Introduce Multiple New Methods of Matrices

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Abstract

In this paper the author introduce multiple methods of determinant, inverse of matrix and Eigen values and Eigen vectors.

Key words
Addition, Multiplication, Subtraction & Division.

2) Find the Inverse of Matrix If

$$A = \begin{pmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{pmatrix}$$

Solve by previous method

$$A^{-1} = \frac{1}{|A|} \text{adj } A$$

$$|A| = 3\,[-3+4] + 3\,[2-0] + 4\,[-2+0]$$
$$= 3\,[1] + 3\,[2] + 4\,[-2]$$
$$= 3 + 6 - 8$$
$$= 1$$

To find adj A

$$a_{11} = +[-3+4] = +[1] = 1$$
$$a_{12} = -[2-0] = -[2] = -2$$
$$a_{13} = +[-2-0] = +[-2] = -2$$
$$a_{21} = -[-3+4] = -[1] = -1$$
$$a_{22} = +[3-0] = +[3] = 3$$
$$a_{23} = -[-3+0] = -[-3] = 3$$
$$a_{31} = +[-12+12] = +[0] = 0$$
$$a_{32} = -[12-8] = -[4] = -4$$
$$a_{33} = +[-9+6] = +[-3] = -3$$

$$\text{Cofactor of } A = \begin{pmatrix} 1 & -2 & -2 \\ -1 & 3 & 3 \\ 0 & -4 & -3 \end{pmatrix}$$

$$\text{adj } A = \begin{pmatrix} 1 & -2 & -2 \\ -1 & 3 & 3 \\ 0 & -4 & -3 \end{pmatrix}$$

$$\therefore \quad A^{-1} = \frac{1}{|A|} \text{adj } A$$

$$= \frac{1}{1} \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$$

Solve by New Method

Inverse of Matrix A^{-1} $= \frac{1}{|A|}$ adj A

Explanation

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{11} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$|A| = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{11} & a_{12} \\ a_{21} & a_{22} & a_{23} & a_{21} & a_{22} \\ a_{31} & a_{32} & a_{33} & a_{31} & a_{32} \end{pmatrix}$$

$= (a_{11} x\, a_{22}\, x\, a_{33} + a_{12}\, x\, a_{23}\, x\, a_{31} + a_{13}\, x\, a_{21}\, x\, a_{32})$ -

$= (a_{12} x\, a_{21}\, x\, a_{33} + a_{11}\, x\, a_{23}\, x\, a_{32} + a_{13}\, x\, a_{22}\, x\, a_{31})$

To find adj A

The given matrix is $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

This can be written as

$$\begin{pmatrix} \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} & \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix} \\ \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} & \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} \end{pmatrix}$$

i) The first diagonal elements are change in place

$$\begin{pmatrix} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} & \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix} \\ \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} & \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \end{pmatrix}$$

ii) The second diagonal elements are change in place

$$\begin{pmatrix} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} & \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} \\ \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix} & \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \end{pmatrix}$$

iii) Then we write rows in to columns (or) columns into rows.

$\therefore$ Adj A =

$$\begin{pmatrix} a_{21} & a_{32} & a_{12} & a_{22} \\ & a_{23} & a_{33} & \\ a_{21} & a_{31} & a_{11} & a_{21} \\ a_{22} & a_{32} & a_{12} & a_{22} \end{pmatrix} \quad a_{13} \quad a_{23}$$

$\begin{vmatrix} a & b \\ c & d \end{vmatrix}$ = ad – bc

Find the inverse of matrix if

$$A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$$

Soe :

$A^{-1} = \frac{1}{|A|}$ adj A

$$|A| = \begin{pmatrix} 3 & -3 & 4 & 3 & -3 \\ 2 & -3 & 4 & 2 & -3 \\ 0 & -1 & 1 & 0 & -1 \end{pmatrix}$$

$= [(-9 - 0 - 8) - (-6 - 12 + 0]$

= -17 + 18

|A| = 1

To find adj A

Adj A =

$$\begin{pmatrix} -3 & -1 & -3 & -3 \\ 4 & 1 & 4 & 4 \\ 2 & 0 & 3 & 2 \\ -3 & -1 & -3 & -3 \end{pmatrix}$$

$\begin{vmatrix} a & b \\ c & d \end{vmatrix}$ = ad – bc

` $= \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|}$ adj A

$= \frac{1}{1} \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$

Solve by previous method

Introduction – II

Find the Eigen values and eigenvectors of

$$\begin{bmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{bmatrix}$$

Sol

$$A = \begin{bmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{bmatrix}$$

The characteristic equ is $\boldsymbol{\lambda^3 - a_1\lambda^2 + a_2\lambda - a_3 = 0}$

a_1 = Sum of leading diagonal elements

= 1 + 1 – 3

= 0

a_2 = Sum of the minors of the leading diagonal elements

$= \begin{vmatrix} 1 & 1 \\ 2 & -3 \end{vmatrix} + \begin{vmatrix} 2 & 0 \\ -7 & -3 \end{vmatrix} + \begin{vmatrix} 2 & 2 \\ 2 & 1 \end{vmatrix}$

= -5 – 6 – 2

= -13

a_3 = $|A|$ = 2 (-3-2) -2 (-6+7) + 0 (4+7)

= -10-2 +6

= -12

∴ The characteristic equ is $\lambda^3 - 0\lambda^2 + 13\lambda + 12 = 0$

To find Eigen values

λ = 1, $(\lambda + 4)(\lambda - 3) = 0$ $\lambda^2 + \lambda - 12 = 0$

λ = 1, 3, - 4

12
4⋀3

1	1	0	-13	12
	0	1	1	-12
	1	1	-12	0

∴ Eigen values are λ = 1, 3, -4

To find Eigenvectors $|A - \lambda I|x = 0$

$$= \begin{bmatrix} 2-\lambda & 2 & 0 \\ 2 & 1-\lambda & 1 \\ -7 & 2 & -3-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

When λ =1

$$= \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ -7 & 2 & -4 \end{bmatrix} \text{x} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

1	2	0	1
2	0	1	2

$= \quad x_1 + 2x_2 + 0x_3 = 0$

$2x_1 + 0x_2 + x_3 = 0 \qquad = \frac{x_1}{2} = \frac{x_2}{-1} = \frac{x_3}{-4}$

$-7x_1 + 2x_2 - 4x_3 = 0 \qquad x_1 = \begin{pmatrix} 2 \\ -1 \\ -4 \end{pmatrix}$

When λ =3

$$= \begin{bmatrix} -1 & 2 & 0 \\ 2 & -2 & 1 \\ -7 & 2 & -6 \end{bmatrix} \text{x} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

-1	2	0	-1
2	-2	1	2

$= \quad -x_1 + 2x_2 + 0x_3 = 0$

$2x_1 + 2x_2 + x_3 = 0 \qquad = \frac{x_1}{2} = \frac{x_2}{-1} = \frac{x_3}{-2}$

$-7x_1 + 2x_2 - 6x_3 = 0 \qquad x_1 = \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$

When $\lambda = 4$

$$= \begin{bmatrix} 6 & 2 & 0 \\ 2 & 5 & 1 \\ -7 & 2 & 1 \end{bmatrix} \text{x} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

6	2	0	6
2	5	1	2

$= \quad 6x_1 + 2x_2 + 0x_3 = 0$

$2x_1 + 5x_2 + x_3 = 0 \qquad = \frac{x_1}{2} = \frac{x_2}{-6} = \frac{x_3}{26} = \frac{x_1}{2} = \frac{x_2}{-3} = \frac{x_3}{13}$

$-7x_1 + 2x_2 + x_3 = 0 \qquad x_1 = \begin{pmatrix} 1 \\ -3 \\ 13 \end{pmatrix}$

Introduction : Solve by New method

Working Rule to find Eigen values and vectors by new method.

Step :1

Find the determinant value.

Step :2

Dividing determinant value by long division method, take the values in acceding order.

Step : 3

The values are stratified the following properties.

1) Sum of the diagonal elements = sum of Eigen values

2) Product of Eigen values = determinant value

Step : 4

The find Eigen values solve (A - λI) X = 0

sing Cofactor method.

Find the Eigen values and Eigen vectors of the matrix

$$A = \begin{bmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{bmatrix}$$

Explanation

Sub :

Without using characteristic equations we can find the Eigen values of the matrix

Find |A|

$$|A| = \begin{pmatrix} 2 & 2 & 0 & 2 & 2 \\ 2 & 1 & 1 & 2 & 1 \\ -7 & 2 & -3 & -7 & 2 \end{pmatrix}$$

$= [(-6 - 14 + 0) - (-12 + 4 + 0]$

= -20 + 8

|A| = -12

Dividing |A| value by long division method

2	12	1 is a common divisor of all elements
2	6	[1, 3, 2 x 2]
3	3	
	1	

The matrix of order 3 x 3

So we have 3 Eigen values $\therefore \lambda = 1, 3, 2\,x\,2 =$ 1, 3, -4

Which Elements are satisfied the following property then the elements are called Eigen values.

Property : 1

Sum of the Eigen values is equal to the sum of the diagonal elements.

Sum of diagonal elements = 2 + 1-3 = 0

Sum of Eigen values = 1 + 3-4 = 0

$\therefore \lambda = 1, 3, -4$

Property : 2

Product of Eigen values is equal to its determinant value.

|A| = -12

Product of Eigen values = 1 x 3 x -4 = -12

$\therefore$ The Eigen values of the given matrix is = $\therefore \lambda = 1, 3, -4$

Note :1

If |A| = 0 then the Eigen values is choose in the above properties. The Eigen vectors are linearly dependents.

To find Eigen Vectors

$[A - \lambda I]\, x = 0$

$$= \begin{bmatrix} 2-\lambda & 2 & 0 \\ 2 & 1-\lambda & 1 \\ -7 & 2 & -3-\lambda \end{bmatrix} x \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

When λ =1 ***Explanation***

$$= \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ -7 & 2 & -4 \end{bmatrix} x \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Using Co factor method.

+ (0 -2) – (-8+7) + (4+0) = -2, 1, 4 = 2, -1, -4

When $\lambda = 3$

$$= \begin{bmatrix} -1 & 2 & 0 \\ 2 & -2 & 1 \\ -7 & 2 & -6 \end{bmatrix} x \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Using Co factor method.

+ (12 -2) – (-12+7) + (4-14) = 10, 5, -10 = 2, 1, -2

When $\lambda = -4$

$$= \begin{bmatrix} 6 & 2 & 0 \\ 2 & 5 & 1 \\ -7 & 2 & 1 \end{bmatrix} x \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Using Co factor method.

+ (5-2) – (2+7) + (4+35) = 3, -9, 39 = 1, -3, 13

therefore Eigen Vectors.

$$\begin{bmatrix} 2 \\ -1 \\ -4 \end{bmatrix} \quad \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix} \begin{bmatrix} 1 \\ -3 \\ 13 \end{bmatrix}$$

Conclusion

This method is very useful to school students, Arts & Science College Students and Engineering Students. It is very useful to gate exam and competitive examinations.

Reference

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