Path Union and Double Path Union of Cordialgraphs.

Mukund V.Bapat[1]

1.Abstract:

In this paper we show path union families of kite and flag of S_4 (also called as dart) are cordial graphs. Pm(flag-S_4) has 8 different structures and all are cordial.We also show double path union of graph G i.e.Pm(2-G) are cordial for G =C_4,flag of C_3,flag of C_4 .In allcases we consider all possible structures and establish invariance under cordiality.

Key words :

path union, cordial,graph,double path,flag, kite,invariance.

Subject classification : 05C78.

2.Introduction:

The graphs we consider are simple ,finite,undirected and connected.For terminology and definitions we depend on Graph Theory by Harary [6],A dynamic survey of graph labeling by J.Gallian [8] and Douglas West.[9].I.Cahit introduced the concept of cordial labeling[5].f:V(G)→{0,1} be a function.From this label of any edge (uv) is given by |f(u)-f(v)|.Further number of vertices labeled with 0 i.e$v_f(0)$ and the number of vertices labeled with 1 i.e.$v_f(1)$ differ at most by one .Similarly number of edges labeled with 0 i.e.$e_f(0)$ and number of edges labeled with 1 i.e.$e_f(1)$ differ by atmostone.Then the function f is called as cordial labeling.Cahit has shown that : every tree is cordial; Kn is cordial if and only if $n \leq 3$; $K_{m,n}$ is cordial for all m and n; the friendship graph $C_3^{(t)}$ (i.e., the one-point union of t copies of C_3) is cordial if and only if tis not congruent to 2 (mod 4); all fans are cordial; the wheel W_n is cordial if and only if n is not congruent to 3 (mod 4).A lot of work has been done in this type of labeling.One may refer dynamic survey by J.Gallian[8].

We have defined path unions on different graphs inparticular on bull graph.For the same given graph there are many path union $P_m(G)$ structures possible.It depends on which point on G is used to fuse with vertex on P_m.If this point is changed and path union is designed then it may be a different (up to isomorphism) structure.We have shown that for G = bull on C_3,bull on C_4,C_3^+,C_4^+-e the different path union $P_m(G)$ are cordial[4].It is called as invariance under cordial labeling.Weusethe convention that $v_f(0,1)$ = (a,b) to indicate the number of vertices labeled with 0 are a and that number of vertices labeled with 1 are b.Further $e_f(0,1)$ = (x,y) we mean the number of edges labeled with o are x and number of edges labeled with 1 are y.

3.Preliminaries:

The Bull graph on C_3 is defined in [9].We generalize the definition of bull graph. Let G be a (p,q) graph.Choose any two adjacent vertices of G say u and v. Attach an pendent edge each to u and v .It is denoted by bull(G). Thus bull graph of G has P+2 vertices and q+2 edges.

4.Definitions:

4.1 Fusion of vertices.Let u ≠v be any two vertices of G. We replace these two vertices by a single vertex say x and all edges incident to u and v are now incident to x.If loop is formed then it is deleted.

4.2 Flag of a graph G denoted by FL(G) is obtained by taking a graph G=G(p,q).At sutable vertex of G attach a pendent edge.It has p+1 vertices and q+1 edges.

4.3 Path union : $P_m(G)$ is obtained by taking a path on m points and m copies of G are taken.At each vertex of path a copy each of G is fused. The point of fusion on G is same and fixed for all copies of G.

4.4 Double path union :Let G be a (p,q) graph.$P_m(2\text{-}G)$ is obtained by taking a path on m points and 2m copies of G are taken.At each vertex of path two copies of G are fused. The point of fusion on G is same and fixed for all copies of G.It has m(2p-1) vertices and m(2q)+m-1 edges.

5.Theorems Proved:

5.1Theorem : double Path union of C_4i.e $P_m(2\text{-}C_4)$ is cordial.

Proof:

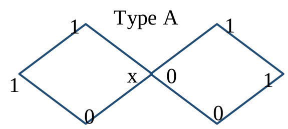

Fig 5.1 $v_f(0,1)=(3,4)$, $e_f(0,1)=(4,4)$

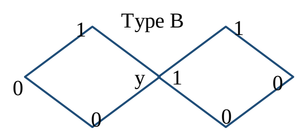

Fig 5.2 $v_f(0,1)=(4,3)$, $e_f(0,1)=(4,4)$

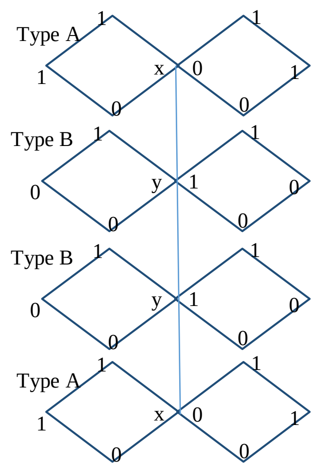

Fig 5.3 : $P_4(2\text{-}C_4)$: $v_f(0,1)=(14,14)$, $e_f(0,1)=(4,4)$ edgelabel (xy)=edge label(yx) = 1;edge label (xx) = edge label (yy) =0

Define a function f:V(G)$\rightarrow${0,1} as follows:

We define two types of labels Type A and Type B as above. Both are cordial labelings but they differ in label of x and y and on label numbers on vertices. Both types has same label numbers (4,4) on edges.To obtain a $P_m(2\text{-}C_4)$ we start with a path $p_m=(v_1,v_2,\ldots v_m)$ and at each of it's node fuse one of the two types of libeling. Type A is fused at vertex 'x'and Type B is fused at vertex 'y' on it.For i≡ 1,4 (mod4) use type A labeling for i≡ 2,3(mod4) use type B labeling.

The label numbers observed are for vertices we have $v_f(0,1)=(7x,7x)$.when m = 2x.

$v_f(0,1)=(14x+3,14x+4)$.For… m ≡1(mod 4),(m = 4x+1)

$v_f(0,1)=(14x+11,14x+10)$….for m≡3 (mod 4),(m=4x+3)

For edges $e_f(0,1)=$ (8+9x,9+9x) for m ≡2,0 (mod 4),write m =4x or 4x+2, x= 0,1,

$e_f(0,1)=$ (4+9x,4+9x) if m≡1,3 (mod4),write m = 4x+1 or 4x+ 3,,x=0,1,..

Thus$P_m(2\text{-}C_4)$ is cordial. #

5.2 Theorem : Double path union on flag of C_4 (i.e. G = $P_m(2\text{-}flag\ C_4)$) is cordial.(We consider all possible fournon isomorphic structures)

Proof: To obtain G we start with path $P_m = (v_1, v_2, \ldots v_m)$ and at each vertex on P_m fuse two copies of flag of C_4. There are four possible structures of path union depending on the vertex on flag of C_4 used to fuse with vertex on P_m.

Refer fig 5.4.
If the vertex **a** or**c** on 2-flag C_4 is used to fuse with v on P_m we get structure 1.
If the vertex **b**on 2-flag C_4 is used to fuse with v on P_m we get structure 2.
If the vertex **d** on 2-flag C_4 is used to fuse with v on P_m we get structure 3.
If the vertex **e**on 2-flag C_4 is used to fuse with v on P_m we get structure 4.

Define a function $f:V(G) \rightarrow \{0,1\}$ as follows:

We give below two types of labeling units type A and type B and use them in a certain way to obtain labeled copy of G. To obtain any of the 4 structures we proceed as follows. Westart with path P_m and type **A** is fused (at vertex x on it) at vertex v_i of $P_m = (v_1, v_2, \ldots v_m)$ when $i \equiv 1,0 \pmod 4$; type **B** is fused (at vertex y on it) at vertex v_i of P_m when $i \equiv 2,3 \pmod 4$.

The labelnumber distribution is as follows:on vertices when m = 2x we have $v_f(0,1)=(9x,9x)$.and when m is odd number given by m= 2x+1 we get $v_f(0,1)=(18x+4,18x+5)$.
For edges $e_f(0,1) = (5m+x1,5m+x)$ for m is even number 2x and $e_f(0,1)= (5m+x,5m+x)$ if m is odd number given by 2x+1. Thus P_m (2-C_4) is cordial.

Structure 1:

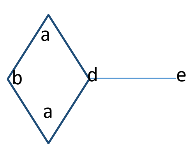

Fig 5.4:Flag(C_4)

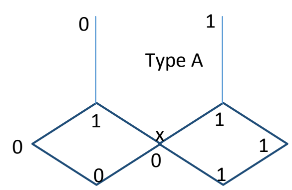

Fig 5.5: P_1(2-Flag(C_4)); $v_f(0,1)=(4,5), e_f(0,1)=(5,5)$

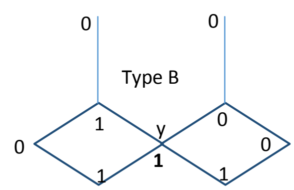

Fig 5.6: P_1(2-Flag(C_4)); $v_f(0,1)=(5,4), e_f(0,1)=(5,5)$

Structure 2:

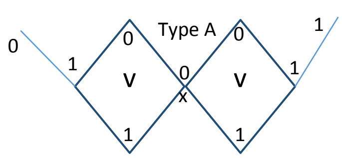

Fig 5.7: P_1(2-Flag(C_4)); $v_f(0,1)=(4,5), e_f(0,1)=(5,5)$

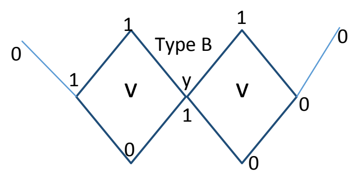

Fig 5.8: P_1(2-Flag(C_4)); $v_f(0,1)=(5,4), e_f(0,1)=(5,5)$

Structure 3:

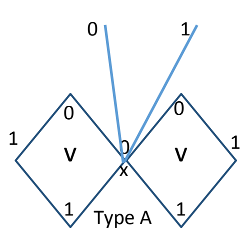

Fig 5.9: P_1(2-Flag(C_4)); $v_f(0,1)=(4,5), e_f(0,1)=(5,5)$

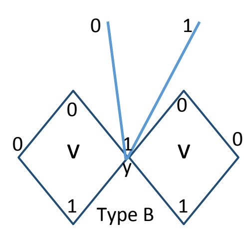

Fig 5.10: P_1(2-Flag(C_4)); $v_f(0,1)=(5,4), e_f(0,1)=(5,5)$

Structure 4:

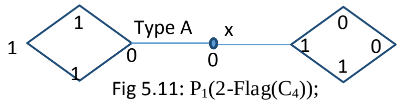

Fig 5.11: P_1(2-Flag(C_4)); $v_f(0,1)=(4,5)$, $e_f(0,1)=(5,5)$

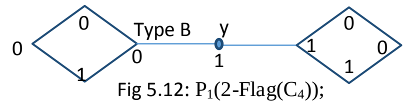

Fig 5.12: P_1(2-Flag(C_4)); $v_f(0,1)=(5,4)$, $e_f(0,1)=(5,5)$

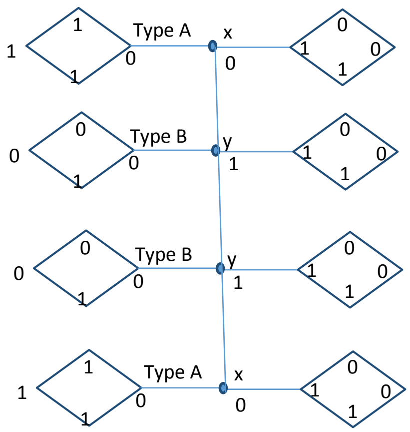

Fig 5.12: P_4(2-Flag(C_4)); $v_f(0,1)=(5,4)$, $e_f(0,1)=(18,18)$

5.3Theorem: Double Path union of C_3-flag (i.e.G = P_m(2-flag C_3) is cordial.

Proof :To obtain a double path union of C_3 flag i.e.P_m(2 -Flag(C_3)) we start with a path $P_m=(v_1,v_2,\ldots v_m)$ and at each of it's vertex we fuse two copies of C_3 flag,with same fixed point on flag.There are three structurespossible on P_m(2-Flag(C_3)) namely structure 1 ,structure2,structure3.For structure 1 the two degree vertex on Flag(C_3) is used to fuse with vertex of path Pm. Vertices and edges of two copies of flag(C_3) attached at i^{th}(i = 1,2,..m) vertex v_i of path P_m is as shown in figure 5.1.

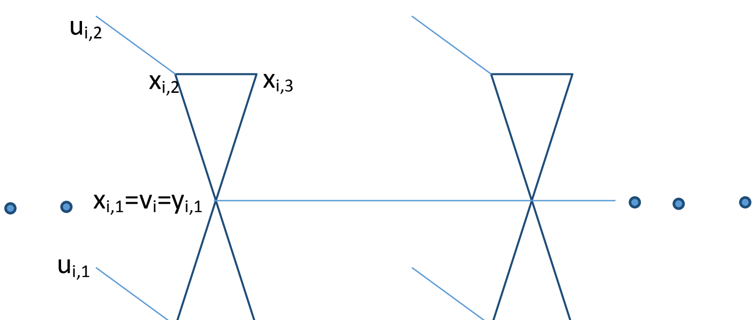

Fig 5.13 P_m(2-Flag(C_3)){unlabeled copy}

Define a function f : V(G)→ {0,1} as follows

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Wedefine two types of labels type A and type B as follows.All are cordial labelings but they differ either in label of x and y or on label numbers on vertices.All four types has same label numbers (4,4) on edges.Type A and Type B is used as follows:For i≡1,0 (mod 4) use Type A and for i≡2,3(mod 4) use B type label at the vertex vi

Structure 1:

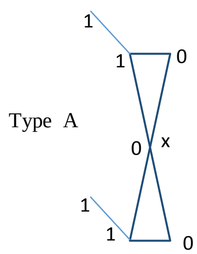


Fig 5.14: $P_m(Flag(C_3))$; $vf(0,1)=(3,4), ef(0,1)=(4,4)$

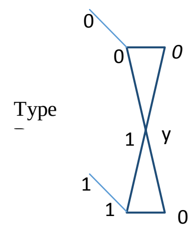


Fig 5.15: $P_m(Flag(C_3))$; $vf(0,1)=(4,3), ef(0,1)=(4,$

Structure 2:

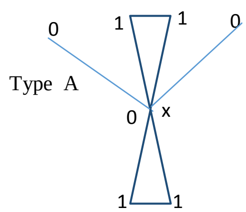


Fig 5.16: $P_m(Flag(C_3))$; $vf(0,1)=(3,4), ef(0,1)=(4,4)$

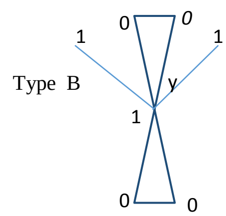


Fig 5.17: $P_m(Flag(C_3))$; $vf(0,1)=(4,3), ef(0,1)=(4,4)$

Structure 3

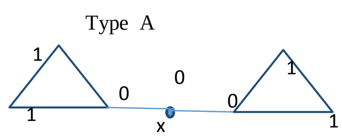


Fig 5.18: $P_1(Flag(C_3))$; $v_f(0,1)=(3,4), e_f(0,1)=(4,4)$

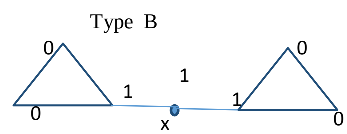


Fig 5.19: $P_1(Flag(C_3))$; $v_f(0,1)=(4,3), e_f(0,1)=(4,4)$

The label number distribution is as follows:

On vertices when $m\equiv 2x$ we have , $v_f(0,1) = (7x,7x)$ and when $m \equiv 1 (\mathrm{mod}\ 4)$ write $m = 1+4x$ We have $v_f(0,1)=(3+14x,4+14x)$ and when $m\equiv 3(\mathrm{mod}\ 4)$ write $m = 3+4x$ and we have $v_f(0,1)=(11+14x,10+14x)$.On edges we have $e_f(0,1)=(4+9x,4+9x)$ when $m =2x+1$,x=0,1,2…. And when $m = 2x$ we have $e_f(0,1)=(8+9x,9+9x)$

5.4Theorem: Path union of kite (i.e. P_m(kite) graph is cordial

Proof: A path union on Kite is obtained by first taking a path $P_m=(v_1,v_2,\dots v_m)$ and m labeled copies of Kite as shown below.We fuse a suitable labeled copy of kite at each vertex of path P_m. Define a function $f:V(G)\rightarrow\{0,1\}$ as follows.:A labeled copy of kite has 4- vertices named as a, b,c,d with labels 1,1,0,0 respectively. Thereare two structures possible on path union .Structure 1 is obtained by fusing vertex b or d at the vertex of path $P_m =(v_1,v_2,\dots v_m)$ And structure 2 is obtained by fusing vertex a or c on kiteat the vertex of path $P_m =(v_1,v_2,\dots v_m)$

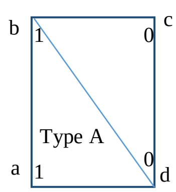


Fig 5.20 : labeled copy of kite label numbers $vf(0,1) = (2,2)$, $e_f(0,1) =(2,3)$

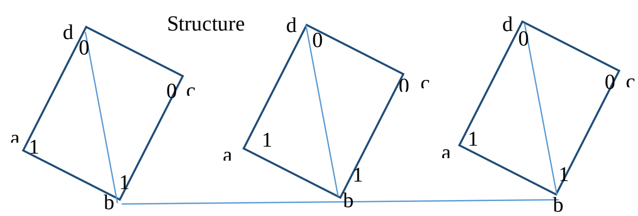


Fig 5.21 : P_3(kite): labeled copy :label of edge (bb) = 0

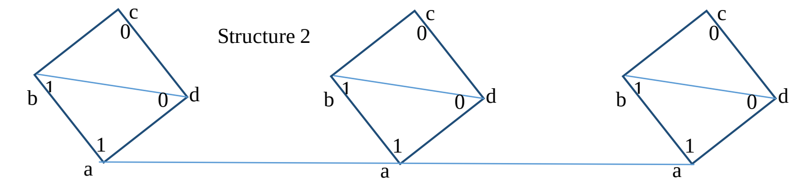


Fig 5.22: P_3(kite): labeled copy :label of edge (aa) = 0

For both structures the label number distribution is $v_f(0,1)=(2m,2m)$ and $e_f(0,1)=(3m\text{-}1,3m)$.The graph is cordial.

Theorem 4 :Double path union of kite (i.e.G = P_m(2-kite))is cordial.

Proof.There are two structures possible on G depending on which vertex of kite is used to fuse with the vertex of path P_m.Define a function $f:V(G)\rightarrow\{0,1\}$ as follows:

Structure 1

We give two types (units) of labels which are used suitably to fuse at vertex on path Pm.Point of fusion is point x on the unit.

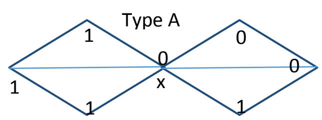


Fig 5.23 : labeled copy of P_1(2- kite) label numbers $v_f(0,1) = (3,4)$, $e_f(0,1) =(5,5)$

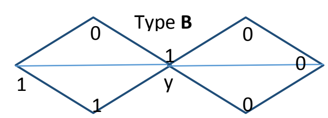


Fig 5.24 : labeled copy of P_1(2- kite) label numbers $v_f(0,1) = (4,3)$, $e_f(0,1) = (5,5)$

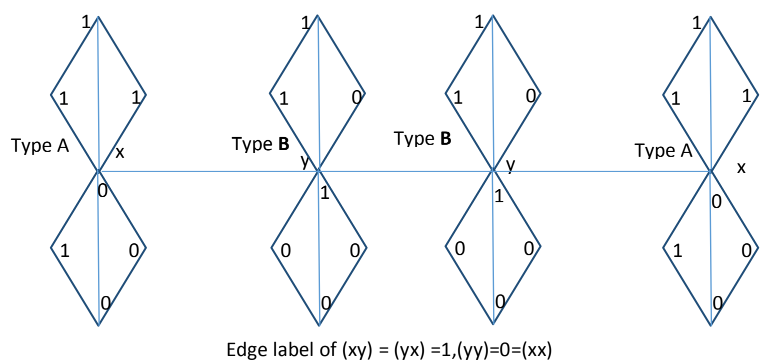


Fig 5.25: P_4(2-kite):label numbers $v_f(0,1) = (14,14)$, $e_f(0,1) =(21,22)$

Structure 2:

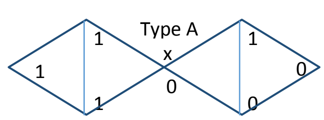


Fig 5.26 : labeled copy of P_1(2- kite) label numbers $v_f(0,1) = (3,4)$, $e_f(0,1) =(5,5)$

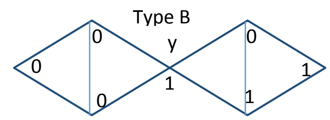
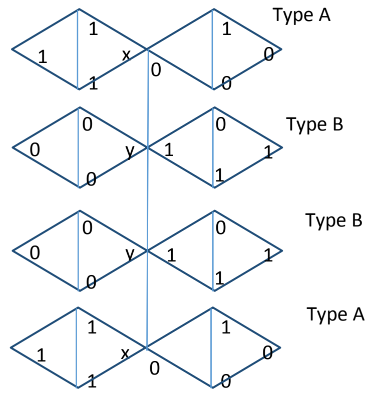


Fig 5.27 : labeled copy of P_1(2- kite) label numbers $v_f(0,1) = (4,3)$, $e_f(0,1) =(5,5)$



Edge label of (xy) = (yx)

Fig 5.28: P_4(2-kite):label numbers $v_f(0,1)$ = (14,14), $e_f(0,1)$ =(21,22)

To construct both structureswe use type A label on vi of path Pm $=(v_1,v_2,\ldots v_m)$ when i≡1,0(mod 4) and type B is used when i≡2,3(mod 4).

For both structures we observe number distribution as follows:

$v_f(0,1)$ =(7x,7x)for m is even number 2x and when m is of type 4x + 1 we have $v_f(0,1)$ = (3+14x,4+14x)and when mis of type m = 4x +3 we have $v_f(0,1)$ = (11+14x,10+14x) To obtain edge numbers case 1:$e_f(0,1)$=(5+11x,5+11x) when m is odd number given by m =2x case 2:m is of type m = 2x we have $e_f(0,1)$=(11x+10,11x+11). Thus the graph is cordial.#

5.5Theorem :Path union of flag of S_4 (i.e.G = P_m(flag S_4) is cordial

Proof: Define a function f:V(G)$\rightarrow$\{0,1\} as follows: We obtain two types of labeled units type A and type B as shown below. In all 8 structure we fuse Type A at vertex v_i of Pm=$(v_1,v_2, v_3, \ldots,v_m)$ where i≡1,3 (mod4) and Type B when i≡2, 3 (mod 4) .There are eight non- isomorphic structures possible.

Structure 1

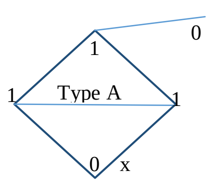


Fig 5.29$v_f(0,1)$=(2,3),$e_f(0,1)$=(3,3)

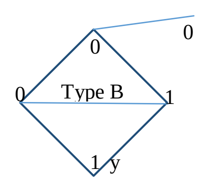


Fig 5.30$v_f(0,1)$=(2,3),$e_f(0,1)$=(3,3)

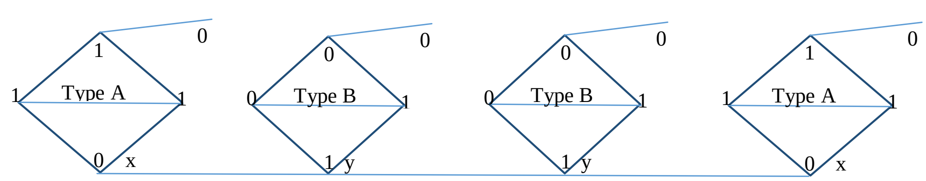


Fig 5.31P_4(flag S_4):label(xy)=label(yx) = 1;label(yy)=label(xx)=0; structure 1

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Structure 2

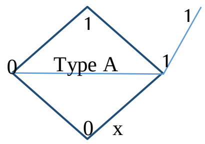

Fig 5.32 $v_f(0,1)= (2,3),e_f(0,1)= (3,3)$

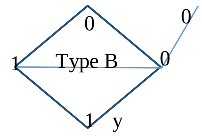

Fig 5.33 $v_f(0,1)=(3,2),e_f(0,1)=(3,3)$

Structuire 3

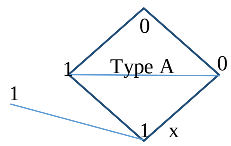

Fig 5.34 $v_f(0,1)= (2,3),e_f(0,1)= (3,3)$

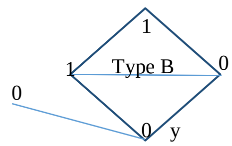

Fig 5.34 $v_f(0,1)= (3,2),e_f(0,1)= (3,3)$

Structure 4

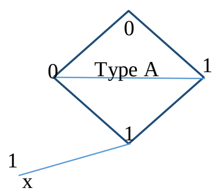

Fig 5.35 $v_f(0,1)= (2,3),e_f(0,1)= (3,3)$

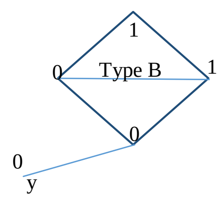

Fig 5.36 $v_f(0,1)= (3,2),e_f(0,1)= (3,3)$

Structure 5

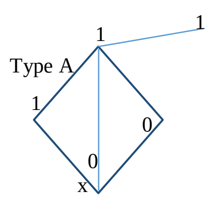

Fig 5.37 $v_f(0,1)= (2,3),e_f(0,1)= (3,3)$

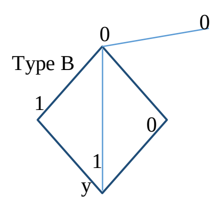

Fig 5.38 $v_f(0,1)= (3,2),e_f(0,1)= (3,3)$

Structure 6

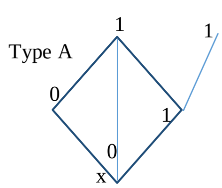

Fig 5.39 $v_f(0,1)= (2,3),e_f(0,1)= (3,3)$

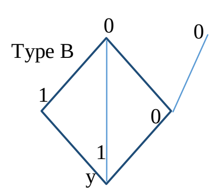

Fig 5.40 $v_f(0,1)= (3,2),e_f(0,1)= (3,3)$

Structure 7

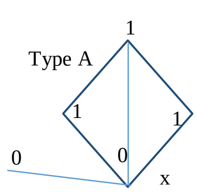

Fig 5.41 $v_f(0,1)= (2,3),e_f(0,1)= (3,3)$

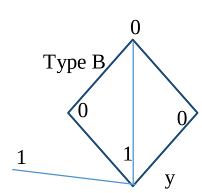

Fig 5.42 $v_f(0,1)= (3,2),e_f(0,1)= (3,3)$

Structure 8

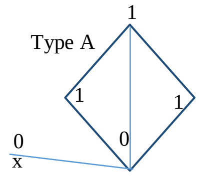

Fig 5.43 $v_f(0,1)$= (2,3),$e_f(0,1)$= (3,3)

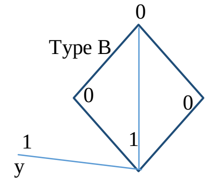

Fig 5.44 $v_f(0,1)$= (3,2),$e_f(0,1)$= (3,3)

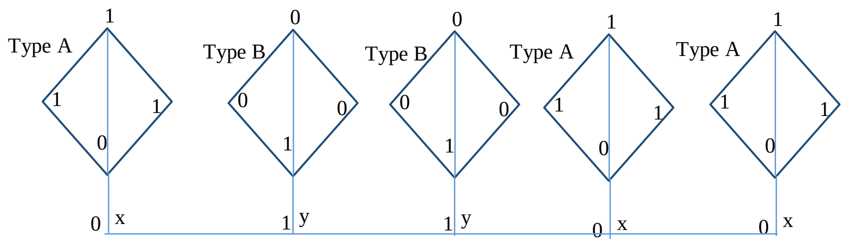

Fig 5.45: labeled copy P_5((flag S_4) : $v_f(0,1)$=(12,13): ,$e_f(0,1)$= (17,17): structure 8

To construct labeled copy of path union from labeled units typeAandTypeB the vertex 'x' on type A and vertex 'y ' on type B is used. We observe label number distribution as follows:

On edges $e_f(0,1)$=(3+7x),3+7x) …..for odd m = 2x+1.and when m=2x we have ef(0,1) =(6+7x,7+7x)

On vertices $v_f(0,1)$ =(5x+2,5x+3) for m≡1 (mod 4) write m=4x+1 ,x=0,1,2.. And$v_f(0,1)$ =(5x+8,5x+7)…. for m≡3 (mod 4),m=4x+3, x= 0,1,2… vf(0,1) = (5x,5x) when m is even number given by 2x,x=1,2,3…

Thus G is cordial #

6.Conclusions: We define double path union and obtain particular cordial labeling of some families of graph .Also note that double path union Pm(2-G) don't include double path but two copies of G at each node of path Pm. Similarly it is interesting to define similar path unions such as Pm(3-G),..Pm(k-G) etc.and study it for cordial labeling.

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