotating disk.

Thermal radiation effect on steady laminar hydromagnetic convective flow past a porous rotating infinite disk with the consideration of heat and mass transfer in the presence of Soret and Dufour diffusion effects was investigated by Anjali Devi and Uma Devi (2011). Turkyilmazoglu (2012) presented an exact solution for the incompressible viscous magnetohydrodynamic fluid flow over a permeable disk rotating about its axis of rotation.

Due to enormous industrial, transportation, electronics, biomedical applications, such as in advanced nuclear systems, cylindrical heat pipes, automobiles, fuel cells, drug delivery, biological sensors, and hybrid-powered engines, the convective heat transfer in nanofluids has become a topic of great interest. Recently, the term 'nanofluid' was first proposed by Choi (1995) to indicate engineered colloids composed of nanoparticles dispersed in a base fluid.

A comprehensive study of convective transport in nanofluids was made by Buongiorno (2006) who said that a satisfactory explanation for the increase of the thermal conductivity and viscosity is yet to be found. He considered in turn seven slip mechanisms: inertia, Brownian diffusion, thermophoresis, diffusiophoresis, magnus effect, fluid drainage and gravity, and claimed that out of these seven, only Brownian diffusion and thermophoresis are important slip mechanisms in nanofluids.

Khan and Pop (2010) investigated boundary layer flow of a nanofluid past a stretching sheet. The model used by them for the nanofluid incorporates the effects of Brownian motion and Thermophoresis. Chamkha et al. (2011) investigated the mixed convection flow of a nanofluid past a stretching surface in the presence of Brownian motion and thermophoresis effects. Haddad et al. (2012) experimentally investigated natural convection in nanofluid by considering the role of thermophoresis and Brownian motion in heat transfer enhancement. They indicated that neglecting the role of Brownian motion and thermophoresis deteriorate the heat transfer and this deterioration elevates when the volume fraction of nanoparticles increases.

Anjali Devi and Suriyakumar (2013) investigated numerically the laminar hydromagnetic boundary layer flow of nanofluids over an inclined flat plate. Numerical modeling of fluid flow and thermal behaviour of different Nanofluids on a rotating disk was scrutinized by Masoudmofarahi et al. (2014). The flow and heat transfer characteristics over a rotating disk immersed in five distinct nanofluids has been investigated by Turkyilmazoglu (2014). Hayat et al. (2015) studied magnetohydrodynamic flow of copper water nanofluid due to a rotating disk with partial slip. Anjali Devi and Elakkiyapriya (2017) worked on the slip effects on hydromagnetic flow of nanofluids due to a rotating porous disk with internal heat absorption.

A close observation of the literature reveals that, to the best of author’s knowledge, so far no one has considered the role of Brownian motion and thermophoresis on steady, laminar, hydromagnetic flow of a Nanofluid over a rotating disk in the presence of suction. The fluid is water - based nanofluid containing two types of nanoparticles, namely, Copper and Silver. The governing partial differential equations of the flow are converted into nonlinear coupled ordinary differential equations by using similarity transformation. Nachtsheim-Swigert shooting scheme for satisfaction of asymptotic boundary conditions along with Fourth order Runge - Kutta Fehlberg Method is employed to yield the numerical solutions for the model. A parametric study is conducted to illustrate the influence of various governing physical parameters on the velocities, temperature and nanoparticle volume fraction concentration and also on the radial and tangential skin friction coefficients and non-dimensional heat transfer rate.

II. MATHEMATICAL MODEL

The problem of steady, laminar, three dimensional, axisymmetric, hydromagnetic flow of incompressible nanofluid due to a rotating porous disk in the presence of Thermophoresis and Brownian motion has been considered. Fig. 1 shows the physical model and geometrical coordinates. The disk at $z = 0$ rotates with constant angular velocity Ω (where z is the vertical axis in the cylindrical coordinate system with r and ϕ as the radial and tangential axes respectively) and is subjected to uniform suction W_0 ($W_0 > 0$). The components of the flow velocity are (u, v, w) in the directions of increasing (r, ϕ, z) , respectively. The pressure is p and the density of the fluid is ρ . T and Φ are the fluid temperature and Nanoparticle volume fraction, respectively and the surface of the rotating disk is maintained at a uniform temperature T_w and the normal flux of the nanoparticles is assumed to be zero at the boundary in order to cope up with the physical reality. Far away from the surface, the free stream is kept at a constant temperature and pressure, T_∞ and p_∞ , respectively. The fluid is viscous and electrically conducting.

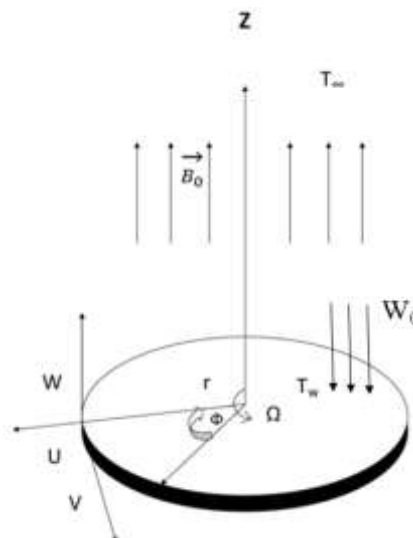


Fig. 1 Schematic diagram of the Problem

An external uniform magnetic field is applied in the z – direction. The induced magnetic field due to the motion of the electrically conducting fluid is assumed to be negligible by considering magnetic Reynolds number ($R_m \ll 1$) as small. Since the induced magnetic field is neglected and \vec{B}_0 is independent of time, $\text{curl } \vec{E} = 0$. Also, $\text{div } \vec{E} = 0$ in the absence of surface charge density. Hence $\vec{E} = 0$. The model used for the nanofluid incorporates the effects of Brownian motion and Thermophoresis. The fluid is water based Nanofluid containing different types of nanosolid particles, say Copper (Cu), Silver (Ag). Under the above assumptions, the conservation equations for mass, momentum, thermal energy and nanoparticle volume fraction for nanofluids can be written as, [see Buongiorno (2006)]

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0 \tag{1}$$

$$u \frac{\partial u}{\partial r} - \frac{v^2}{r} + w \frac{\partial u}{\partial z} + \frac{1}{\rho_{nf}} \frac{\partial P}{\partial r} = \nu_{nf} \left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} - \frac{u}{r^2} + \frac{\partial^2 u}{\partial z^2} \right) - \frac{\sigma B_0^2}{\rho_{nf}} u \tag{2}$$

$$u \frac{\partial v}{\partial r} + \frac{uv}{r} + w \frac{\partial v}{\partial z} = \nu_{nf} \left(\frac{\partial^2 v}{\partial r^2} + \frac{1}{r} \frac{\partial v}{\partial r} - \frac{v}{r^2} + \frac{\partial^2 v}{\partial z^2} \right) - \frac{\sigma B_0^2}{\rho_{nf}} v \tag{3}$$

$$u \frac{\partial w}{\partial r} + w \frac{\partial w}{\partial z} + \frac{1}{\rho_{nf}} \frac{\partial P}{\partial z} = \nu_{nf} \left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial z^2} \right) \tag{4}$$

$$u \frac{\partial T}{\partial r} + w \frac{\partial T}{\partial z} = \alpha_{nf} \left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} \right) + \frac{(\rho C_p)_s}{(\rho C_p)_{nf}} \left(D_B \frac{\partial \phi}{\partial z} \frac{\partial T}{\partial z} + \frac{D_T}{T_\infty} \left(\frac{\partial T}{\partial z} \right)^2 \right) \tag{5}$$

$$u \frac{\partial \phi}{\partial r} + w \frac{\partial \phi}{\partial z} = D_B \left(\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r} \frac{\partial \phi}{\partial r} + \frac{\partial^2 \phi}{\partial z^2} \right) + \frac{D_T}{T_\infty} \left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} \right) \tag{6}$$

These equations are to be solved under the following boundary conditions

$$u = 0, \quad v = \Omega r, \quad w = -W_0, \quad T = T_w, \quad D_B \frac{\partial \phi}{\partial z} + \frac{D_T}{T_\infty} \frac{\partial T}{\partial z} = 0 \quad \text{at } z = 0 \tag{7}$$

$$u \rightarrow 0, \quad v \rightarrow 0, \quad T \rightarrow T_\infty, \quad p \rightarrow p_\infty, \quad \phi \rightarrow \phi_\infty \quad \text{as } z \rightarrow \infty$$

In the above expressions D_B and D_T represent the Brownian diffusion coefficient and the Thermophoresis diffusion coefficient, ρ_{nf} is the nanofluid density, ν_{nf} is the kinematic viscosity of the nanofluid, α_{nf} is the thermal diffusivity of nanofluid, $(\rho C_p)_{nf}$ is the heat Capacitance of the nanofluid and $(\rho C_p)_s$ is the heat capacitance of the solid particle and all these are defined as follows

$$\nu_{nf} = \frac{\mu_{nf}}{\rho_{nf}}, \quad \mu_{nf} = \frac{\mu_f}{(1 - \phi)^{2.5}}, \quad \alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}}, \quad \rho_{nf} = (1 - \phi)\rho_f + \phi\rho_s, \tag{8}$$

$$(\rho C_p)_{nf} = (1 - \phi)(\rho C_p)_f + \phi(\rho C_p)_s, \quad \frac{k_{nf}}{k_f} = \frac{(k_s + 2k_f) - 2\phi(k_f - k_s)}{(k_s + 2k_f) + \phi(k_f - k_s)}$$

where, μ_f is the viscosity of the base fluid, ϕ is the nanoparticle volume fraction, ρ_f is the density of the base fluid, ρ_s is the density of the solid particle, k_{nf} is the effective thermal conductivity of the nanofluid.

The governing Equations (1) - (7) can be transformed by introducing the similarity transformations

$$\eta = \left(\frac{\Omega}{\nu_f} \right)^{\frac{1}{2}} z, \quad u = \Omega r F(\eta), \quad v = \Omega r G(\eta), \quad w = (\Omega \nu_f)^{\frac{1}{2}} H(\eta), \quad (9)$$

$$P - P_\infty = 2 \mu_f \Omega P(\eta), \quad T - T_\infty = \theta(\eta)(T_w - T_\infty), \quad \Phi(\eta) = \frac{\phi - \phi_\infty}{\phi_\infty}$$

The transformed momentum, energy and nanoparticle volume fraction equations together with the boundary conditions can be written as:

$$2F + H' = 0 \quad (10)$$

$$F'' - [A(F^2 + HF' - G^2) + CM^2F] = 0 \quad (11)$$

$$G'' - [A(HG' + 2FG) + CM^2G] = 0 \quad (12)$$

$$H'' - [AH H' + 2CP'] = 0 \quad (13)$$

$$\frac{k_{nf}}{k_f} \theta'' - Pr BH \theta' + Nb \Phi' \theta' + Nt \theta'^2 = 0 \quad (14)$$

$$\frac{1}{Le} \Phi'' - Pr H \Phi' + \frac{Nt}{Nb} \frac{1}{Le} \theta'' = 0 \quad (15)$$

The transformed boundary conditions are:

$$F(0) = 0, \quad G(0) = 1, \quad H(0) = -W_s, \quad \theta(0) = 1, \quad \Phi'(0) = -\frac{Nt}{Nb} \theta'(0) \quad (16)$$

$$F(\eta) \rightarrow 0, \quad G(\eta) \rightarrow 0, \quad P(\eta) \rightarrow 0, \quad \theta(\eta) \rightarrow 0, \quad \Phi(\eta) \rightarrow 0 \quad \text{as } \eta \rightarrow \infty$$

where primes denote differentiation with respect to η and the non-dimensional parameters appearing in Equations (10) - (16) are defined as follows:

$$M^2 = \frac{\sigma B_0^2}{\Omega \rho_f}$$
 is the magnetic interaction parameter,

$$Pr = \frac{\alpha_f}{\nu_f}$$
 is the Prandtl number,

$$Nb = \frac{(\rho C_p)_s D_B \phi_\infty}{(\rho C_p)_f \alpha_f}$$
 is the Brownian motion parameter,

$$Nt = \frac{(\rho C_p)_s D_T (T_w - T_\infty)}{(\rho C_p)_f \alpha_f T_\infty}$$
 is the Thermophoretic parameter,

$$Le = \frac{\alpha_f}{D_B}$$
 is the Lewis Number,

$$W_s = \frac{W_0}{(\Omega \nu_f)^{\frac{1}{2}}}$$
 is the uniform suction parameter and

$$A = (1 - \phi)^{2.5} (1 - \phi + \phi \rho_s / \rho_f), \quad B = (1 - \phi + \phi (\rho C_p)_s / (\rho C_p)_f) \quad \text{and} \quad C = (1 - \phi)^{2.5}$$

Skin friction coefficient

The radial shear stress and tangential shear stress are given by the Newtonian formulae:

$$\tau_t = \left[\mu_{nf} \left(\frac{\partial v}{\partial z} + \frac{1}{r} \frac{\partial w}{\partial \theta} \right) \right]_{z=0} = \mu_{nf} G'(\Omega / \nu_f)^{1/2}$$

$$\tau_r = \left[\mu_{nf} \left(\frac{\partial u}{\partial z} \right) + \left(\frac{\partial w}{\partial r} \right) \right]_{z=0} = \mu_{nf} F'(\Omega / \nu_f)^{1/2}$$

Hence the tangential and radial skin friction coefficients are respectively given by

$$(Re)^{1/2} C_{ft} = \frac{1}{(1 - \phi)^{2.5}} G'(0) \quad \text{and} \quad (Re)^{1/2} C_{fr} = \frac{1}{(1 - \phi)^{2.5}} F'(0)$$

Nusselt number

The Nusselt number is defined as,

$$Nu = \frac{r q_w}{k_f (T_w - T_\infty)}$$

where the net surface heat flux q_w is given by

$$q_w = -k_{nf} \left(\frac{\partial T}{\partial z} \right)_{z=0}$$

Using (9), the Nusselt number is obtained as,

$$Nu (Re)^{-1/2} = - \frac{k_{nf}}{k_f} \theta'(0)$$

where $Re = (\Omega r^2 / \nu_f)$ is the local rotational Reynolds number.

III. NUMERICAL RESULTS

The governing nonlinear partial differential equations of the problem are converted into set of non-linear ordinary differential equations using similarity transformations. Equations (10) - (13) and (15) constitute the nonlinear boundary value problem which is difficult to solve analytically. Hence the system of transformed equations together with the boundary conditions are solved numerically using Nachtsheim-Swigert shooting iteration technique for the satisfaction of the asymptotic boundary conditions along with Runge-Kutta Fehlberg method. The objective of the technique is to make an initial guess for the values of $F'(0), G'(0), \theta'(0)$ and $\Phi(0)$. In this shooting method, the success of the procedure depends on the appropriate choice of the guess. The different initial guesses were made taking into account of the convergence. The process is repeated until the results are corrected upto the desired accuracy of 10^{-5} level. The numerical computations have been carried out for different values of the parameters involved, namely, Magnetic interaction parameter, Suction parameter, Brownian motion parameter, thermophoresis parameter and Lewis number.

**TABLE I
THERMOPHYSICAL PROPERTIES OF BASE FLUID WATER, COPPER AND SILVER AT 25° C**

	ρ (Kg/m ³)	c_p (J/Kg.K)	k (W/m.K)
Water	997.1	4179	0.613
Copper	8933	385	400
Sliver	10500	235	429

IV. RESULTS AND DISCUSSION

The ultimate goal of this work is to establish the influence of Brownian motion and thermophoresis effects and other physical parameters over the hydromagnetic forced convective Nanofluid flow past a rotating porous disk. The numerical analysis has been carried out for various values of Physical parameters and the computed results are displayed graphically through Fig.2 - Fig.17. The thermophysical properties of the base fluid water and the two different nanoparticles (copper (Cu) and silver (Ag)) are listed in Table I.

In the absence of Brownian motion parameter, thermophoresis parameter, Lewis number, magnetic interaction parameter and suction parameter with $Pr=6.2$, the results are identical to those of Turkyilmazoglu (2014) which are justified through Table II. The comparisons are found to be in very good agreement which validates the adopted numerical scheme.

**TABLE II
COMPARISON OF NUMERICAL VALUES OF $F'(0), -G'(0)$ AND $\theta'(0)$ WHEN $M^2=0$ AND $W_s=0$ WITH $PR = 6.2$**

	$F'(0)$	$-G'(0)$	$-\theta'(0)$
Turkyilmazoglu	0.51023262	0.61592201	0.93387794
Present Work	0.51021	0.61591	0.93301

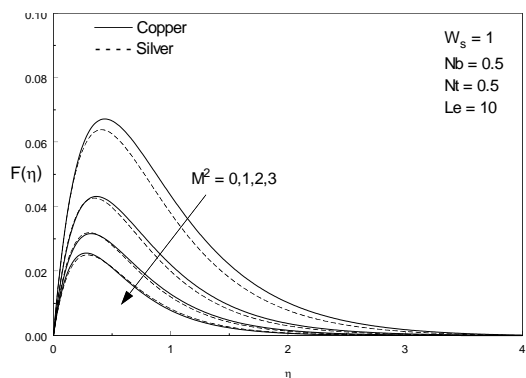


Fig. 2 Effect of magnetic field on radial velocity profiles for copper-water nanofluid and silver-water nanofluid

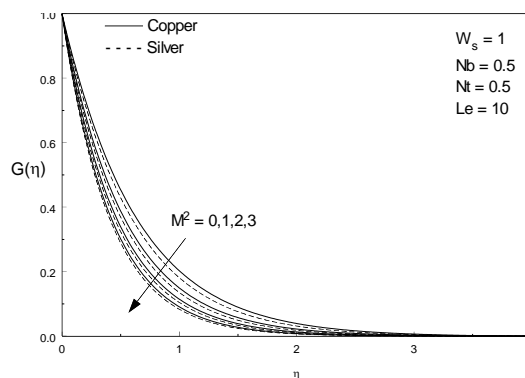


Fig. 3 Tangential velocity profiles for different values of magnetic interaction parameter for copper-water nanofluid and silver-water nanofluid

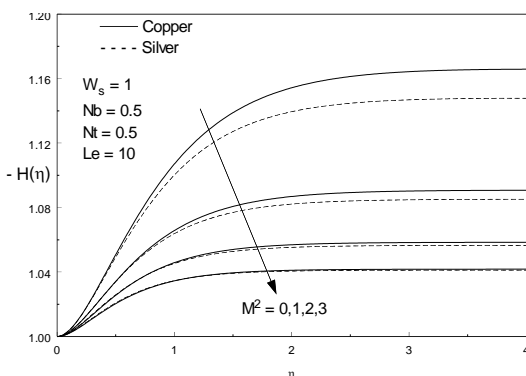


Fig. 4 Effect of magnetic field on axial velocity profiles for copper-water nanofluid and silver-water nanofluid

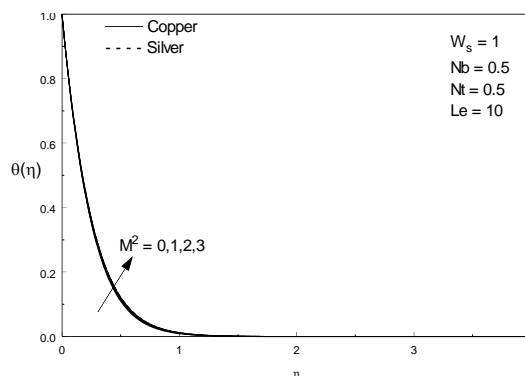


Fig. 5 Temperature distribution for various values of magnetic interaction parameter for copper-water nanofluid and silver-water nanofluid

Figures (2)-(5) elicit the effect of the magnetic interaction parameter M^2 on the velocity components (radial, tangential and axial) and temperature distribution for copper - water nanofluid and silver - water nanofluid. Imposition of a magnetic field in an electrically conducting fluid generally creates a drag like force called Lorentz force that has the tendency to slow down the flow around the disk at the same time increasing fluid temperature. As the magnetic field promotes, the radial, tangential and axial velocities decelerate while the temperature upholds as shown in Figs. 2-5, respectively. Fig.6 is the graphical representation of nanoparticle volume fraction for various values of magnetic interaction parameter for both types nanofluid. It is noted that M^2 increases, $\phi(\eta)$ sublimates. Increasing magnetic interaction parameter leads to reduction in the velocity boundary-layer thickness and propagation of both temperature and nanoparticle volume fraction distributions.

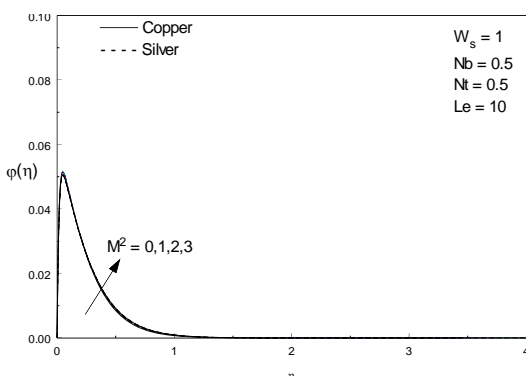


Fig. 6 Nanoparticle volume fraction distribution for various values of M^2 for copper-water nanofluid and silver-water nanofluid

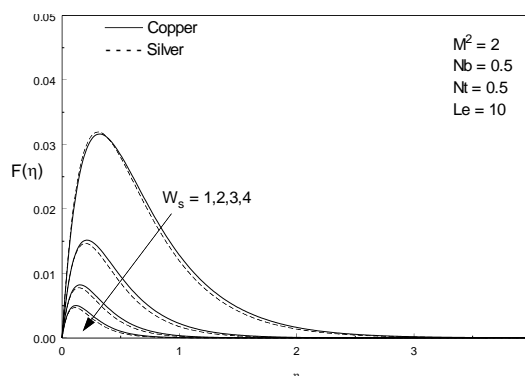


Fig. 7 Radial velocity profiles for different values of suction parameter for copper-water nanofluid and silver-water nanofluid

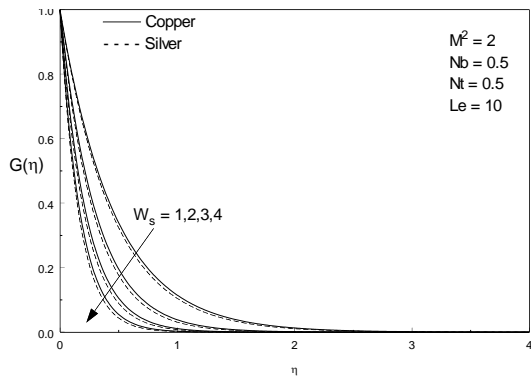


Fig. 8 Effect of suction parameter on tangential velocity profiles for copper-water nanofluid and silver-water nanofluid

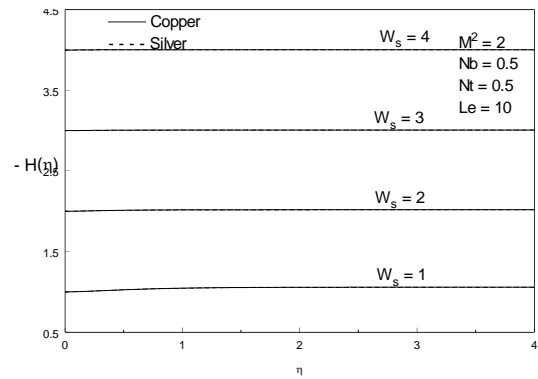


Fig. 9 Axial velocity profiles for different values of suction parameter for copper-water nanofluid and silver-water nanofluid

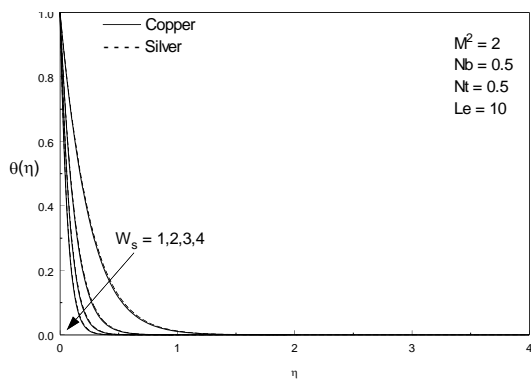


Fig. 10 Influence of suction parameter on temperature distribution for copper-water nanofluid and silver-water nanofluid

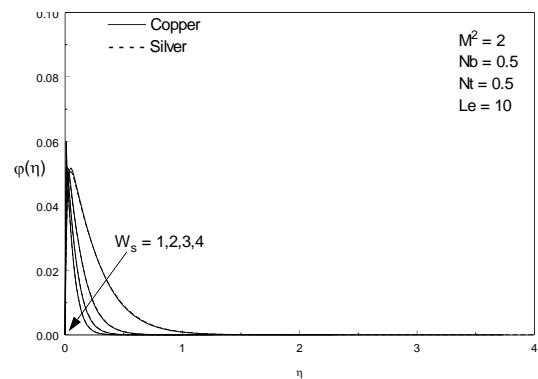


Fig. 11 Nanoparticle volume fraction distribution for different values of W_s for copper-water nanofluid and silver-water nanofluid

For strong suction, the radial velocity is small. The fact that suction stabilizes the boundary layer is also apparent from Fig.7 for both types of nanofluids. As suction increases, radial velocity decelerates. The effect of suction on the tangential velocity for two types nanofluids are shown in Fig.8. For strong suction, the tangential velocity decay rapidly away from the surface. Growing values of suction leads to decelerate the tangential velocity. As the wall suction increase, escape through the wall becomes easier and easier. Therefore, as suction parameter increases, axial velocity enlarges in magnitude. Further the value of axial velocity remains uniform with respect to η when W_s takes values 1, 2, 3 and 4. All these are noted in Fig.9 for both copper – water nanofluid and silver – water nanofluid.

The effect of suction parameter on temperature distribution is depicted in Fig.10. Imposition of wall suction leads to the reduction in the temperature and as well as in the thermal boundary layer thickness. The graph of the nanoparticle volume fraction distribution for variation of suction parameter W_s for both types of nanofluid are represented through Fig.11. As suction parameter increases, the nanoparticle volume fraction distribution increases gradually from the disk surface to its highest value in the vicinity of the disk and then decreases gradually to the zero value of quiescent fluid. The explanation for such behaviour is that the fluid is brought closer to the surface and reduce the thermal and nanoparticle volume fraction boundary layer thickness due to the Suction parameter.

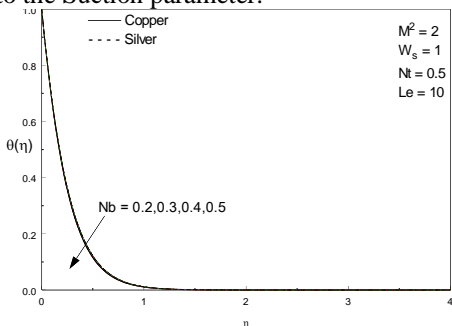


Fig. 12 Effect of Brownian motion parameter on temperature distribution for copper-water nanofluid and silver-water nanofluid

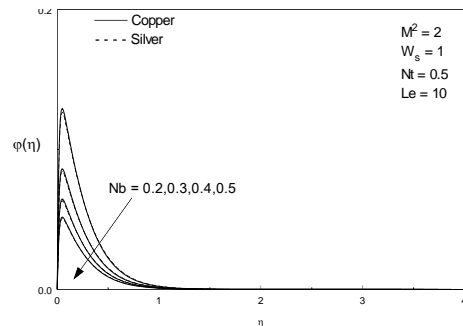


Fig.13 Nanoparticle volume fraction distribution for different values of Nb forcopper-water nanofluid and silver-water nanofluid

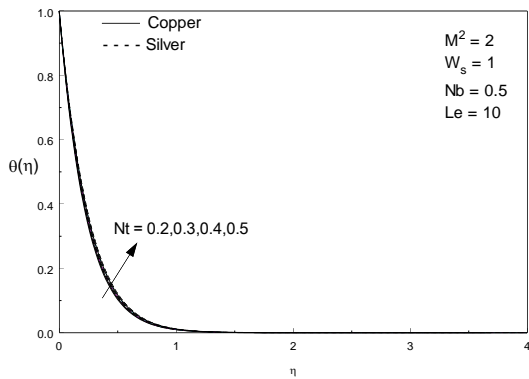


Fig. 14 Temperature distribution for various values of Thermophoresis parameter for copper-water nanofluid and silver-water nanofluid

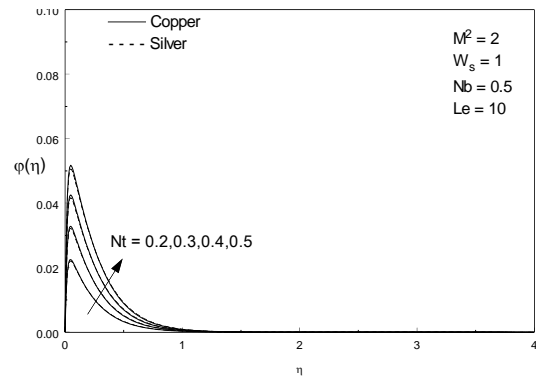


Fig.15 Effect of Nt on Nanoparticle volume fraction distribution for copper-water nanofluid and silver-water nanofluid

Figs. 12 and 13 portray the influence of the change of Brownian motion parameter (Nb) on temperature and nanoparticle volume fraction when $M^2 = 1$ and $W_s = 1$. It is found that the nanoparticle volume fraction distribution gets delayed due to Brownian motion parameter for both copper - water and silver - water nanofluids respectively. It seems that the Brownian motion acts to warm the fluid in the boundary layer and at the same time exacerbates particle deposition away from the fluid regime to the surface which results in the reduction of the thermal boundary layer thickness in both types of nanofluids. However, from Fig.12 it is revealed that the temperature does not change much when the values of Nb increases.

Figs. 14 - 15 present the typical distributions for temperature (θ) and nanoparticle volume fraction (ϕ) for various values of Thermophoresis parameter (Nt) for both types of nanofluids. It is observed that an upraise in the Thermophoresis parameter leads to elevate both the fluid temperature and Nanoparticle volume fraction distributions. Increase in Nt causes an increment in the thermophoretic force which tends to move nanoparticles from hot to cold areas and consequently it enhances the temperature and Nanoparticle volume fraction distributions.

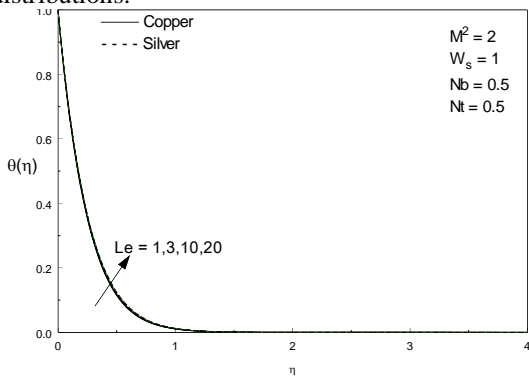


Fig.16 Effect of Lewis number on temperature distribution for copper-water nanofluid and silver-water nanofluid

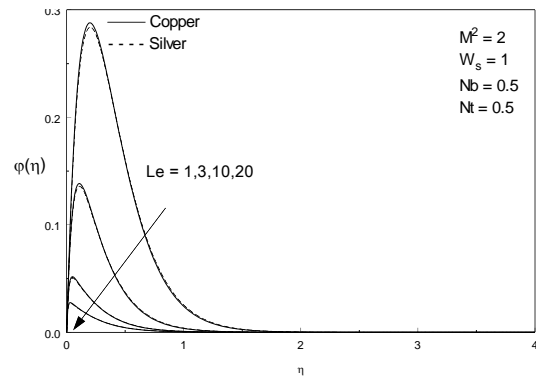


Fig.17 Nanoparticle volume fraction distribution for different values of Le for copper-water nanofluid and silver-water nanofluid

TABLE III

VARIATION OF $\frac{1}{(1-\phi)^{2.5}} F'(0)$, $\frac{-1}{(1-\phi)^{2.5}} G'(0)$ AND $-\frac{k}{k_f} \theta'(0)$ FOR DIFFERENT VALUES OF M^2, W_s, Nb, Nt AND Le FOR COPPER-WATER NANOFLUID

M^2	W_s	Nb	Nt	Le	$\frac{1}{(1-\phi)^{2.5}} F'(0)$	$\frac{-1}{(1-\phi)^{2.5}} G'(0)$	$-\frac{k}{k_f} \theta'(0)$
0	1	0.5	0.5	10	0.54816	2.00137	5.22927
1					0.42293	2.43269	5.20183
2					0.35449	2.79142	5.18723
3					0.31108	3.09958	5.17831
2	1	0.5	0.5	10	0.35449	2.79142	5.18723
	2				0.25685	4.22858	10.29073
	3				0.19251	5.84177	15.40970
	4				0.15159	7.53316	20.39080

2	1	0.2	0.5	10	0.35449	2.79142	5.18718
		0.3			0.35449	2.79142	5.18720
		0.4			0.35449	2.79142	5.18722
		0.5			0.35449	2.79142	5.18723
2	1	0.5	0.2	10	0.35449	2.79142	5.73602
			0.3		0.35449	2.79142	5.54471
			0.4		0.35449	2.79142	5.36191
			0.5		0.35449	2.79142	5.18723
2	1	0.5	0.5	1	0.35449	2.79142	5.52648
				3	0.35449	2.79142	5.30696
				10	0.35449	2.79142	5.18723
				20	0.35449	2.79142	5.15666

The variation in the dimensionless temperature with η is shown in Fig. 16 for copper - water nanofluid and silver - water nanofluid for some particular values of Lewis number (1, 3, 10 and 20). It is observed from the figure that the temperature enhances and it noticeable only in a region closer to the disk surface with a slight increase in the temperature. As it is noticed from Fig.17, as Lewis number amplifies, the nanoparticle volume fraction distribution falls down and the nanoparticle volume fraction boundary layer thickness shortens. This is probably due to the fact that molecular diffusivity decreases as Lewis number increases. Moreover, the nanoparticle volume fraction at the surface of a disk inhibits as the values of Lewis number increase.

Table III shows the influence of the magnetic interaction parameter (M^2), suction parameter (W_s), Brownian motion parameter (Nb), Thermophoresis parameter (Nt) and Lewis Number (Le) on radial skin friction coefficient, tangential skin friction coefficient, and non dimensional rate of heat transfer respectively for copper - water nanofluid. In this Table, tangential skin friction coefficient in magnitude enhances as the magnetic interaction parameter and suction parameter increases and opposite trend is observed for radial skin friction coefficient. Also, it is inferred that the dimensionless rate of heat transfer gets remitted with the growing effect in the magnetic interaction parameter, whereas it gets amplified with rise in the value of W_s . It is clear that the non-dimensional heat transfer rate demotes with the influence in Thermophoresis parameter (Nt), and also defect with a rise in Lewis Number (Le).

The behaviour of the quantities of chief physical interest such as the radial and tangential skin friction coefficients and non dimensional rate of heat transfer rate for silver - water nanofluid are illustrated in Table IV. From the table, it is clear that the radial skin friction coefficient remits due to the influence of magnetic interaction parameter, suction parameter. But the tangential skin friction coefficient intensifies the effect of M^2 and W_s . As M^2 increases, the non dimensional heat transfer rate gets abates and propagated for the increasing effect of suction parameter. The impact of Nt and Le over the dimensionless rate of heat transfer have the tendency to suppress it.

TABLE IV

VARIATION OF $\frac{1}{(1-\phi)^{2.5}} F'(0)$, $\frac{-1}{(1-\phi)^{2.5}} G'(0)$ AND $-\frac{k}{k_f} \theta'(0)$ FOR DIFFERENT VALUES OF M^2, W_s, Nb, Nt AND Le FOR SILVER-WATER NANOFLUID

M^2	W_s	Nb	Nt	Le	$\frac{1}{(1-\phi)^{2.5}} F'(0)$	$\frac{-1}{(1-\phi)^{2.5}} G'(0)$	$-\frac{k}{k_f} \theta'(0)$
0	1	0.5	0.5	10	0.55853	2.15056	5.01429
					0.44075	2.56497	5.07897
					0.37363	2.91338	5.06495
					0.32994	3.21506	5.05616
2	1	0.5	0.5	10	0.37363	2.91338	5.06495
					0.26437	4.50495	10.04413
					0.19568	6.28168	15.04089
					0.15314	8.13594	19.91014
2	1	0.2	0.5	10	0.37363	2.91338	5.06491
		0.3			0.37363	2.91338	5.06492
		0.4			0.37363	2.91338	5.06494
		0.5			0.37363	2.91338	5.06495
2	1	0.5	0.2	10	0.37363	2.91338	5.60161
			0.3		0.37363	2.91338	5.41451
			0.4		0.37363	2.91338	5.23576
			0.5		0.37363	2.91338	5.06495

2	1	0.5	0.5	1	0.37363	2.91338	5.39204
				3	0.37363	2.91338	5.17984
				10	0.37363	2.91338	5.06495
				20	0.37363	2.91338	5.03572

V. CONCLUSION

In this work, the effects of Brownian motion and thermophoresis effects on nonlinear, hydromagnetic flow of nanofluid over a rotating disk with suction has been investigated. The effects of magnetic interaction parameter, suction parameter and nanofluid parameters including Brownian motion number, Thermophoresis number, Lewis number on the velocity components, temperature and Nanoparticle volume fraction are analyzed and discussed using figures and tables. The results can be summarized as follows:

- When $M^2 = 0$, $W_s = 0$ and $Pr=6.2$, the numerical values are identical to that of Turkyilmazoglu (2014) which confirms the validity of our numerical simulation.
- The effect of the magnetic interaction parameter and suction parameter is to decelerate the radial velocity, tangential velocity and radial skin friction coefficient. Also, Tangential skin friction coefficient in magnitude is enlarged due to the lifting effects of magnetic interaction parameter and suction parameter.
- Axial velocity in magnitude and dimensionless rate of heat transfer are diminished while the temperature and nanoparticle volume fraction distributions are amplified due to the influence of magnetic field. The opposite trend is noticed in the case of the suction parameter.
- An increase in Brownian motion parameter leads to reduction in the temperature. A rise in the Thermophoresis parameter and Lewis Number upholds the temperature. But the opposite trend is occurred in non-dimensional rate of heat transfer for both types of nanofluid.
- It is observed that the effect of Thermophoresis parameter is to promote the nanoparticle volume fraction distribution whereas it reduces the Brownian motion parameter and Lewis number.

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