P-waves interaction with crack at orthotropic interface

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Abstract: *The paper is concerned with the problem of diffraction of P-waves by a Griffith crack situated at the interface of two different orthotropic media has been analyzed. Using Fourier transform and integral equation method, the mixed boundary value problem has been reduced to the solution of dual integral equations which has finally been reduced to the solution of a Fredholm integral equation of the second kind which are finally solved by using perturbation method. Stress intensity factor(SIF) and crack opening displacement (COD) around the crack tip are derived. The values of SIF and crack opening displacement have been calculated for various type of orthotropic materials and depicted graphically.*

Key Words: *Griffith crack, Orthotropic media, P-wave, Stress intensity factor, Crack opening displacement.*

I. Introduction

In recent years the problems of diffraction of elastic waves by cracks are of considerable importance in view of their application in Engineering Mathematics. The primary objective in engineering structure is to avoid the growth of the crack once it initiated. It was found that the stress has a square root singularity at the tip of the crack. In this prospect a non-dimensional quantity called stress intensity factor is used to predict the stress state near the crack tip caused by an applied load. Many researchers did their work on wave propagation on bonded media containing an interfacial crack. Fracture mechanics and study of crack propagation can be considered as an interesting branch in elastic theory. The performance of engineered systems is affected by inhomogeneities such as cracks and inclusions present in the material. Theory and problems related to crack geometry in orthotropic materials have emerged as a very interesting area of research in recent times mostly due to the rapid growth in construction engineering. Anisotropy and flaw at the interface of two bonded materials are of great importance in designing engineering structures and machines. Composite materials are becoming an essential commodity in modern era as they offer advantages such as low weight to strength ratio, corrosion resistance, and high fatigue strength. The growing use of composite materials in many engineering applications demand the fundamental understanding of the response of cracked orthotropic bodies under stress. Many composite materials are used in making aircraft structures to golf clubs, electronic packaging to medical equipment, and space

vehicles to home building. It has been observed that applications of composite materials in the commercial market are also increasing day by day. The crack problem in fracture mechanics has a wide range of applications in civil engineering. It has a big application on manufacturing engineering for designing metal and polymer forming processes, machining, etc.

Srivastava *et al.* [22] solved the problem of interaction of shear waves with Griffith crack situated at the interface of two bonded dissimilar elastic half spaces. The problem becomes more difficult and complicated when boundaries are present in the media. Li [14] obtained the analytical solution for a static problem of two bonded orthotropic strips containing an interfacial crack. The dynamical problem was studied by Matbuly [18] and obtained the analytical expression of stress intensity factor. Diffraction problems involving multiple cracks had been studied by many authors. But most of the problems were either involving diffraction of shear waves or diffraction in infinite media. E. Lira-Vergara and C. Rubio-Gonzalez [12-13] obtained the dynamic stress intensity factor of interfacial finite cracks in orthotropic materials. Itou [7] considered the diffraction problem of an antiplane shear wave by two coplanar Griffith cracks in an infinite elastic medium. Stress distribution near periodic cracks at the interface of two bonded dissimilar orthotropic half planes was studied by Garg [6]. The problem of diffraction of elastic waves by two coplanar Griffith cracks in an infinite elastic medium was solved by Jain and Kanwal [10]. The transient response of two cracks at the interface of a layered half space had been investigated by Kundu [11]. Mandal and Ghosh [15] solved the problem of interaction of elastic waves with a periodic array of coplanar Griffith cracks in an orthotropic medium. Diffraction problem of three coplanar Griffith cracks in an orthotropic medium was considered by Sarkar *et al.* [20]. Das [3] and others considered the problems containing a Griffith crack in an transversly orthotropic medium. Mukherjee *et al.* [17] have studied the interaction of three interfacial Griffith cracks between bonded dissimilar orthotropic half planes and find out the stress intensity factor. Satapathy *et al.* [19] considered an orthotropic strip containing a Griffith crack which is finally solved. Das *et al.* [4]are finding the Stress intensity factor of an edge crack in bonded orthotropic materials. Dynamic stress intensity factors of multiple cracks in an orthotropic strip with FGM coating was studied by Monfared and Ayatollahi [16]. Sinharoy [21] solved elastostatic problem of an infinite row of parallel cracks in an orthotropic medium under general loading. Itou [9] solved the problem of dynamic stress intensity factors of three collinear cracks in an orthotropic plate subjected to time-harmonic disturbance

In most of the above discussed papers, the problem involving the P-waves on a finite crack with two semi-infinite elastic half-spaces. Therefore we state the problem as diffraction of P-waves in two bonded dissimilar containing a Griffith crack at the interface. The Fourier transform is used to reduce the problem to a system of dual integral equations . Then the set of dual integral equation is further reduced to a Fredholm integral equation of second kind by using Abel's transform technique. The solution of this integral equation has been obtained for low frequency by using perturbation method. An iterative procedure is adopted to obtain the low frequency solution of the problem. This procedure leads to obtain the analytical expressions of the stress intensity factor (SIF) and crack opening displacement. Finally the effect of material constants, on stress intensity factor and crack opening displacement have been shown by virtue of the graphs.

II. Formulation of the problem

Let us consider the elastodynamic plane problem of diffraction of normally incident longitudinal wave by a Griffith cracks situated at the interface of two orthotropic half spaces. Let x, y, z be the cartesian co-ordinate axes which are the axes of symmetry of the orthotropic materials. The crack is assumed to occupy the region $|x_1| \leq a$, $-\infty < z_1 < \infty$, $y_1 = 0$. If we

consider the transformation $x_1/a = x$, $y_1/a = y$ and $z_1/a = z$, then we find that the location of the crack at the interface becomes $|x| < 1$, $-\infty < z < \infty$, $y = 0$.
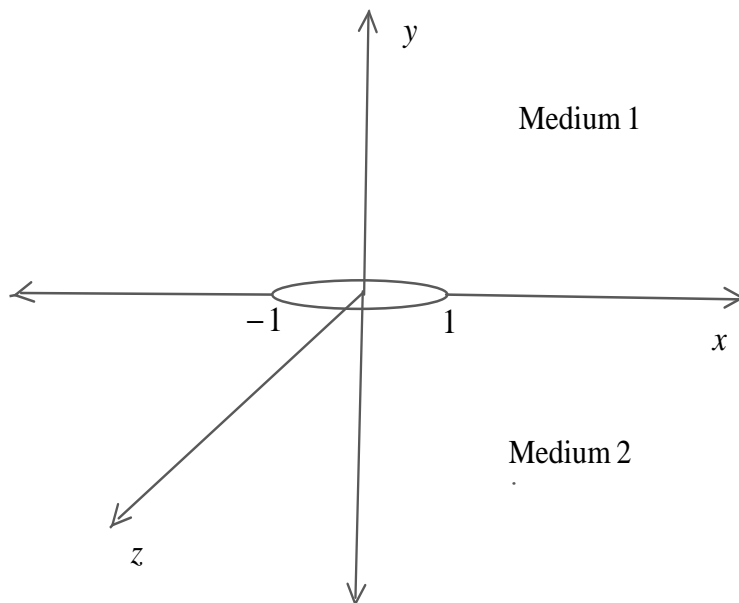

Fig.1   Configuration of the Crack

The displacements $u_x(x,y)$ and $u_y(x,y)$ along the x and y axes respectively are given by the following equations

$$c_{11}^{(k)}\frac{\partial^2 u_x^{(k)}}{\partial x^2} + \frac{\partial^2 u_x^{(k)}}{\partial y^2} + (1 + c_{12}^{(k)})\frac{\partial^2 u_y^{(k)}}{\partial x \partial y} = \frac{a^2}{c_s^2}\frac{\partial^2 u_x^{(k)}}{\partial t^2}, \quad k = 1,2. \tag{1}$$

$$c_{11}^{(k)}\frac{\partial^2 u_y^{(k)}}{\partial x^2} + \frac{\partial^2 u_y^{(k)}}{\partial y^2} + (1 + c_{12}^{(k)})\frac{\partial^2 u_x^{(k)}}{\partial x \partial y} = \frac{a^2}{c_s^2}\frac{\partial^2 u_y^{(k)}}{\partial t^2}, \quad k = 1,2. \tag{2}$$

where c_{11}, c_{22} and c_{12} are non-dimensional parameters related to the elastic constants by the relations

$$c_{11} = \frac{E_1}{\mu_{12}[1 - (\frac{E_2}{E_1})\mu_{12}^2]}$$

$$c_{22} = \frac{E_2}{E_1}c_{11}$$

$$c_{12} = \nu_{12}c_{22} = \nu_{21}c_{11}$$

In the above equations E_i, μ_{ij} and ν_{ij} $(i, \ j = 1, \ 2, \ 3)$ denote the engineering elastic constants of the material where the subscript 1, 2, 3 correspond to the x, y, z directions which coincide with the axis of material orthotropy and the constants E_i and ν_{ij} satisfy the Maxwell's relation $\nu_{ij}/E_i = \nu_{ji}/E_j$. Therefore, substituting $u_x(x,y,t) = u_x(x,y)e^{-i\omega t}$ and $u_y(x,y,t) = u_y(x,y)e^{-i\omega t}$ in equations (1) and (2) we obtain

$$c_{11}^{(k)}\frac{\partial^2 u_x^{(k)}}{\partial x^2} + \frac{\partial^2 u_x^{(k)}}{\partial y^2} + (1 + c_{12}^{(k)})\frac{\partial^2 u_y^{(k)}}{\partial x \partial y} + k_s^2 u_x^{(k)} = 0, \quad k = 1,2. \tag{3}$$

$$c_{11}^{(k)}\frac{\partial^2 u_y^{(k)}}{\partial x^2} + \frac{\partial^2 u_y^{(k)}}{\partial y^2} + (1+c_{12}^{(k)})\frac{\partial^2 u_x^{(k)}}{\partial x \partial y} + k_s^2 u_y^{(k)} = 0, \quad k=1,2. \tag{4}$$

The stresses and displacements are related by the following equations

$$\sigma_{yy}^{(k)}/\mu_{12}^{(k)} = c_{12}^{(k)}\frac{\partial u_x^{(k)}}{\partial x} + c_{22}^{(k)}\frac{\partial u_y^{(k)}}{\partial y}$$

$$\sigma_{xy}^{(k)}/\mu_{12}^{(k)} = \frac{\partial u_x^{(k)}}{\partial y} + \frac{\partial u_y^{(k)}}{\partial x}$$

$$\sigma_{xx}^{(k)}/\mu_{12}^{(k)} = c_{11}^{(k)}\frac{\partial u_x^{(k)}}{\partial x} + c_{12}^{(k)}\frac{\partial u_y^{(k)}}{\partial y}$$

Henceforth the time factor $e^{-i\omega t}$ which is common to all field variables would be omitted in the sequence but to be understood.

The boundary conditions are given by

$$\sigma_{yy}^{(1)}(x,0^+) = \sigma_{yy}^{(2)}(x,0^-) = -\sigma_0, \qquad\qquad |x| < 1 \tag{5}$$

$$u_y^{(1)}(x,0^+) = u_y^{(2)}(x,0^-), \qquad\qquad |x| > 1 \tag{6}$$

$$u_x^{(1)}(x,0^+) = u_x^{(2)}(x,0^-), \qquad\qquad |x| > 1 \tag{7}$$

$$\sigma_{yy}^{(1)}(x,0^+) = \sigma_{yy}^{(2)}(x,0^-), \qquad\qquad |x| > 1 \tag{8}$$

$$\sigma_{xy}^{(1)}(x,0^+) = \sigma_{xy}^{(2)}(x,0^-), \qquad\qquad |x| < \infty \tag{9}$$

The displacement component u_x in x direction is negligible in comparison to the displacement component u_y in y direction in the case of normally incidence. We consider that the displacement on the crack faces $u_x(x,0^+)$ and $u_y(x,0^-)$ are identical. So for that in lieu of (7) we consider

$$u_x^{(1)}(x,0^+) = u_x^{(2)}(x,0^-) \text{ for } |x| < \infty. \tag{7a}$$

An appropriate integral solutions of (3) and (4) can be taken as

$$u_x^{(1)}(x,y) = \frac{2}{\pi}\int_0^\infty [A_1(\zeta)e^{-\delta_1 y} + A_2(\zeta)e^{-\delta_2 y}]\sin(\zeta x)d\zeta \qquad y > 0 \tag{10}$$

$$u_x^{(2)}(x,y) = \frac{2}{\pi}\int_0^\infty [A_1'(\zeta)e^{\delta_1 y} + A_2'(\zeta)e^{\delta_2 y}]\sin(\zeta x)d\zeta \qquad y < 0 \tag{11}$$

$$u_y^{(1)}(x,y) = \frac{2}{\pi}\int_0^\infty \frac{1}{\zeta}[\omega_1 A_1(\zeta)e^{-\delta_1 y} + \omega_2 A_2(\zeta)e^{-\delta_2 y}]\cos(\zeta x)d\zeta \qquad y > 0 \tag{12}$$

$$u_y^{(2)}(x,y) = \frac{2}{\pi}\int_0^\infty \frac{1}{\zeta}[\omega_1 A_1'(\zeta)e^{\delta_1 y} + \omega_2 A_2'(\zeta)e^{\delta_2 y}]\cos(\zeta x)d\zeta \qquad y < 0 \tag{13}$$

and the non vanishing stress components are given by

$$\sigma_{xy}^{(1)}(x,y) = \mu_{12}^{(1)}\left[\frac{2}{\pi}\int_0^\infty \{-A_1(\zeta)\delta_1 e^{-\delta_1 y} - A_2(\zeta)\delta_2 e^{-\delta_2 y}\}\sin(\zeta x)d\zeta\right.$$
$$\left. -\frac{2}{\pi}\int_0^\infty \{\omega_1 A_1(\zeta)e^{-\delta_1 y} + \omega_2 A_2(\zeta)e^{-\delta_2 y}\}\sin(\zeta x)d\zeta\right] \tag{14}$$

$$\sigma_{xy}^{(2)}(x,y) = \mu_{12}^{(2)}\left[\frac{2}{\pi}\int_0^\infty \{A_1^{'}(\zeta)\delta_1 e^{\delta_1 y} + A_2^{'}(\zeta)\delta_2 e^{\delta_2 y}\}\sin(\zeta x)d\zeta\right.$$
$$\left. -\frac{2}{\pi}\int_0^\infty \{\omega_1 A_1^{'}(\zeta)e^{\delta_1 y} + \omega_2 A_2^{'}(\zeta)e^{\delta_2 y}\}\sin(\zeta x)d\zeta\right] \tag{15}$$

$$\sigma_{yy}^{(1)}(x,y) = \mu_{12}^{(1)}\left[\frac{2c_{12}^{(1)}}{\pi}\int_0^\infty \{A_1(\zeta)e^{-\delta_1 y} + A_2(\zeta)e^{-\delta_2 y}\}\zeta\cos(\zeta x)d\zeta\right.$$
$$\left. +\frac{2c_{22}^{(1)}}{\pi}\int_0^\infty \frac{1}{\zeta}\{-\omega_1 A_1(\zeta)e^{-\delta_1 y}\delta_1 - \omega_2 A_2(\zeta)e^{-\delta_2 y}\delta_2\}\cos(\zeta x)d\zeta\right] \tag{16}$$

$$\sigma_{yy}^{(2)}(x,y) = \mu_{12}^{(2)}\left[\frac{2c_{12}^{(2)}}{\pi}\int_0^\infty \{A_1^{'}(\zeta)e^{\delta_1 y} + A_2^{'}(\zeta)e^{\delta_2 y}\}\zeta\cos(\zeta x)d\zeta\right.$$
$$\left. -\frac{2c_{22}^{(2)}}{\pi}\int_0^\infty \frac{1}{\zeta}\{\omega_1 A_1^{'}(\zeta)e^{\delta_1 y}\delta_1 + \omega_2 A_2^{'}(\zeta)e^{\delta_2 y}\delta_2\}\cos(\zeta x)d\zeta\right] \tag{17}$$

where
$$\omega_i = \frac{c_{11}^{(k)}\zeta^2 - k_s^2 - \delta_i^2}{\delta_i(1+c_{12}^{(k)})} \qquad i = 1,2$$

and $A_i(\zeta)(i=1,2)$, $A_i^{'}(\zeta)(i=1,2)$ are the unknown functions to be determined, δ_1^2 and δ_2^2 are the roots of the equation

$$c_{22}^{(k)}\delta^4 + \{((c_{12}^{(k)})^2 + 2c_{12}^{(k)} - c_{11}^{(k)}c_{22}^{(k)})\zeta^2 + (1+c_{22}^{(k)})k_s^2\}\delta^2 + (c_{11}^{(k)}\zeta^2 - k_s^2)(\zeta^2 - k_s^2) = 0 \tag{18}$$

III. Derivation of the integral equation

From the boundary conditions $\sigma_{yy}^{(1)}(x,0^+) - \sigma_{yy}^{(2)}(x,0^-) = 0$ and $\sigma_{xy}^{(1)}(x,0^+) - \sigma_{xy}^{(2)}(x,0^-) = 0$ we obtain the relations between $A_2, A_1^{'}, A_2^{'}$ and $A_1, A_1^{'}, A_2^{'}$ as

$$A_2 = A_1^{'}T_{11} + A_2^{'}Q_{11} \tag{19}$$

$$A_1 = A_1^{'}R_{11} + A_2^{'}S_{11} \tag{20}$$

where

$$T_{11} = \frac{(\mu_{12}^{(2)}c_{12}^{(2)}\zeta + \mu_{12}^{(2)}c_{22}^{(2)}\frac{\omega_1\delta_1}{\zeta}) + (\mu_{12}^{(1)}c_{12}^{(1)}\zeta + \mu_{12}^{(1)}c_{22}^{(1)}\frac{\omega_1\delta_1}{\zeta})\frac{(\delta_1\mu_{12}^{(2)} - \omega_1\mu_{12}^{(2)})}{(\delta_1\mu_{12}^{(2)} - \omega_1\mu_{12}^{(2)})}}{(\mu_{12}^{(1)}c_{12}^{(1)}\zeta + \mu_{12}^{(1)}c_{22}^{(1)}\frac{\omega_1\delta_1}{\zeta}) + (\mu_{12}^{(1)}c_{12}^{(1)}\zeta + \mu_{12}^{(1)}c_{22}^{(1)}\frac{\omega_1\delta_1}{\zeta})\frac{(\delta_2+\omega_2)}{(\delta_1+\omega_1)}}$$

$$Q_{11} = \frac{(\mu_{12}^{(2)} c_{12}^{(2)} \zeta + \mu_{12}^{(2)} c_{22}^{(2)} \frac{\omega_2 \delta_2}{\zeta}) + (\mu_{12}^{(1)} c_{12}^{(1)} \zeta + \mu_{12}^{(1)} c_{22}^{(1)} \frac{\omega_1 \delta_1}{\zeta}) \frac{(\delta_2 \mu_{12}^{(2)} - \omega_2 \mu_{12}^{(2)})}{(\delta_1 \mu_{12}^{(2)} - \omega_1 \mu_{12}^{(2)})}}{(\mu_{12}^{(1)} c_{12}^{(1)} \zeta + \mu_{12}^{(1)} c_{22}^{(1)} \frac{\omega_1 \delta_1}{\zeta}) + (\mu_{12}^{(1)} c_{12}^{(1)} \zeta + \mu_{12}^{(1)} c_{22}^{(1)} \frac{\omega_1 \delta_1}{\zeta}) \frac{(\delta_2 + \omega_2)}{(\delta_1 + \omega_1)}}$$

$$R_{11} = -\frac{\delta_2 \mu_{12}^{(1)} + \omega_2 \mu_{12}^{(1)}}{\delta_1 \mu_{12}^{(1)} + \omega_1 \mu_{12}^{(1)}} \cdot \frac{(\mu_{12}^{(2)} c_{12}^{(2)} \zeta + \mu_{12}^{(2)} c_{22}^{(2)} \frac{\omega_1 \delta_1}{\zeta}) + (\mu_{12}^{(1)} c_{12}^{(1)} \zeta + \mu_{12}^{(1)} c_{22}^{(1)} \frac{\omega_1 \delta_1}{\zeta}) \frac{(\delta_1 \mu_{12}^{(2)} - \omega_1 \mu_{12}^{(2)})}{(\delta_1 \mu_{12}^{(2)} - \omega_1 \mu_{12}^{(2)})}}{(\mu_{12}^{(1)} c_{12}^{(1)} \zeta + \mu_{12}^{(1)} c_{22}^{(1)} \frac{\omega_1 \delta_1}{\zeta}) + (\mu_{12}^{(1)} c_{12}^{(1)} \zeta + \mu_{12}^{(1)} c_{22}^{(1)} \frac{\omega_1 \delta_1}{\zeta}) \frac{(\delta_2 + \omega_2)}{(\delta_1 + \omega_1)}}$$
$$- \frac{\delta_1 \mu_{12}^{(2)} - \omega_1 \mu_{12}^{(2)}}{\delta_1 \mu_{12}^{(1)} + \omega_1 \mu_{12}^{(1)}}$$

$$S_{11} = -\frac{\delta_2 \mu_{12}^{(1)} + \omega_2 \mu_{12}^{(1)}}{\delta_1 \mu_{12}^{(1)} + \omega_1 \mu_{12}^{(1)}} \cdot \frac{(\mu_{12}^{(2)} c_{12}^{(2)} \zeta + \mu_{12}^{(2)} c_{22}^{(2)} \frac{\omega_2 \delta_2}{\zeta}) + (\mu_{12}^{(1)} c_{12}^{(1)} \zeta + \mu_{12}^{(1)} c_{22}^{(1)} \frac{\omega_1 \delta_1}{\zeta}) \frac{(\delta_2 \mu_{12}^{(2)} - \omega_2 \mu_{12}^{(2)})}{(\delta_1 \mu_{12}^{(2)} - \omega_1 \mu_{12}^{(2)})}}{(\mu_{12}^{(1)} c_{12}^{(1)} \zeta + \mu_{12}^{(1)} c_{22}^{(1)} \frac{\omega_1 \delta_1}{\zeta}) + (\mu_{12}^{(1)} c_{12}^{(1)} \zeta + \mu_{12}^{(1)} c_{22}^{(1)} \frac{\omega_1 \delta_1}{\zeta}) \frac{(\delta_2 + \omega_2)}{(\delta_1 + \omega_1)}}$$
$$- \frac{\delta_2 \mu_{12}^{(2)} - \omega_2 \mu_{12}^{(2)}}{\delta_1 \mu_{12}^{(1)} + \omega_1 \mu_{12}^{(1)}}$$

Applying the boundary condition $(7a)$ and using the equations (19) and (20) in (10) and (11) we obtain the relation between A_1' and A_2' as

$$A_2' = -\beta A_1' \tag{21}$$

where $\qquad \beta = \frac{T_{11} + R_{11} - 1}{S_{11} + Q_{11} - 1}$

The boundary conditions (5) and (6) lead to the following dual integral equations

$$\int_0^\infty G(\zeta) D(\zeta) \cos(\zeta x) d\zeta = -\frac{\sigma_0}{2\mu_{12}^{(1)}}, \qquad\qquad |x| < 1 \tag{22}$$

$$\int_0^\infty D(\zeta) \cos(\zeta x) d\zeta = 0 \qquad\qquad |x| > 1 \tag{23}$$

where

$$G(\zeta) = \frac{1}{(\omega_1 R_{11} + \omega_2 T_{11} - \omega_1 S_{11} \beta - \omega_2 Q_{11} \beta - \omega_1 + \omega_2 \beta)} \left[\frac{c_{12}^{(1)}}{\pi} (\zeta^2 R_{11} + \right.$$
$$\left. \zeta^2 T_{11} - \beta \zeta^2 Q_{11}) - \frac{c_{22}^{(1)}}{\pi} (\omega_1 \delta_1 R_{11} + \omega_2 \delta_2 T_{11} - \omega_1 \delta_1 \beta S_{11} - \omega_2 \delta_2 Q_{11} \beta) \right] \tag{24}$$

and

$$D(\zeta) = \left[\omega_1 R_{11} + \omega_2 T_{11} - \omega_1 S_{11} \beta - \omega_2 Q_{11} \beta - \omega_1 + \omega_2 \beta \right] \frac{A_1'(\zeta)}{\zeta} \tag{25}$$

IV. Method of solution

We consider

$$D(\zeta) = \frac{1}{\zeta} \int_0^1 h(t) \sin(\zeta t) dt \qquad (26)$$

to obtain the solution of integral equation (22) and (23) in which $h(t)$ is the unknown function which has been determined from the boundary conditions. Now using (26) in (22), we have obtained

$$\frac{d}{dx} \int_0^1 h(t) dt \int_0^\infty \frac{\zeta \sin(\zeta t) \sin(\zeta x) G(\zeta)}{\zeta} d\xi = -\frac{\sigma_0}{2\mu_{12}^{(1)}}, \qquad 0 < x < 1 \qquad (27)$$

Using the relations

$$\int_0^\infty \frac{\sin(\zeta t) \sin(\zeta x)}{\zeta} d\zeta = \frac{1}{2} \log \left| \frac{t+x}{t-x} \right| \qquad (28)$$

and

$$\frac{\sin(\zeta x) \sin(\zeta t)}{\zeta^2} = \int_0^x \int_0^t \frac{wv J_0(\zeta w) J_0(\zeta v)}{\sqrt{x^2 - w^2} \sqrt{t^2 - v^2}} dv dw \qquad (29)$$

in $eq.(27)$ and after some manipulation we obtain

$$\frac{d}{dx} \int_0^1 h(t) \log \left| \frac{t+x}{t-x} \right| dt = 2 \left[\tau_1 - \frac{d}{dx} \int_0^1 h(t) dt \int_0^x \int_0^t \frac{vw K(v,w)}{\sqrt{x^2 - w^2} \sqrt{t^2 - v^2}} dv dw \right]$$
$$0 < x < 1 \qquad (30)$$

where $\sigma_1 = -\dfrac{\sigma_0}{\zeta \mu_{12}^{(1)} \phi}$

$$\phi = \frac{\frac{c_{12}^{(k)}}{\pi}(R_{11} + T_{11} - \beta S_{11} - \beta Q_{11}) - \frac{c_{22}^{(k)}}{\pi}(\frac{c_{11}^{(k)} - n_1^2}{(1+c_{12}^{(k)})} R_{11} + \frac{c_{11}^{(k)} - n_2^2}{(1+c_{12}^{(k)})} T_{11} - \frac{c_{11}^{(k)} - n_1^2}{(1+c_{12}^{(k)})} S_{11} - \frac{c_{11}^{(k)} - n_2^2}{(1+c_{12}^{(k)})} Q_{11})}{\frac{c_{11}^{(k)} - n_1^2}{(1+c_{12}^{(k)})n_1} R_{11} + \frac{c_{11}^{(k)} - n_2^2}{(1+c_{12}^{(k)})n_2} T_{11} - \frac{c_{11}^{(k)} - n_1^2}{(1+c_{12}^{(k)})n_1} S_{11}\beta - \frac{c_{11}^{(k)} - n_2^2}{(1+c_{12}^{(k)})n_2} Q_{11}\beta - \frac{c_{11}^{(k)} - n_1^2}{(1+c_{12}^{(k)})n_1} + \frac{c_{11}^{(k)} - n_2^2}{(1+c_{12}^{(k)})n_2}\beta}$$

$$n_1^2 = \frac{-((c_{12}^{(k)})^2 + 2c_{12}^{(k)} - c_{11}^{(k)} c_{22}^{(k)}) + \sqrt{((c_{12}^{(1)})^2 + 2c_{12}^{(k)} - c_{11}^{(k)} c_{22}^{(k)})^2 - 4c_{11}^{(k)} c_{22}^{(k)}}}{2c_{22}^{(k)}}$$

$$n_2^2 = \frac{-((c_{12}^{(k)})^2 + 2c_{12}^{(k)} - c_{11}^{(k)} c_{22}^{(k)}) - \sqrt{((c_{12}^{(1)})^2 + 2c_{12}^{(k)} - c_{11}^{(k)} c_{22}^{(k)})^2 - 4c_{11}^{(k)} c_{22}^{(k)}}}{2c_{22}^{(k)}}$$

$$K(v,w) = \int_0^\infty \zeta G_1(\zeta) J_0(\zeta w) J_0(\zeta v) d\zeta. \qquad (31)$$

and $\qquad G_1(\zeta) = \dfrac{G(\zeta)}{\zeta \phi} - 1$

Applying the contour integration technique [15] the semi-infinite integral has therefore been converted to the following finite integral

$$K(v,w) = -ik_s^2 \left[\int_0^{1/\sqrt{c_{11}}} \frac{c_{22}(\bar{\bar{\omega}}_1\bar{\bar{\delta}}_1 R_{11} + \bar{\bar{\omega}}_2\bar{\bar{\delta}}_2 T_{11} - \bar{\bar{\omega}}_1\bar{\bar{\delta}}_1 T_{11} - \bar{\bar{\omega}}_2\bar{\bar{\delta}}_2 Q_{11})}{\pi\phi\{\bar{\bar{\omega}}_1(S_{11}\beta - R_{11} + 1) + \bar{\bar{\omega}}_2(Q_{11}\beta - T_{11} - \beta)\}} \times J_0(k_s\eta v)H_0^{(1)}(k_s\eta w)d\eta \right.$$
$$\left. - \int_{1/\sqrt{c_{11}}}^1 \frac{c_{22}(\hat{\omega}_2\hat{\delta}_2 T_{11} - \hat{\omega}_2\hat{\delta}_2 Q_{11})}{\pi\phi\{\hat{\omega}_1(R_{11} - S_{11}\beta) - \hat{\omega}_2(1 + T_{11} + Q_{11}\beta)\}} \times J_0(k_s\eta v)H_0^{(1)}(k_s\eta w)d\eta \right] \tag{32}$$

where

$$\zeta = k_s\eta$$

$$\bar{\bar{\delta}}_1 = \left[\tfrac{1}{2}\{r_1 - \sqrt{r_1^2 - 4\bar{r}_2}\} \right]^{\frac{1}{2}}$$

$$\bar{\bar{\delta}}_2 = \left[\tfrac{1}{2}\{r_1 + \sqrt{r_1^2 - 4\bar{r}_2}\} \right]^{\frac{1}{2}}$$

$$\hat{\delta}_1 = \left[\tfrac{1}{2}\{-r_1 + \sqrt{r_1^2 + 4r_2'}\} \right]^{\frac{1}{2}}$$

$$\hat{\delta}_2 = \left[\tfrac{1}{2}\{r_1 + \sqrt{r_1^2 + 4r_2'}\} \right]^{\frac{1}{2}}$$

$$r_1 = \frac{1}{c_{22}^{(k)}} \left[(c_{12}^2 + 2c_{12}^{(k)} - c_{11}^{(k)}c_{22}^{(k)})\eta^2 + (1 + c_{22}^{(k)}) \right]$$

$$\bar{r}_2 = \frac{c_{11}^{(k)}}{c_{22}^{(k)}} \left[(1 - \eta^2)(\tfrac{1}{c_{11}^{(k)}} - \eta^2) \right]$$

$$r_2' = \frac{c_{11}^{(k)}}{c_{22}^{(k)}} \left[(1 - \eta^2)(\eta^2 - \tfrac{1}{c_{11}^{(k)}}) \right]$$

$$\bar{\bar{\omega}}_1 = \frac{c_{11}^{(k)}\eta^2 - 1 + \bar{\bar{\delta}}_1^{\,2}}{(1 + c_{12}^{(k)})\bar{\bar{\delta}}_1}$$

$$\bar{\bar{\omega}}_2 = \frac{c_{11}^{(k)}\eta^2 - 1 + \bar{\bar{\delta}}_2^{\,2}}{(1 + c_{12}^{(k)})\bar{\bar{\delta}}_2}$$

$$\hat{\omega}_1 = \frac{c_{11}^{(k)}\eta^2 - 1 - \hat{\delta}_1^{\,2}}{(1 + c_{12}^{(k)})\hat{\delta}_1}$$

$$\hat{\omega}_2 = \frac{c_{11}^{(k)}\eta^2 - 1 - \hat{\delta}_2^{\,2}}{(1 + c_{12}^{(k)})\hat{\delta}_2}$$

$$\bar{\bar{\beta}} = \frac{\bar{\bar{\delta}}_1 - \bar{\bar{\omega}}_1}{\bar{\bar{\delta}}_2 - \bar{\bar{\omega}}_2}$$

$$\hat{\beta} = \frac{\hat{\omega}_1 + \hat{\delta}_1}{\hat{\omega}_1 + \hat{\delta}_1}$$

Employing the series expansion for Bessel function $J_0()$ and Hankel function $H_0^{(1)}()$, from equation (32)

$$K(v,w) = Mk_s^2 \log(k_s) + O(k_s^2) \tag{33}$$

where

$$M = \frac{2}{\pi^2\phi}\left[\int_0^{1/\sqrt{c_{11}}} \frac{c_{22}(\bar{\bar{\omega}}_1\bar{\bar{\delta}}_1 R_{11} + \bar{\bar{\omega}}_2\bar{\bar{\delta}}_2 T_{11} - \bar{\bar{\omega}}_1\bar{\bar{\delta}}_1 T_{11} - \bar{\bar{\omega}}_2\bar{\bar{\delta}}_2 Q_{11})}{\{\bar{\bar{\omega}}_1(S_{11}\beta - R_{11} + 1) + \bar{\bar{\omega}}_2(Q_{11}\beta - T_{11} - \beta)\}} d\eta \right.$$
$$\left. - \int_{1/\sqrt{c_{11}}}^1 \frac{c_{22}(\hat{\omega}_2\hat{\delta}_2 T_{11} - \hat{\omega}_2\hat{\delta}_2 Q_{11})}{\{\hat{\omega}_1(R_{11} - S_{11}\beta) - \hat{\omega}_2(1 + T_{11} + Q_{11}\beta)\}} d\eta\right] \tag{34}$$

Let us expand $h(t)$ in the form

$$h(t) = h_0(t) + k_s^2 \log(k_s)h_1(t) + O(k_s^2) \tag{35}$$

and utilizing the value of $h(t)$ in (30)

$$\frac{d}{dx}\int_0^1 \{h_0(t) + k_s^2 \log(k_s)h_1(t)\}\log\left|\frac{t+x}{t-x}\right|dt = 2\left[\sigma_1 - \right.$$
$$\left.\frac{d}{dx}\int_0^1 \{h_0(t) + k_s^2 \log(k_s)h_1(t)\}\frac{vwK(v,w)}{\sqrt{x^2-w^2}\sqrt{t^2-v^2}}dvdw\right] \tag{36}$$

Equating the co-efficient of power of k_s from both sides of the above equation

$$h_0(t) = -\frac{2t\sigma_0}{\pi^2\sqrt{1-t^2}\mu_{12}^{(1)}\phi\eta} \tag{37}$$

$$h_1(t) = -\frac{2Mt\sigma_0}{\pi^2\sqrt{1-t^2}\eta\phi\mu_{12}^{(1)}} \tag{38}$$

V. Stress Intensity Factor and Crack Opening Displacement

The normal stress $\sigma_{yy}(x,y)$ in the plane $y = 0$ in the periphery of the crack tip can be found and is given by

$$\sigma_{yy}^{(1)}(x,0) = -\frac{\sigma_0(1 + Mk_s^2\log(k_s))}{\theta}\int_0^\infty \frac{G(\zeta)\sigma_1(\zeta)}{\zeta}\cos(\zeta x)d\zeta \tag{39}$$

Applying the formula $\displaystyle\int_0^\infty \tau_\nu(\alpha x)\cos(\beta x)dx = -\frac{\alpha^\nu \sin\frac{\nu\pi}{2}}{\sqrt{\beta^2-\alpha^2}(\beta+\beta^2-\alpha^2)^{1/2}}, \qquad \beta > \alpha$ and using the relation $\frac{G(\zeta)}{\zeta} = \phi$ as $\zeta \to \infty$ in (39) we obtain

$$\sigma_{yy}^{(1)}(x,0) = -\pi\sigma_0(1 + Mk_s^2\log k_s)\left[\frac{\sin\frac{\pi}{2}}{\sqrt{x^2-1}(x+\sqrt{x^2-1})}\right], \qquad x > 1 \tag{40}$$

Dynamic stress intensity factor denoted by SIF (K) at the tip of the crack defined by the relation

$$SIF(K) = \lim_{x\to 1^+}\left|\frac{\sigma_{yy}^{(1)}(x,0^+)(x-1)^{\frac{1}{2}}}{\sigma_0}\right| \tag{41}$$

is obtained as

$$SIF(K) = \frac{\pi(1 + Mk_s^2 \log k_s)}{\sqrt{2}}.$$ (42)

Another quantity of physical interest is the Crack Opening Displacement (COD) defined by

$$COD = u_y(x, 0^+) - u_y(x, 0^-) = 2\int_x^1 h(t)dt, \qquad 0 \leq x \leq 1$$ (43)

$$COD = -\frac{4\sqrt{1-x^2}[1 + Mk_s^2 log(k_s)]}{\pi\phi\mu_{12}^{(1)}}$$ (44)

VI. Numerical results and discussion

From the expression of stress intensity factor (K) at the tip of the crack has been evaluated numerically and it is clear that the SIF depends on the material constants and frequency. Therefore the values of the SIF can be plotted graphically against the dimensionless frequency k_s for various type of materials. The values of engineering elastic constants are given in the following table

Table 1. **Engineering elastic constants**

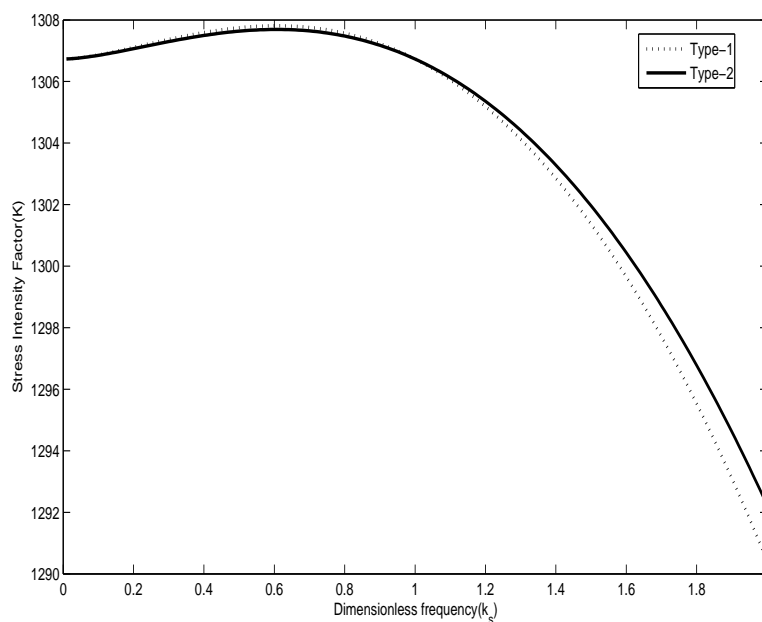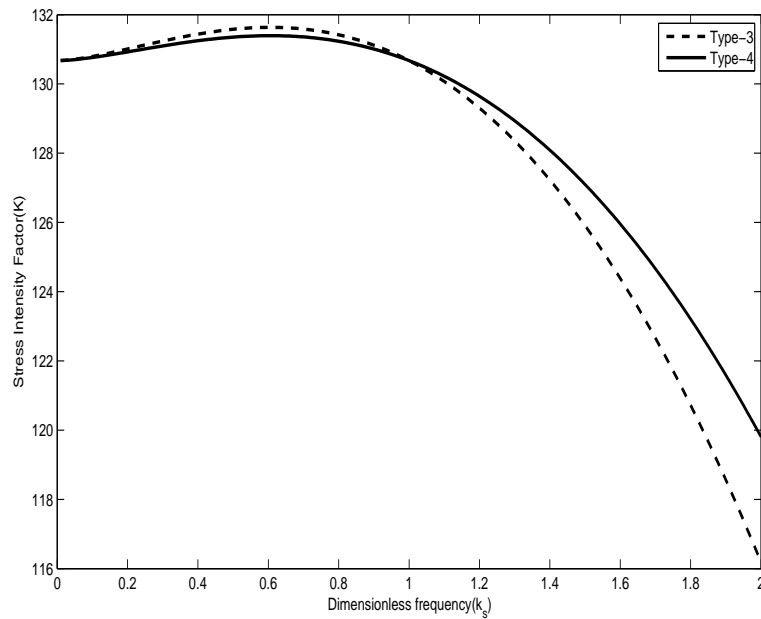|	Elastic constants	c_{11}	c_{22}	c_{12}
Type-1	E-type glass-epoxi composite	2.721	11.759	0.741
Type-2	Steel-Mylar composite	18.7	2.92	1.3
Type-3	Boron-epoxi composite	50.8116	2.8767	0.7364
Type-4	Steel-Mylar composite	18.7	2.92	1.3

Fig.2 SIF versus Frequency

Fig.3 SIF versus Frequency

From the Fig. 2- Fig. 3, it is clear that the SIF decreases with the increasing value of the frequency for any pair of materials selected. For large value of frequency the SIF diminishes and tends to zero which also can be observed from the expression of the SIF given by the equation (42). The effect of material constants on the SIF is also significant here. It has been observed that the values of the SIF change if the materials are interchanged. That difference depends on the elastic constants of the materials.
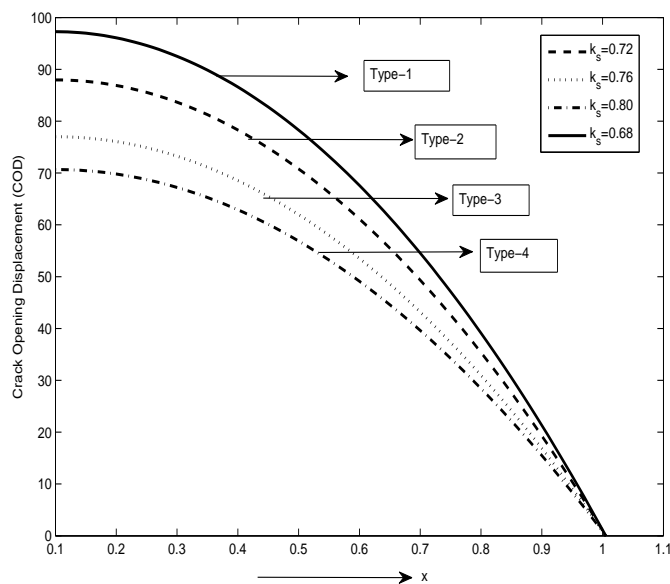

Fig.4 Crack Opening Displacement

The COD has been plotted for different types of material constants. In each case, COD

increases gradually from zero, attains maximum value and then decreases to zero. It is found that the values of COD increases (Fig. 4) for different types of material constants. In all cases the variation of COD is found to be prominent for different orthotropic materials.

VII. Conclusion

An interfacial crack problem with a Griffith Crack between two dissimilar orthotropic media has been solved and numerical computation has been done with a pair of composite materials. The SIF(K) and COD have been obtained at the tip of the crack at the orthotropic bi-material interface subject to P wave incidence. The singularities and discontinuities associated with the incidence P waves and crack have been predicted in the solution. The graphs of Stress Intensity Factor and Crack Opening Displacement have been plotted to show the effects of various parameters on these quantities. The graph of stress intensity factor against the dimensionless frequency initially increases with increasing value of frequency (k_s) and after attaining maximum value it decreases and finally tending to zero. The COD has been plotted for different value of frequency (k_s). The Crack Opening Displacement first increases and then decreased rapidly with increase in frequency k_s and finally tending to 1. From the all graphs it can be concluded that the values of SIF and COD can be controlled and arrested within a certain range by monitoring applied loads.

Acknowledgement

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