The Boundary Layer Flow Over a Linearly Stretching Sheet Under the Influence of the Magnetic Field and Porous Medium

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Abstract:

The present paper studies on the laminar two-dimensional boundary layer flow and heat transfer of an incompressible viscous fluid with thermal radiation in existence of Porous medium and Magnetic field over an linearly stretching sheet is investigated numerically. The governing boundary layer equations are reduced into ordinary differential equations by a similarity transformation. The malformed equations are solved numerically using an embedded finite difference scheme known as the Keller-box method. The numerical solutions for the wall skin friction coefficient, the heat transfer coefficient, and the velocity and temperature profiles are evaluated, analyzed and discussed.

Key words: *Boundary Layer Flow, Linearly Stretching Sheet, Numerical Solution, Thermal Radiation, Porous Medium, Magnetic field*

1. INTRODUCTION

The study of The Numerical solution of the Boundary Layer flow over an Linearly stretching sheet under the influence of the Magnetic field and Porous Medium of considerable interest because of its ever-increasing industrial applications and important bearings on several technological processes. The production of sheeting material arises in many industrial manufacturing processes and includes both metal and polymer sheets. In last few decades, the study of flow and heat transfer in porous media has received much reflection due to its ever-increasing applications in industries and in contemporary technology. As in other porous media problems such as geo-mechanics and insulation engineering, the conventional method is to simulate the pressure drop across the porous regime using Darcy linear model S. Suneetha, N.B. Reddy [1]. It is well known that porous materials can be used to enhance the heat transfer rate from stretching surfaces A. Tamayol, M. Bahrami [2]. Elbashbeshy and Bazid [3,4] analyzed respectively the effects of variable-viscosity and internal heat generation/absorption on flow and heat transfer through a porous medium over a stretching surface. An excellent literature review on flow through porous media can be found from Starov and Zhdanov [5], Kaviany [6], Kiwan and Ali [7] and Tamayol et al. [8].

It is worth stating that the studies of thermal radiation and heat transfer are important inelectrical power generation, astrophysical flows, solar power technology and other industrial areas. Alot of extensive literature that deals with flows in the presence of radiation effects is now available. [9] Elbashbeshy and Dimian (2002) analyzed boundary layer flow in the presence of radiation effect andheat transfer over the wedge with viscous coefficient. Besides that, [10] Cortell (2008) has solved a problemon the effect of radiation on Blasius flow by using fourth-order Runge-Kutta approach. Later, [11] Sajid andHayat (2008) considered the influence of thermal radiation on the boundary layer flow due to an exponentially stretching sheet by solving the problem analytically via homotopy analysis method(HAM). Recently, [12] El-Aziz (2009) and [13] Ishak (2009) also focused on the effects of thermal radiation in their studies

The purpose of this present work is to extend the Numerical Solution of the Boundary Layer Flow Over an Exponentially Stretching Sheet with Thermal Radiation published by Biliana Bidin[14]. A governing continuity, momentum, energy together with associated boundary conditions are first reduced to a set of self-similar non-linear coupled ordinary differential equations by suitable transformations. These equations are solved numerically by using the Keller Box method. Estimation of heat transfer coefficient which is very important from the industrial application point of view is also presented in this analysis. It is hoped that the results obtained will not only provide useful information for applications, but also serve as a complement to the previous studies.

2. MATHEMATICAL FORMULATION

In this paper, the attributes of viscous flow with respect to Nano-fluid on anextracting sheet is considered. Generally, the flow is possible when the condition $y \geq 0$. Here, the term y is the normal coordinate with respect to the expanding sheet. Practically the steady uniform stretching leads to equal and opposite forces along $x - axis$; Hence the sheets will be expanded by fixing the origin.

The following assumptions are made before getting into more mathematical calculations. Assume the temperature at extending surface as a function of x, ambient temperature T as constant and at the sheet nano particle fraction C as a constant with a value C_W. Now assume that the sheet is expanded by stretching with a nonlinearity parameter n along with a velocity of $u_x(x) = a\,x^n$. Here x is the extending surface coordinate where the measurement is in existence.

Now consider a nanofluid is flowing at $y = 0$ and consider fluid to be under the influence of electrical conduction because of the magnetic field $B(x)$, which is normal to stretching sheet. Make a note that at stretching surface the T_W (wall temperature) and C_W are considered to be constant. The ambient value of temperature (T_∞) and nanoparticle fraction (C_∞) are denoted when y tends to infinity. The physical system considered for this study included for nano-technical fabrication and thermal material processing. In the Fig. 1, coordinate system and flow models are shown and governing equations are given below:

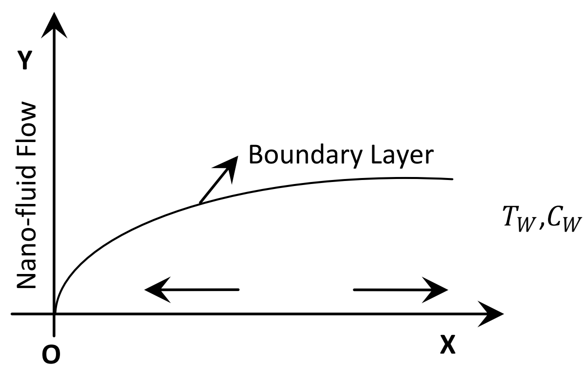

Fig 1: The Physical Model and consequent Coordinate Systems

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (1)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \upsilon\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2(x)}{\rho_f}u - \frac{\nu}{k}u \qquad (2)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2 - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y} \qquad (3)$$

The boundary conditions for the velocity, temperature are given below:

$$y = 0 : \; u_w = ax^n, \quad v=0, \; T= T_w, \qquad (4)$$

$$y = \infty : u = 0, \quad v = 0, \; T= T_\infty, \qquad (5)$$

Here, u and v are the velocity components along the x and y axes, respectively. $\alpha = k/(\rho c)_f$ is the thermal diffusivity, σ is electrical conductivity, ν is the kinematic viscosity, ρ_f is the density Of the base fluid. $\tau = (\rho C)_p/(\rho C)_f$ is the ratio between the effective heat capacity of the nanoparticle material and heat capacity of the fluid, c is the volumetric volume coefficient, ρ_p is the density of the particles, and C is rescaled nanoparticle volume fraction. We assume that the variable magnetic field B(x) is of the form B(x) = $B_0 x^{(n-1)/2}$

Using Rosseland approximation for radiation, We can write

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \tag{6}$$

where k^* is the absorption coefficient, σ^* is the Stefan-Boltzman constant, Assuming the temperature difference within the flow is such that T^4 may be expanded in a Taylor series about T_∞ and neglecting higher orders we get $T^4 \approx 4T_\infty^3 T - 3T_\infty^4$.

Hence Eq. (7), becomes

$$\frac{\partial q_r}{\partial y} = -\frac{16\sigma^* T_\infty^3}{3(\rho_{cp})_f k^*}\frac{\partial^2 T}{\partial y^2} \tag{7}$$

The dimensionless variable can be taken as

$$\eta = y\sqrt{\frac{a(n+1)}{2v}}\,x^{\frac{(n-1)}{2}},\; u = ax^n f'(\eta),$$
$$v = -\sqrt{\frac{a(n+1)}{2}}\,x^{\frac{(n-1)}{2}}\left(f(\eta) + \frac{n-1}{n+1}\eta f'(\eta) \right),$$
$$\theta(\eta) = (T - T_\infty)\big/(T_W - T_\infty) \tag{8}$$

Where, ψ represents stream functions and is defined as $u = \frac{\partial \psi}{\partial y}$, $v = \frac{-\partial \psi}{\partial y}$

so that equation (1) is satisfied identical.

Substituting Eq (8) into Eqs (1) – (3) , We obtain the Ordinary Differential equations as follows:

$$f''' + ff'' - 2f'^2 - (M+G)f' = 0 \tag{9}$$

$$(1 + \frac{4}{3}R)\theta'' + \Pr\left(f\theta' - f'\theta + Ecf''^2\right) = 0 \tag{10}$$

The malformed boundary conditions

$$f(0) = 0,\; f'(0) = 1,\; \theta(0) = 1,$$
$$f'(\infty) = 0,\; \theta(\infty) = 0, \tag{11}$$

Where primes denote differentiation with respect to η , the involved physical parameters are defined as:

$$\Pr = \frac{v}{a} \qquad , M = \frac{2\sigma B_0^2}{a\rho f\,(n+1)},$$

$$Ec = \frac{u_w^2}{c_\rho (T_w - T_\infty)}, \quad G = \frac{v}{k},$$

$$R = \frac{4\sigma^* T_\infty^3}{k k^*} \tag{12}$$

Here Pr, M, Ec, G and R denote the Prandtl number, Magnetic parameter, Eckert number, Porous Medium and thermal radiation respectively. This boundary value problem is condensed to the classical problem of flow and heat transfer due to a stretching surface in a viscous fluid when n = 1 in eqs (10).

The quantities of practical interest, in this study, are the local skin friction C_{fx}, Nusselt number Nu_x which are defined as

$$C_{fx} = \frac{\mu_f}{\rho u_w^2}\left(\frac{\partial u}{\partial y}\right)_{y=0}, Nu_x = \frac{x q_w}{k(T_w - T_\infty)} \tag{13}$$

Where k is the thermal conductivity of the nanofluid and q_w is the heat fluxes at the surface, respectively, given by

$$q_w = -\left[\frac{\partial T}{\partial y}\right]_{y=0}, \tag{14}$$

Substituting Eq (6) into Eqs (14) – (15), we obtain

$$\mathrm{Re}_x^{1/2} C_{fx} = \sqrt{\frac{n+1}{2}} f''(0), \ \mathrm{Re}_x^{-1/2} Nu_x = -\sqrt{\frac{n+1}{2}}\theta'(0), \tag{15}$$

Where $\mathrm{Re}_x = u_w \dfrac{x}{v}$ is the local Reynolds number

3. RESULTS AND DISCUSSION

The reduced Eqs. (9) – (11) are nonlinear and joined, and thus their exact analytical solutions are not possible. They can be solved numerically using Keller Box for different values of parameters such as magnetic parameter, Prandtl number, Eckert number, Porous Medium and Thermal radiation. The effects of the budding parameters on the dimensionless velocity, temperature, skin friction, and the rates of heat transfer are investigated.

The important steps in using the Keller Box method are:

1) Reducing higher order ODEs (systems of ODEs) in to system of first order ODEs;
2) Writing the systems of first order ODEs into difference equations using central differencing scheme;
3) Liberalizing the difference equations using Newton's method and writing it in vector form;
4) Solving the system of equations using block eliminations method.

In order to solve the above differential equations numerically, we adopt Mat lab software which is very efficient in using the well-known Keller Box method.

To authenticate the present solution, comparisons have been made with previously published data in the fiction for $-\theta'(0)$ in Table 1, and they are found to be outstanding.

Table 1: Estimation of Nusselt number for various Values of Ec when Pr , R and Ec.

Pr	R	Ec	Bidin	Present
1	0	0.0	0.9547	0.9559
		0.2	0.8622	0.8634
		0.9	0.5385	0.5395
2	1	0.0	0.8627	0.8122
		0.2	0.7818	0.7171
		0.9	0.4984	0.3843
3	1	0.0	1.1214	1.0191
		0.2	1.0006	0.9929
		0.9	0.6055	0.8974

Various comparisons are presented in Table 1 to authenticate the proposed method with respect to previous method for the equations $-\theta^{'}(0)$. These comparisons resulted to be outstanding and are found to be excellent. At the same time various effects due to magnetic and viscous parameters are shown in the Table 1.

Table 2: Resulting table:

Showing results of Nusselt number $-\theta'(0)$ for the values of M, Pr and G when n= 1,Ec=0 and R=0

Pr	G	M	$-\theta'(0)$
1	0	0.0	0.9559
		0.5	0.9079
		1.0	0.8671
2	0	0.0	1.0205
		0.5	1.0204
		1.0	1.0231
3	0	0.0	1.0207
		0.5	1.0207
		1.0	0.0236

The magnetic field number values. The increase in the tangential velocity as the magnetic parameter M decreases is because the existence of a magnetic field in an electrically conducting fluid introduces a force called the Lorentz force, which acts against the flow if the magnetic field is applied in the normal direction, as in the present study. This resistive force slows down the fluid velocity component as shown in Fig. 2.

For different values of the magnetic parameter M the temperature profiles are plotted in Fig. 3. It is obvious that an increase in the Magnetic parameter M results in an increase in the temperature within the boundary layer.

Figure 4, show effect of porosity parameter G on the velocity profiles. It is experiential that the presence of the porous medium reduces the velocity profile. This is because the porous medium inhibits the fluid not to move generously through the boundary layer.

Figure 5, show effect of porosity parameter G on the temperature profiles, It is observed that the presence of the porous medium. Where as it increases the temperature profile. This is because the porous medium inhibits the fluid not to move freely through the boundary layer. This leads the flow to increase thermal boundary layer thickness.

Fig. 6 depicts for the different values of the radiation parameter R the temperature profiles . It is evident that an increase in the radiation parameter R results in an decrease in the temperature within the boundary layer.

Figures 7 show the behaviour of temperature for different values Prandtl number. The numerical result shows that the effect of rising values of Prandtl number results in a falling velocity. It is observed that an increase in the Prandtl number results a decrease of the thermal boundary layer thickness and in general lower average temperature within the boundary layer. The reason is that smaller values of Pr are correspondent to increase in the thermal conductivity of the fluid and therefore, heat is able to disperse away from the heated surface more quickly for higher values of Pr . Hence in the case of smaller Prandtl number as the thermal boundary later is thicker and the rate of heat transfer is reduced.

For different values of the Eckert number Ec the temperature profiles are plotted in Fig. 8 It is obvious that an intensify in the Eckert number Ec results in an increase in the temperature within the boundary layer.

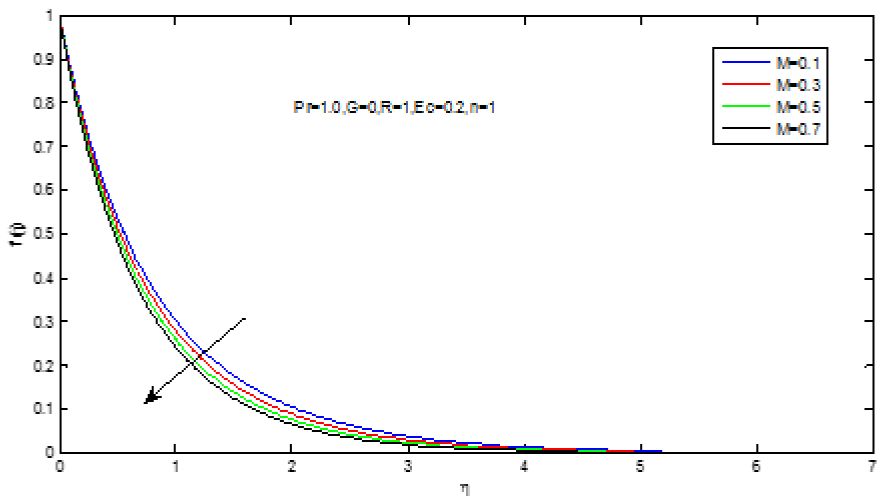

Fig. 2. Velocity profile for M

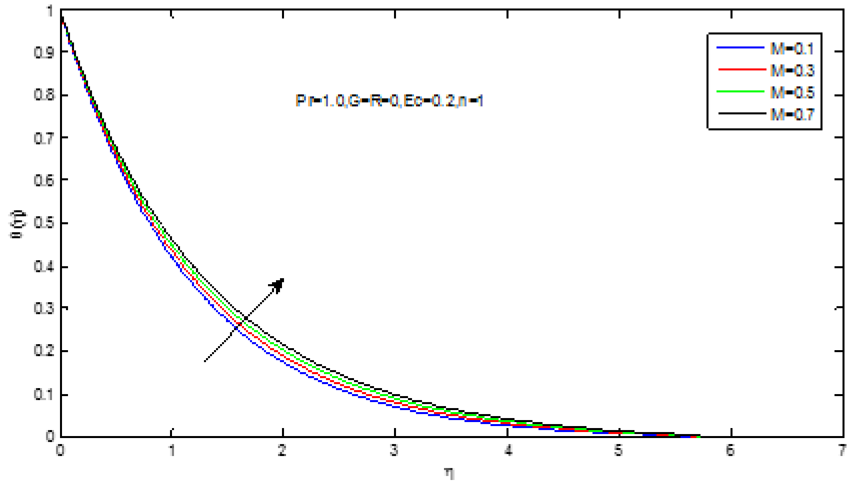

Fig. 3. Temperature profile for M

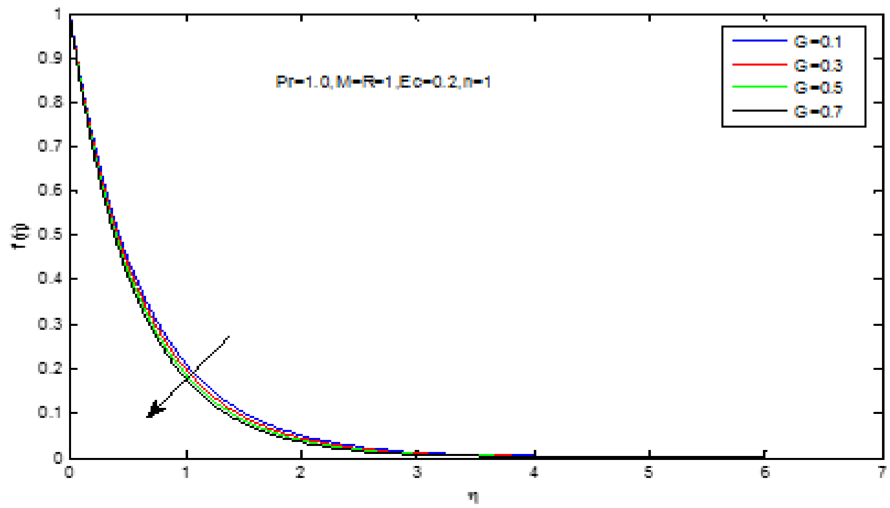

Fig. 4. Velocity profile for G

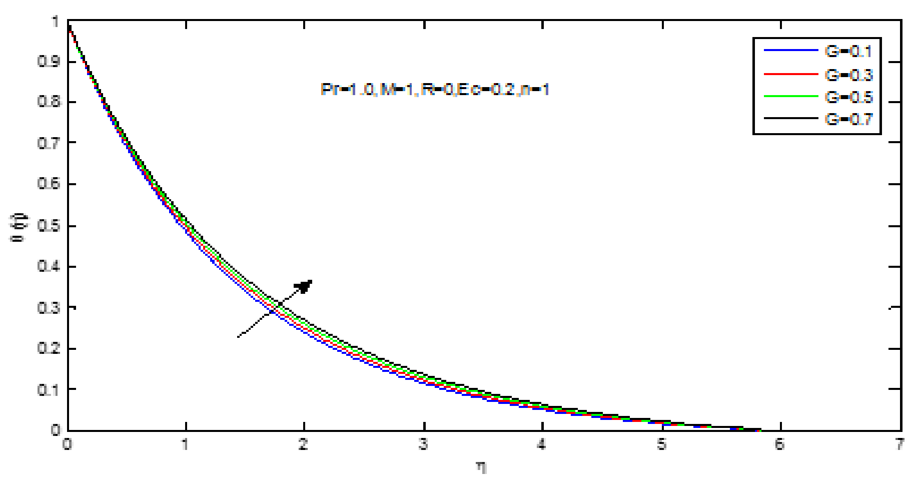

Fig. 5. Temperature profile for G

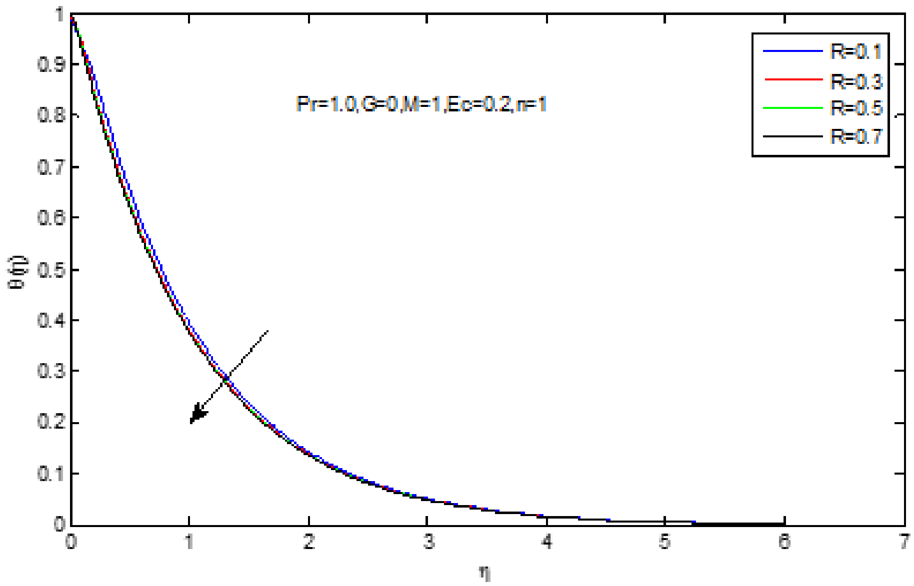

Fig. 6. Temperature profile for R

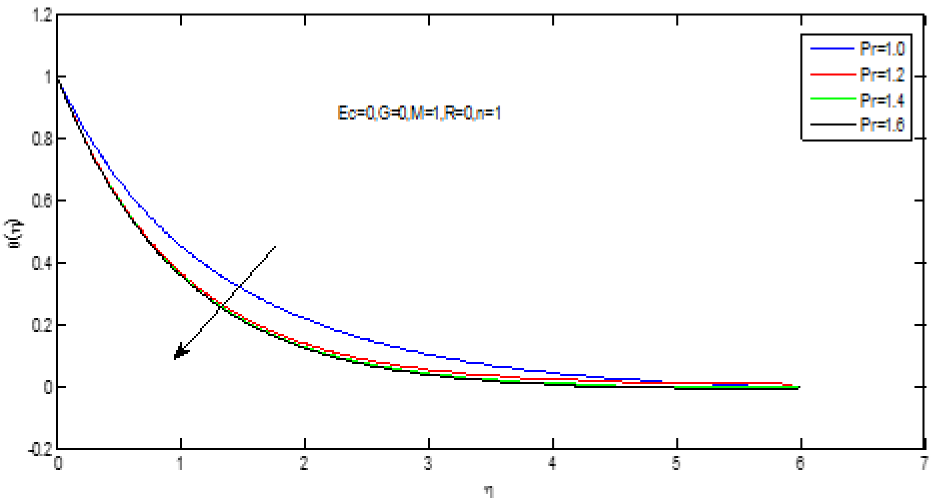

Fig. 7. Temperature profile for Pr

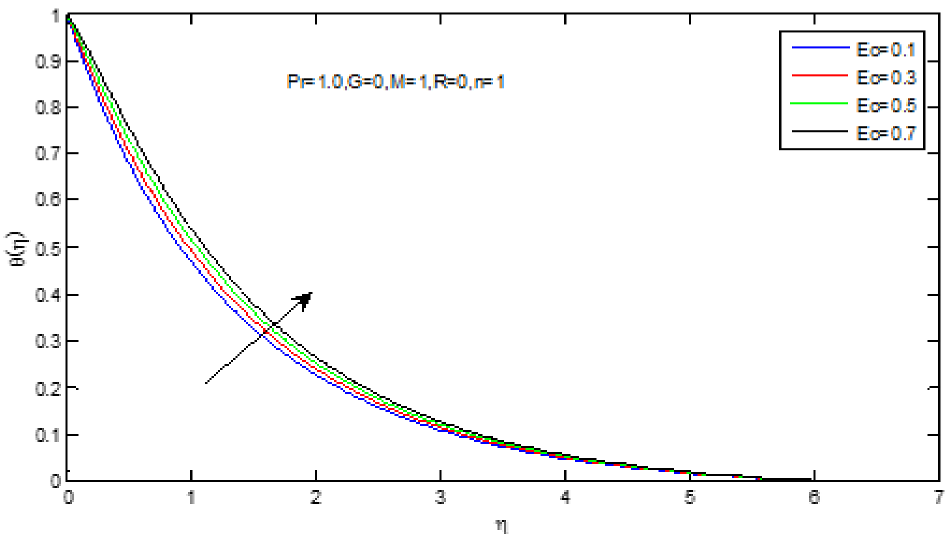

Fig. 8 Temperature profile for Ec

4. CONCLUSION:

The fundamental governing equations are rehabilitated to paired nonlinear usual differential equations. Keller Box method is used to perform the numerical calculations. The effects of linear extending parameter, Prandtl number, radiation parameter, porous medium and Magnetic parameters on the heat transfer features are analyzed. Finally, got an excellent agreement with the previous paper. Briefly the above negotiations can be concise as follows.

1. The Local Nusselt number decreased with the increase in Magnetic parameter and Prandtl number It is also noticed that the linear stretching parameter is to stifle the velocity field.
2. The velocity of the fluid is originate to be decreased with the increase in M where as the temperature is increased in this case.
3. The velocity of the fluid is found to be decreased with the increase in Porous term G where as the temperature is increased in this case.
4. The rising effect of the Radiation parameter decreases the temperature.
5. The growing effect of the Prandtl number decreases the temperature.
6. The mounting effect of the Eckert number increases the temperature.

5. ACKNOWLEDGEMENT:

Dedicated to my dear Kalyani Sharma Kolipaka. S KKKK SSS forever.

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