Symmetric Bi-T-Derivations of Incline Algebra

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Abstract
The concept of derivation in incline algebra was introduced by N.O.Alsherhi[1] . Kyung Ho kim and so Young Park[2] introduced the symmetric bi-f- derivation in incline algebra. In this paper, we introduce the concept of symmetric bi-t-derivation in incline algebras and present some properties of symmetric bi-t-derivations. Also, we characterize $Ker_{dt}(K)$ and $T_a(K)$ by symmetric bi-t-derivations in incline algebra and give some examples. Also we define isotone symmetric bi-t- derivation in incline algebra and analyse its properties.

Key words: *bi-t-derivation, isotone, incline algebra, joinitive, $Ker_{dt}(K)$.*

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1.Introduction

Z.Q.Cao, K.H.Kim and F.W Roush[2] introduced the notation of incline algebras in their book. After that Some authors studied incline algebra and its application. N.O.Alsherhi [1] introduced the notation of derivation in incline algebra. Kyung Ho kim [2] a introduced the symmetric bi-f-derivation in algebra. In this paper we introduced some concept of a symmetric bi-t-derivation of incline algebra and give some properties of incline algebras. Also we characterize Ker_{dt}(K) and $T_a(K)$ by symmetric bi-t-derivation in incline algebra.

2. Preliminaries

Definition 2.1
An *incline algebra* is a set K with two binary operations denoted by " + " and " $*$ " satisfying the following axioms:

$(K1)\ x + y = y + x$

$(K2)\ x + (y + z) = (x + y) + z$

$(K3)\ x * (y * z) = (x * y) * z$

$(K4)\ x * (y + z) = (x * y) + (x * z)$

$(K5)\ (y + z) * x = (y * x) + (z * x)$

$(K6)\ x + x = x$

$(K7)\ x + (x * y) = x$

$(K8)\ y + (x * y) = y$

For all $x, y, z \in K$
For pronounce " + " (resp. " $*$ ") as addition (resp. multiplication).
In an incline algebra K, the following properties hold.

$(K9)$ $x * y \le x$ and $y * x \le x$ for all $x, y \in K$
$(K10)$ $y \le z$ implies $x * y \le x * z$ and $y * x \le z * x$ for all $x, y, z \in K$

$(K11)$ If $x \leq y$ and $a \leq b$ then $x + a \leq y + b$, and $x * a \leq y * b$ $\forall x, y, a, b \in K$
An incline algebra K is said to be commutative if $x * y = y * x$ $\forall x, y \in K$.

Definition 2. 2

Let K be an incline algebra. A mapping $d_t: K \times K \to K$ is called *symmetric* if $d_t(x, y) = d_t(y, x)$ holds $\forall x, y \in K$

Definition 2.3

Let K be an incline algebra and let $d_t: K \to K$ be a function. We call d_t a *t-derivation* of K, if it satisfies the following condition $x, y \in K$

$$d_t(x * y) = (d_t x * y) + (x * d_t y)$$

For all $x, y \in K$

Definition 2.4

Let K be an incline algebra. If $d_t: K \times K \to K$ be a symmetric mapping. We call d_t is joinitive mapping if it satisfies $d_t(x + y, z) = d_t(x, z) + d_t(y, z)$ $\forall x, y \in K$

3 Symmetric bi-t-derivations of incline algebras

let K denote an incline algebra with zero-element unlessotherwise specified

Definition 3.1

Let K be a incline algebra and let $d_t: K \times K \to K$ be a symmetric mapping. We call d_t a *symmetric bi-t-derivation on* K if there exists a function $t: K \times K \to K$ such that

$$d_t(x * y, z) = (d_t(x, z) * t(y)) + (t(x) * d_t(y, z))$$

$\forall x, y, z \in K$
Obviously, a symmetric bi-t-derivation d_t on K satisfies the relation

$$d_t(x, y * z) = (d_t(x, y) * t(z)) + (t(y) * d_t(x, z))$$

$\forall x, y, z \in K$

Example 3.2

Let $K = \{0, a, b, 1\}$ be a set in which " + " and " ∗" is definded by

+	0	a	b	1
0	0	a	b	1
a	a	a	b	1
b	b	b	b	1
1	1	1	1	1

*	0	a	b	1
0	0	0	0	0
a	0	a	a	a
b	0	a	b	b
1	0	a	b	1

Then it i ∗) is an incline algebra.
Define a

$$d_t(x,y) = \begin{cases} 0 & \text{if } (x,y)=(0,0) \\ 0 & \text{if } (x,y)=(0,a),(a,0) \\ 0 & \text{if } (x,y)=(0,b),(b,0) \\ 0 & \text{if } (x,y)=(0,1),(1,0) \\ 0 & \text{if } (x,y)=(a,a) \\ b & \text{if } (x,y)=(b,b) \\ b & \text{if } (x,y)=(1,1) \\ 0 & \text{if } (x,y)=(a,b) \text{ or } (b,a) \\ 0 & \text{if } (x,y)=(a,1) \text{ or } (1,a) \\ b & \text{if } (x,y)=(b,1) \text{ or } (1,b) \end{cases}$$

and $t : K \to K$ by

$$t(x,y) = \begin{cases} 0 & \text{if } x=0 \\ 0 & \text{if } x=a \\ b & \text{if } x=b \\ b & \text{if } x=1 \end{cases}$$

Then it is easily checked that d_t symmetric bi-t-derivation of an incline algebra K.
But d_t is not a symmetric bi-derivation since

$$0 = d_t(b*a,b) = d_t(a,b) \neq (d_t(b,b)*a) + (b*d_t(a,b) = (b*a) + (b*0) \qquad = a+0 = a$$

Proposition 3.3

Let K be an incline algebra and let d_t be a symmetric bi-t-derivation on K. Then the following identities hold for all $\forall x, y, z \in K$
$(i)\ d_t(x*y,z) \leq t(x) + t(y)\ \forall x, y, z \in K$
$(ii) d_t(x*y,z) \leq d_t(x,z) + d_t(y,z)\ \forall x, y, z \in K$

Proof:

Let K be an incline algebra.
let d_t be a symmetric bi-t-derivation on K.
(i) Let $x, y, z \in K$
By using $(K9)$we have
$d_t(x,z)*t(y) \leq t(y)$ and $t(x)*d_t(y,z) \leq t(x)$
By using$(K11)$ we have

$$d_t(x,z)*t(x) + t(x)*d_t(y,z) \leq t(x) + t(y)$$

Hence $\quad d_t(x*y,z) \leq t(x) + t(y)$

(ii) Let $x, y, z \in K$
By using $(K9)$we have
$d_t(x,z)*t(y) \leq d_t(x,z)$
and $t(x)*d_t(y,z) \leq d_t(y,z)$
By using $(K11)$ we have
$d_t(x,z)*t(y) + t(x)*d_t(y,z) \leq d_t(x,z) + d_t(y,z)$

Hence $d_t(x*y,z) \le d_t(x,z) + d_t(y,z)$

Proposition 3.4

Let K be an incline algebra. If K is a distributive lattice,
we have $d_t(x,y) \le t(x)$ and $d_t(x,y) \le t(y)$ for all $x, y \in K$.

Proof:
Let K be a distributive lattice.

Then $d_t(x,y) = d_t(x*x,y)$
$= d_t(x,y)*t(x) + (t(x)*d_t(x,y))$

And so $d_t(x,y)+t(x) = (d_t(x,y)*t(x) + \big(t(x)*d_t(x,y)\big) + t(x)$
$= \big(d_t(x,y)*t(x) + d_t(x,y)*t(x)\big) + t(x)$
$= \big(d_t(x,y)*t(x)\big) + t(x) for all x, y \in K.$

By using $(K8)$,
We get $d_t(x,y)+t(x) = t(x)$
Hence we obtain $d_t(x,y) \le t(x)$.
Similarly, we have $d_t(x,y) \le t(y)$.

Definition 3.5

Let K be an incline algebra and let d_t be a symmetric bi-t-derivation on K. If $x \le w$ implies $d_t(x,y) \le d_t(w,y)$. d_t is called an *isotone* symmetric bi-t-derivation for all $x, y, z \in K$.

Note
Every distributive lattice is an incline algebra under addition (resp. multiplication).
An incline algebra is a distributive lattice if and only if $x*x = x$ for all $x \in K$.

Proposition 3.6

Let K be an incline algebra and let d_t a joinitive symmetric bi-t-derivation d_t of K. Then d_t is an isotone symmetric bi-t-derivation of K

Proof:

Let x and w be such that $x \le w$.

Then $x + w = w$ and so

$d_t(w,y)$=$d_t(w+x,y) = d_t(w,y) + d_t(x,y)$

This implies that $d_t(x,y) \le d_t(w,y)$.

This completes the proof.

Let d_t be a symmetric bi-t-derivation of K.

Fix $a \in K$ and define a set $T_a(K)$ by

$T_a(K) = \{x \in K / d_t(x,a) = t(x)\} \quad \forall\, x \in K.$

Definition 3.7

Let K be an incline algebra and $d_t: K \times K \to K$ be a symmetric mapping then $Ker_{dt}(K)$ is defined by

$$Ker_{dt}(K) = \{x \in K / d_t(0,x) = 0\}$$

Proposition 3.8

Let K be an incline algebra and let$d_t$be a joinitive symmetric bi-t-derivation of K. If $x \leq y$ and $y \in Ker_{dt}(K)$ then we have $x \in Ker_{dt}(K)$.

Proof:

Let $x, y \in Ker_{dt}(K)$.
Then $d_t(0, x) = d_t(0, y) = 0$ and so

$$d_t(0, x * y) = d_t(x * y, 0) = d_t(x, 0) * t(y) + t(x) * d_t(y, 0)$$

$$= 0 * t(y) + t(x) * 0 = 0 + 0$$

$$= 0$$

Proposition 3.9
Let K be an incline algebra and let d_t be a joinitive symmytric bi-t-derivation of K. Then Ker_{d_t} is sub incline of K.

Proof:
Let $x, y \in Ker_{d_t}(K)$.
Then $d_t(0, x) = d_t(0, y) = 0$, and so

$$d_t(0, x * y) = d_t(x * y, 0) = d_t(x, 0) * t(y) + t(x) * d_t(y, 0)$$
$$= 0 * t(y) + t(x) * 0 = 0 + 0$$
$$= 0.$$

Which implies $x * y \in Ker_{dt}(K)$.
Now $d_t(x + y, 0) = d_t(x, 0) + d_t(y, 0) = 0 + 0 = 0$
Hence $x + y \in Ker_{dt}(K)$.
This completes the proof.

Theorem 3.10
Let $d_t$be a joinitive symmetric bi-t-derivation of K. Then $Ker_{dt}(K)$ is an ideal of K.

Proof:
By Proposition 3.9, $Ker_{dt}(K)$ is an subincline of K. Let $x \leq y$ and $y \in Ker_{dt}(K)$.
Then $x + y = x$ and $d_t(0, y) = 0$.
Thus

$$0 = d_t(0, y) = d_t(0, x + y) = d_t(0, x) + d_t(0, y)$$
$$= d_t(0, x) + 0 = d_t(0, x)$$

Which implies $x \in Ker_{dt}(K)$.

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