Prime Labeling to Brush Graphs

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Abstract-*A graph $G = (V, E)$ with n vertices is said to admit prime labeling if its vertices can be labeled with distinct positive integers not exceed n such that the label of each pair of adjacent vertices are relatively prime. A graph G which admits prime labeling is called a prime graph. In this paper, we investigate prime labeling for some graphs related to Brush graph. We discuss prime labeling in the context of some graph operations namely duplication, fusion, switching in Brush graph (B_n).*

Keywords-*Prime labeling, Prime graph, Brush graph, Duplication, Fusion, Switching, coloring.*

[I] INTRODUCTION

we consider only simple, finite, undirected and non – trivial graph $G = (V(G), E(G))$ with the vertex set $V(G)$ and the edge set $E(G)$. For notations and terminology we refer to Bondy and Murthy[1]. Many researchers have studied prime graph for example in Fu. H[8] have proved that the path P_n on *n* vertices is a prime graph. In [10] S. Meena and K. Vaithilingam have proved the prime labeling for some Fan related graphs. For latest survey on graph labeling we refer to [9] (Gallian. J. A., 2017). Vast amount of literature is available on different types of graph labeling and more than 1000 research papers have been published so far in past four decades. In Edward Samuel A and Kalaivani S [3] have proved the Prime labeling for some octopus related graphs. In Edward Samuel A and Kalaivani S [4] have proved the Prime labeling for some planter related graphs. In Edward Samuel A and Kalaivani S [5] have proved the Prime labeling for some vanessa related graphs. In Edward Samuel A and Kalaivani S [6] have proved the Square sum labeling for some lilly related graphs. In Edward Samuel A and Kalaivani S [7] have proved the Prime labeling to drums graphs.

[II] PRELIMINARY DEFINITIONS

Definition [10] **:** Let $G = (V(G), E(G))$ be a graph with *p* vertices. A bijection $f : V(G) \to \{1, 2, \dots, p\}$ is called a *prime labeling* if for each edge $e = uv$, $\gcd\{f(u), f(v)\} = 1$. A graph which admits prime labeling is called a *prime graph*.

Definition [10] **:** *Duplication* of a vertex v_k of a graph G produces a new graph G_1 by adding a vertex $v_k{}'$ with $N(v_k{}') = N(v_k)$. In other words a vertex $v_k{}'$ is said to be a duplication of v_k if all the vertices which are adjacent to v_k are now adjacent to $v_k{}'$ also.

Definition [10] **:** Let *u* and *v* be two distinct vertices of a graph G. A new graph G_1 is constructed by *identifying(fusing)* two vertices *u* and *v* by a single vertex *x* is such that every edge which was incident with either *u* or *v* in G is now incident with *x* in G_1.

Definition [10] **:** A *vertex switching* G_v of a graph G is obtained by taking a vertex *v* of G, removing all the entire edges incident with *v* and adding edges joining *v* to every vertex which are not adjacent to *v* in G.

Definition [2] **:** A *k* – coloring of a graph $G = (V, E)$ is a function $c : V \to C$, where $|c| = k$. (Most often we use $c = [k]$). Vertices of the same color form a color class. A coloring is *proper* if adjacent vertices have different colors. A graph is *k – colorable* if there is a proper *k* – coloring. The chromatic number $\chi(G)$ of a graph G is the minimum *k* such that G is *k* – colorable.

[III] PRIME LABELING TO BRUSH GRAPHS

3.1. Brush graph : The *Brush graph* $B_n, (n \geq 2)$ can be constructed by path graph $P_n, (n \geq 2)$ by joining the star graph $K_{1,1}$ at each vertex of the path. i.e., $B_n = P_n + nK_{1,1}$.

Example 3.2.

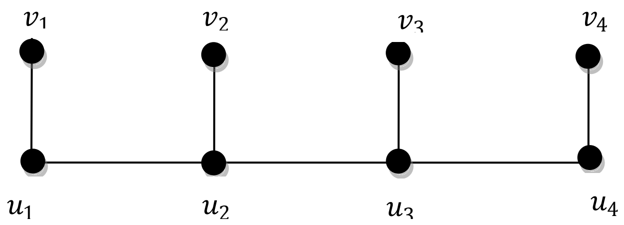

Figure. 1 Brush graph B_4.

Theorem 3.3. The Brush graph B_n is a prime graph, where n is any positive integer.
Proof. Let $V(B_n) = \{u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_n\}$

$E(B_n) = \{u_i u_{i+1} / 1 \le i \le n-1\} \cup \{u_i v_i / 1 \le i \le n\}$. Here $|V(B_n)| = 2n$.

Define a labeling $f : V(B_n) \to \{1, 2, \dots, 2n\}$ as follows.

$f(u_i) = 2i - 1$ for $1 \le i \le n$

$f(v_i) = 2i$ for $1 \le i \le n$

Clearly vertex labels are distinct. Then f admits prime labeling. Thus B_n is a prime graph.

Example 3.4.

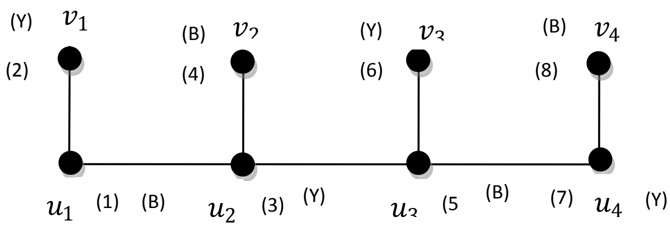

Figure. 2 Prime labeling for B_4.

Theorem 3.5. The graph obtained by duplication of any vertex in the brush graph B_n is a prime graph, where n is any positive integer.
Proof. Let $V(B_n) = \{u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_n\}$

$E(B_n) = \{u_i u_{i+1} / 1 \le i \le n-1\} \cup \{u_i v_i / 1 \le i \le n\}$. Here $|V(G_k)| = 2n+1$.

Define a labeling $f : V(G_k) \to \{1, 2, \dots, 2n+1\}$ as follows.

$f(u_i) = 2i - 1$ for $1 \le i \le n$

$f(v_i) = 2i$ for $1 \le i \le n$

$f(u_1') = 11$

Clearly vertex labels are distinct. Then f admits prime labeling. Thus B_n is a prime graph.

Example 3.6.

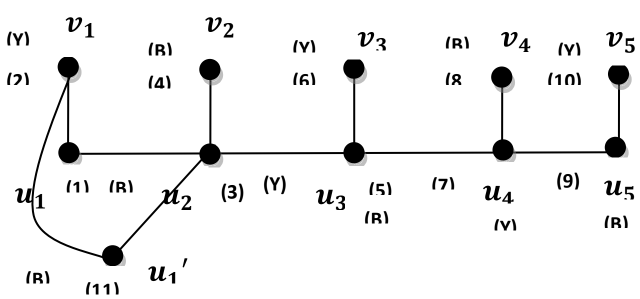

Figure. 3 Duplication of u_1 in B_5.

Theorem 3.7. The graph obtained by identifying(fusing) of any two vertices in a brush graph B_n is a prime graph, where n is any positive integer.
Proof. Let $V(B_n) = \{u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_n\}$

$E(B_n) = \{u_i u_{i+1} / 1 \le i \le n-1\} \cup \{u_i v_i / 1 \le i \le n\}$. Let G_k be the graph obtained by fusing any two vertices in B_n. Here $|V(G_k)| = 2n - 1$.

Define a labeling $f : V(G_k) \to \{1, 2, \ldots, 2n-1\}$ as follows.

$$f(u_i) = 2i - 1 \quad \text{for } 1 \le i \le n-2$$
$$f(v_i) = 2i \quad \text{for } 1 \le i \le n-1$$
$$f(u_5 = u_6) = 9$$
$$f(v_6) = 11$$

Clearly vertex labels are distinct. Then f admits prime labeling. Thus G_k is a prime graph.

Example 3.8.

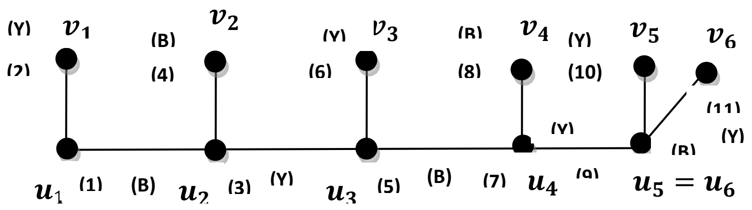

Figure. 4 Fusion of u_5 and u_6 in B_6.

Theorem 3.9. The graph obtained by switching of any vertex u_k in a brush graph B_n produces a prime graph, where n is any positive integer.

Proof. Let $V(B_n) = \{u_1, u_2, \ldots, u_n, v_1, v_2, \ldots, v_n\}$

$E(B_n) = \{u_i u_{i+1} / 1 \le i \le n-1\} \cup \{u_i v_i / 1 \le i \le n\}$. Let G_k be the graph obtained by switching of any arbitrary vertex u_k in B_n. Here $|V(G_k)| = 2n$.

Define a labeling $f : V(G_k) \to \{1, 2, \ldots, 2n\}$ as follows.

$$f(u_i) = 2i - 1 \quad \text{for } 1 \le i \le n$$
$$f(v_i) = 2i \quad \text{for } 1 \le i \le n$$

Clearly vertex labels are distinct. Then f admits prime labeling. Thus G_k is a prime graph and it is a disconnected graph.

Example 3.10.

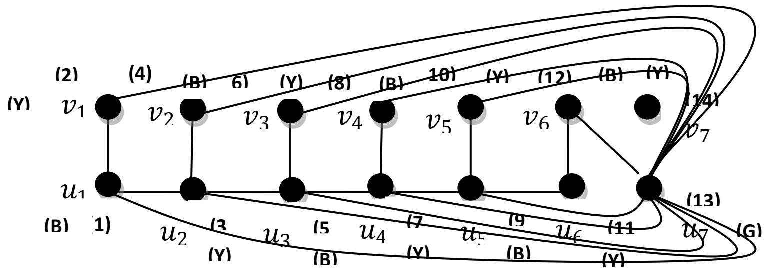

Figure. 5 Switching of u_7 in B_7.

Theorem 3.11. The graph obtained by switching of any vertex v_k in a brush graph B_n produces a prime graph, where n is any positive integer.

Proof. Let $V(B_n) = \{u_1, u_2, \ldots, u_n, v_1, v_2, \ldots, v_n\}$

$E(B_n) = \{u_i u_{i+1} / 1 \le i \le n-1\} \cup \{u_i v_i / 1 \le i \le n\}$. Let G_k be the graph obtained by switching of any arbitrary vertex v_k in B_n. Here $|V(G_k)| = 2n$.

Define a labeling $f : V(G_k) \to \{1, 2, \ldots, 2n\}$ as follows.

$$f(u_i) = 2i - 1 \quad \text{for } 1 \le i \le n-1$$
$$f(v_i) = 2i \quad \text{for } 1 \le i \le n-1$$
$$f(u_k) = 2n$$
$$f(v_k) = 2n - 1$$

Clearly vertex labels are distinct. Then f admits prime labeling. Thus G_k is a prime graph.

Example 3.12.

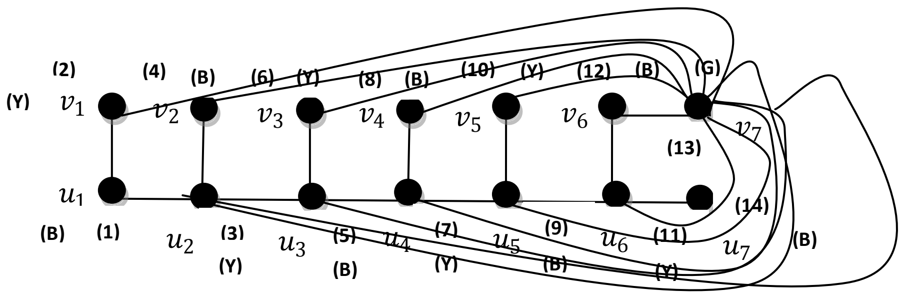

Figure. 6 Switching of v_7 in B_7.

[IV] CONCLUSION

In this paper we proved that the Brush graph B_n, duplication of the Brush graph B_n, fusing of the Brush graph B_n, switching of the Brush graph B_n are prime graphs. There may be many interesting prime graphs can be constructed in future.

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