Ratio Cum Median Based Modified Ratio Estimators with Known Skewness, and Coefficient of Variation

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Abstract

The present paper deals with some ratio cum median based modified ratio estimators forestimating the finite population mean with the linear combinations of known parameters of the auxiliary variable such as skewness, and Coefficient of variation. The efficiencies of the proposed estimators are assessed with that of simple random sampling without replacement (SRSWOR) sample mean and ratio estimator both by algebraically and numerically. From the numerical comparison it is observed that the proposed estimators perform better than the SRSWOR sample mean as well as the Ratio estimator.

Keywords: *Auxiliary variable;Bias;Mean squared error; Natural population; Percentage relative efficiency; Simple random sampling.*

1. INTRODUCTION

Let the finite population with N distinct and identifiable units be represented by $U = \{U_1, U_2, \ldots, U_N\}$. Let Y and X denote the study and auxiliary variables and are taking the values Y_i *and* X_i respectively measured on the population unit $U_i, i = 1, 2, \ldots, N$. In general the purpose of any sample survey is to estimate the unknown population mean $\overline{Y} = \frac{1}{N}\sum_{i=1}^{N} Y_i$ of the study variable by selecting a random sample of fixed size n from the populationU with some desirable properties like unbiasedness, minimum variance etc. The widely used estimator is the simplerandom sampling without replacement (SRSWOR) sample mean which provides an unbiased estimatorfor the population mean in the absence of auxiliary variable. In the presence of an auxiliary variable onecan always to improve the efficiency of SRSWOR sample mean by introducing Ratio estimator provided the auxiliary variable X is positively correlated with that of the study variableY .For a more detailed discussion on ratio estimator and its related problems the readers are referred to Cochran (1977) and Murthy (1967). The efficiency of the ratio estimator can be improved further with the help of known parameters of the auxiliary variable such as, correlation coefficient, coefficient of variation, Skewness, Kurtosis, Quartiles etc. The resulting estimators are called in literature as modified ratio estimators. See for example Adepoju and Shittu (2013), Das and Tripathi (1978), Diana, Giordan and Perri (2011), Gupta and Shabbir (2008a,b), Kadilar and Cingi (2003a,b, 2004, 2005a,b, 2006a,b,c), Koyuncu (2012), Koyuncu and Kadilar (2009a,b), Shittu and Adepoju (2013), Singh and Agnihotri (2008), Singh and Tailor(2003), Sisodia and Dwivedi (1981), Subramani and Kumarapandiyan (2012a,b,c, 2013) and the references cited there in.

Recently Subramani (2013b) has introduced a new median based ratio estimator that uses the population median of the study variableY and shown that the median based ratio estimator performs better than SRSWOR sample mean, ratio estimator and also the linear regression estimator. Later the median based ratio estimator of Subramani (2013b) has been extended and developed the median based modified ratio estimators by Subramani and Prabavathy (2014a,b,c, 2015). Recently Jayalakshmi et.al (2016) and Subramani et.al (2016) have introduced some ratio cum median based modified ratio estimators for estimation of finite population mean with known parameters of the auxiliary variable such as kurtosis, skewness, coefficient of variation and correlation coefficient and their linear combinations. In this paper an attempt is made to introduce some more ratio cum median based modified ratio estimators using some more linear combinations known parameters of the auxiliary variable such asskewness, coefficient of variation .

1.2 **Notations to be used**

- **N − Population size**
- **n − Sample size**

- $\mathbf{f = n/N}$, **Sampling fraction**
- $\delta = \frac{1-f}{n}$, **finite population correction**
- $\bar{Y} = \frac{1}{N}\sum_{i=1}^{N} Y_i$, **Population mean of the study variable**
- $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$, **SRSWOR Sample mean of the study variable (Unbiased)**
- $\bar{X} = \frac{1}{N}\sum_{i=1}^{N} X_i$, **Population mean of the Auxiliary variable**
- $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$, **SRSWOR Sample mean of the Auxiliary variable (Unbiased)**
- $S_y = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(Y_i - \bar{Y})^2}$, **Population standard deviation of Y**
- $S_X = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(X_i - \bar{X})^2}$, **Population standard deviation of X**
- $S_{xy} = E(X-\bar{X})(Y-\bar{Y}) = \frac{1}{N}\sum_{i=1}^{N} X_i Y_i - \bar{X}\bar{Y}$, **Covariance between X and Y**
- $C_X = \frac{S_x}{\bar{X}}$ & $C_y = \frac{S_y}{\bar{Y}}$ − **Co-efficient of variations**
- $\rho = \frac{S_{xy}}{S_x S_y}$ − **Co-efficient of correlation between X and *Y***
- $\beta_{1(x)} = \frac{\mu_3^2}{\mu_2^3}$, **Skewness of the auxiliary variable**
- $\beta_{2(x)} = \frac{\mu_4}{\mu_2^2}$, **Kurtosis of the auxiliary variable, where** $\mu_r = \frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{X})^r$
- Q_1 −**First (lower) quartile of the auxiliary variable**
- Q_3 −**Third (upper) quartile of the auxiliary variable**
- $Q_r = Q_3 - Q_1$, **Inter-quartile range of the auxiliary variable**
- $Q_d = \frac{Q_3 - Q_1}{2}$, **Semi-quartile range of the auxiliary variable**
- $Q_a = \frac{Q_3 + Q_1}{2}$ − **Semi-quartile average of the auxiliary variable**
- $B(.)$ − **Bias of the estimator**
- $MSE(.)$ − **Mean squared error of the estimator**
- $V(.)$ − **Variance of the estimator**
- $\bar{y}$ − **SRSWOR Sample Mean (Unbiased)**
- $\hat{\bar{Y}}_R$ − **Ratio Estimator (Biased)**
- $\hat{\bar{Y}}_i$ − i^{th}**Existing modified ratio estimator of** $\bar{Y}$ **(Biased)**
- $\hat{\bar{Y}}_{p_i}$ − i^{th}**Proposed modified ratio estimator of** $\bar{Y}$ **(Biased)**

Existing Estimators

Simple Random Sampling without Replacement (SRSWOR)

In case of simple random sampling without replacement (SRSWOR), the sample mean $\bar{y}_r$ is used to estimate population mean $\bar{Y}$ which is an unbiased estimator and its variance is given below:

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i \quad (1)$$

$$V(\bar{y}) = \frac{(1-f)}{n} S_y^2 \quad (2)$$

where $f = \frac{n}{N}, S_y^2 = \frac{1}{N-1}\sum_{i=1}^{N}(Y_i - \bar{Y})^2$

Ratio Estimator for Estimation of Population Mean

In the presence of an auxiliary variable X and is positively correlated with the study variable Y the ratio estimator

$$\hat{\bar{Y}}_R = \frac{\bar{y}}{\bar{x}}\bar{X} = \hat{R}\bar{X} \quad (3)$$

The mean squared error of $\hat{\bar{Y}}_R$ is given below:

$$MSE\left(\hat{\bar{Y}}_R\right) = \bar{Y}^2\{C'_{yy} + C'_{xx} - 2C'_{yx}\} \quad (4)$$

where $C'_{yy} = \frac{V(\bar{y})}{\bar{Y}^2}, C'_{xx} = \frac{V(\bar{x})}{\bar{X}^2}, \; C'_{yx} = \frac{Cov(\bar{y},\bar{x})}{\bar{X}\bar{Y}}$

2.PROPOSED ESTIMATORS

In this section, some more ratio cum median based modified ratio estimators with known linear combinations of the known parameters of the auxiliary variable likeskewnessand coefficient of Variation, in line with the ratio cum median based modified ratio estimators by Jayalakshmiet. al (2018). The proposed estimators together with their mean squared errors are given below:

Case i: The proposed estimator with known skewnessβ_1 and thecoefficient of variation $C_x$is

$$\hat{\hat{Y}}_{P_1} = \bar{y}\left\{\alpha_1\left(\frac{C_x M + \beta_1}{C_x m + \beta_1}\right) + \alpha_2\left(\frac{C_x \bar{X} + \beta_1}{C_x \bar{x} + \beta_1}\right)\right\}$$

Case ii: The proposed estimator with known coefficient of variation $C_x$and the skewness $\beta_1$is

$$\hat{\hat{Y}}_{P_2} = \bar{y}\left\{\alpha_1\left(\frac{\beta_1 M + C_x}{\beta_1 m + C_x}\right) + \alpha_2\left(\frac{\beta_1 \bar{X} + C_x}{\beta_1 \bar{x} + C_x}\right)\right\}$$

A.DERIVATION OF BIAS AND MEAN SQUARED ERRORS OF PROPOSED ESTIMATORS

The derivation of the bias and mean squared error of the proposed median based modified ratio estimator $\hat{\hat{Y}}_{p_i}$, $i = 1,2$, are given below:

Consider $\hat{\hat{Y}}_{p_i} = \bar{y}\left[\alpha_1\left(\frac{M+T_i}{m+T_i}\right) + \alpha_2\left(\frac{\bar{X}+T_i}{\bar{x}+T_i}\right)\right]$ (A1)

where $\alpha_1 + \alpha_2 = 1$

$\bar{y}$is the SRSWOR sample mean of the study variable Y
mis the sample median of the study variable Y
Mis the population median of the study variable Y
$T_i$is the parameter or ratio of parameters of the auxiliary variableX

Let $e_0 = \frac{\bar{y}-\bar{Y}}{\bar{Y}}$, $e_1 = \frac{m-M}{M}$ and $e_2 = \frac{\bar{x}-\bar{X}}{\bar{X}}$

$\Rightarrow E(e_0) = 0; \; E(e_1) = \frac{\bar{M}-M}{M} = \frac{B(m)}{M}andE(e_2) = 0$

$$\Rightarrow E(e_0^2) = \frac{V(\bar{y})}{\bar{Y}^2}; E(e_1^2) = \frac{V(m)}{M^2}; E(e_2^2) = \frac{V(\bar{x})}{\bar{X}^2}$$

$$E(e_0 e_1) = \frac{Cov(\bar{y}, m)}{\bar{Y}M};$$

$E(e_0 e_2) = \frac{Cov(\bar{y},\bar{x})}{\bar{Y}\bar{X}}; \; E(e_1 e_2) = \frac{Cov(\bar{x},m)}{\bar{X}M}$

The estimator $\hat{\hat{Y}}_{p_i}$ can be written in terms of e_0, e_1 and e_2 as

$$\hat{\hat{Y}}_{p_i} = \bar{Y}(1+e_0)\left\{\alpha_1\left(\frac{M+T_i}{M(1+e_1)+T_i}\right) + \alpha_2\left(\frac{\bar{X}+T_i}{\bar{X}(1+e_2)+T_i}\right)\right\}$$

$$\Rightarrow \hat{\hat{Y}}_{p_i} = \bar{Y}(1+e_0)\left\{\alpha_1\left(\frac{M+T_i}{(M+T_i)+Me_1}\right) + \alpha_2\left(\frac{\bar{X}+T_i}{(\bar{X}+T_i)+\bar{X}e_2}\right)\right\}$$

$$\Rightarrow \hat{\hat{Y}}_{p_i} = \bar{Y}(1+e_0)\left\{\alpha_1\left(\frac{1}{1+\left(\frac{M}{M+T_i}\right)e_1}\right) + \alpha_2\left(\frac{1}{1+\left(\frac{\bar{X}}{\bar{X}+T_i}\right)e_2}\right)\right\}$$

$$\Rightarrow \hat{\hat{Y}}_{p_i} = \bar{Y}(1+e_0)\left\{\alpha_1\left(\frac{1}{1+\theta_i e_1}\right) + \alpha_2\left(\frac{1}{1+\varphi_i e_2}\right)\right\};$$

$$\text{where } \theta_i = \frac{M}{M+T_i}, \varphi_i = \frac{\bar{X}}{\bar{X}+T_i}$$

$$\Rightarrow \hat{\bar{Y}}_{p_i} = \bar{Y}(1+e_0)\left\{\alpha_1(1+\theta_i e_1)^{-1} + \alpha_2(1+\varphi_i e_2)^{-1}\right\}$$

Neglecting the terms of higher order, we have

$$\hat{\bar{Y}}_i = \bar{Y}\{\alpha_1(1+e_0-\theta_i e_1-\theta_i e_0 e_1+\theta_i^2 e_1^2)+\alpha_2(1+e_0-\varphi_i e_2-\varphi_i e_0 e_2+\varphi_i^2 e_2^2)\}$$

$$\Rightarrow \hat{\bar{Y}}_{p_i} = \bar{Y}\{1+e_0+\alpha_1(-\theta_i e_1-\theta_i e_0 e_1+\theta_i^2 e_1^2)+\alpha_2(-\varphi_i e_2-\varphi_i e_0 e_2+\varphi_i^2 e_2^2)\}$$

$$\Rightarrow \hat{\bar{Y}}_{p_i} - \bar{Y} = \bar{Y}\{e_0-\alpha_1(\theta_i e_1+\theta_i e_0 e_1-\theta_i^2 e_1^2)-\alpha_2(\varphi_i e_2+\varphi_i e_0 e_2-\varphi_i^2 e_2^2)\} \quad \text{(A2)}$$

Taking expectations on both sides of (A2) one can have,

$$E(\hat{\bar{Y}}_{p_i} - \bar{Y}) = \bar{Y}\{E(e_0)-\alpha_1 E(\theta_i e_1+\theta_i e_0 e_1-\theta_i^2 e_1^2)-\alpha_2 E(\varphi_i e_2+\varphi_i e_0 e_2-\varphi_i^2 e_2^2)\}$$

$$\Rightarrow E\left(\hat{\bar{Y}}_{p_i} - \bar{Y}\right) = \bar{Y}\left\{\alpha_1\left(\theta_i^2\frac{V(m)}{M^2}-\theta_i\frac{B(m)}{M}-\theta_i\frac{Cov(\bar{y}, m)}{\bar{Y}M}\right)+\alpha_2\left(\varphi_i^2\frac{V(\bar{x})}{\bar{X}^2}-\varphi_i\frac{Cov(\bar{y},\bar{x})}{\bar{Y}\bar{X}}\right)\right\}$$

$$\Rightarrow B(\hat{\bar{Y}}_{p_i}) = \bar{Y}\left\{\alpha_1\left(\theta_i^2 C'_{mm}-\theta_i C'_{ym}-\theta_i\frac{B(m)}{M}\right)+\alpha_2\left(\varphi_i^2 C'_{xx}-\varphi_i C'_{yx}\right)\right\} \quad \text{(A3)}$$

Squaring on both sides of (A2), neglecting the terms of higher order and taking expectation on both sides one can get,

$$MSE\left(\hat{\bar{Y}}_{p_i}\right) = E\left(\hat{\bar{Y}}_{p_i}-\bar{Y}\right)^2 = \bar{Y}^2 E\{e_0-\alpha_1\theta_i e_1-\alpha_2\varphi_i e_2\}^2$$

$$E\left(\hat{\bar{Y}}_{p_i}-\bar{Y}\right)^2 = \bar{Y}^2\{E(e_0^2)+\alpha_1^2\theta_i^2 E(e_1^2)+\alpha_2^2\varphi_i^2 E(e_2^2)-2\alpha_1\theta_i E(e_0 e_1)-2\alpha_2\varphi_i E(e_0 e_2)+2\alpha_1\alpha_2\theta_i\varphi_i E(e_1 e_2)\}$$

After a little algebra, the mean squared error of $\hat{\bar{Y}}_{p_i}$ is obtained as

$$MSE\left(\hat{\bar{Y}}_{p_i}\right) = \bar{Y}^2\left\{\frac{V(\bar{y})}{\bar{Y}^2}+\alpha_1^2\theta_i^2\frac{V(m)}{M^2}-2\alpha_1\theta_i\frac{Cov(\bar{y},m)}{\bar{Y}M}+\alpha_2^2\varphi_i^2\frac{V(\bar{x})}{\bar{X}^2}-2\alpha_2\varphi_i\frac{Cov(\bar{y},\bar{x})}{\bar{Y}\bar{X}}+2\alpha_1\alpha_2\theta_i\varphi_i\frac{Cov(m,\bar{x})}{M\bar{X}}\right\}$$

That is,

$$MSE\left(\hat{\bar{Y}}_{p_i}\right) = \bar{Y}^2\{C'_{yy}+\alpha_1^2\theta_i^2 C'_{mm}+\alpha_2^2\varphi_i^2 C'_{xx}-2\alpha_1\theta_i C'_{ym}-2\alpha_2\varphi_i C'_{yx}+2\alpha_1\alpha_2\theta_i\varphi_i C'_{xm}\} \quad \text{(A4)}$$

Theorem 1:In SRSWOR, ratio cum median based modified ratio estimator

$\hat{\bar{\mathbf{Y}}}_{\mathbf{P_5}} = \bar{\mathbf{y}}\left\{\alpha_1\left(\frac{C_x M+\beta_1}{C_x m+\beta_1}\right)+\alpha_2\left(\frac{C_x\bar{X}+\beta_1}{C_x\bar{x}+\beta_1}\right)\right\}$ where $\alpha_1+\alpha_2=1$ for the known parameter β_1 and C_x is not an unbiased estimator for its population mean $\bar{Y}$ and its bias and MSE are respectively given as:

$$B\left(\hat{\bar{Y}}_{p_5}\right) = \bar{Y}\left\{\alpha_1\left(\theta_i^2 C'_{mm}-\theta_i C'_{ym}-\theta_i\frac{B(m)}{M}\right)+\alpha_2\left(\varphi_i^2 C'_{xx}-\varphi_i C'_{yx}\right)\right\}$$

$$MSE\left(\hat{\bar{Y}}_{p_5}\right) = \bar{Y}^2\{C'_{yy}+\alpha_1^2\theta_i^2 C'_{mm}+\alpha_2^2\varphi_i^2 C'_{xx}-2\alpha_1\theta_i C'_{ym}-2\alpha_2\varphi_i C'_{yx}+2\alpha_1\alpha_2\theta_i\varphi_i C'_{xm}\},$$

$$\text{where } \theta_i = \frac{C_x M}{C_x M+\beta_1}, \varphi_i = \frac{C_x\bar{X}}{C_x\bar{X}+\beta_1}$$

Proof: By replacing $T_i = \beta_1/C_x$ in Theorem A the proof follows.

Theorem 2: In SRSWOR, ratio cum median based modified ratio estimator

$\hat{\bar{Y}}_{P_6} = \bar{y}\left\{\alpha_1\left(\frac{\beta_1 M + C_x}{\beta_1 m + C_x}\right) + \alpha_2\left(\frac{\beta_1 \bar{X} + C_x}{\beta_1 \bar{x} + C_x}\right)\right\}$ **where** $\alpha_1 + \alpha_2 = 1$ **for the known parameter** C_x **and** β_1 **is not an unbiased estimator for its population mean** $\bar{Y}$ **and its bias and MSE are respectively given as**:

$$B\left(\hat{\bar{Y}}_{p6}\right) = \bar{Y}\left\{\alpha_1\left(\theta_i^{\,2} C'_{mm} - \theta_i C'_{ym} - \theta_i \frac{B(m)}{M}\right) + \alpha_2\left(\varphi_i^{\,2} C'_{xx} - \varphi_i C'_{yx}\right)\right\}$$

$$MSE\left(\hat{\bar{Y}}_{p6}\right) = \bar{Y}^2\{C'_{yy} + \alpha_1^2\theta_i^2 C'_{mm} + \alpha_2^2\varphi_i^2 C'_{xx} - 2\alpha_1\theta_i C'_{ym} - 2\alpha_2\varphi_i C'_{yx} + 2\alpha_1\alpha_2\theta_i\varphi_i C'_{xm}\}, \text{ where}$$

$$\theta_i = \frac{M}{M+\rho}, \varphi_i = \frac{\bar{X}}{\bar{X}+\rho}$$

Proof: By replacing $T_i = C_x/\beta_1$ Theorem A the proof follows.

The above proposed estimators are represented as a more generalized form as given below:

$$\hat{\bar{Y}}_{P_i} = \bar{y}\left\{\alpha_1\left(\frac{M+T_i}{m+T_i}\right) + \alpha_2\left(\frac{\bar{X}+T_i}{\bar{x}+T_i}\right)\right\} \quad (5)$$

where $\alpha_1 + \alpha_2 = 1, \quad i = 1,2$

The MSE of proposed estimator $\hat{\bar{Y}}_{P_i}$ is given as

$$MSE\left(\hat{\bar{Y}}_{p_i}\right) = \bar{Y}^2\{C'_{yy} + \alpha_1^2\theta_i^2 C'_{mm} + \alpha_2^2\varphi_i^2 C'_{xx} - 2\alpha_1\theta_i C'_{ym} - 2\alpha_2\varphi_i C'_{yx} + 2\alpha_1\alpha_2\theta_i\varphi_i C'_{xm}\} \quad (6)$$

$$\text{where } \theta_i = \frac{M}{M+T_i}, \varphi_i = \frac{\bar{X}}{\bar{X}+T_i}, \; C'_{xm} = \frac{Cov(\bar{x}, m)}{M\bar{X}},, T_1 = \beta_1/C_x, T_2 = C_x/\beta_1$$

3.EFFICIENCY COMPARISON

The efficiencies of the proposed estimators are assessed with that of SRSWOR sample mean and ratio estimator in terms of variance/mean squared error. The results are as follows:

3.1Comparison with that of SRSWOR sample mean

Comparing (6) and (2), it is noticed that the proposed estimators perform better than the SRSWOR sample mean

That is

$$MSE\left(\hat{\bar{Y}}_{P_i}\right) \le V(\bar{y}) \text{if} \alpha_1^2\theta_i^2 C'_{mm} + \alpha_2^2\varphi_i^2 C'_{xx} + 2\alpha_1\alpha_2\theta_i\varphi_i C'_{xm} \le 2(\alpha_1\theta_i C'_{ym} + \alpha_2\varphi_i C'_{yx}) \quad (7)$$

3.2Comparison with that of Ratio Estimator

Comparing (4.6) and (1.6), it is noticed that the proposed estimators perform better than the ratio estimator

That is $MSE\left(\hat{\bar{Y}}_{P_i}\right) \le MSE\left(\hat{\bar{Y}}_R\right)$

$$\text{if } \alpha_1^2\theta_i^2 C'_{mm} + (\alpha_2^2\varphi_i^2 - 1)C'_{xx} + 2\alpha_1\alpha_2\theta_i\varphi_i C'_{xm} \le 2[\alpha_1\theta_i C'_{ym} + (\alpha_2\varphi_i - 1)C'_{yx}] \quad (8)$$

3.3NUMERICAL COMPARISON

The efficiencies of proposed ratio cum median based modified ratio estimators with that of existing estimators SRSWOR sample mean and ratio estimator are derived algebraically in the sections 3.1 and 3.2. To support it by means of numerical comparison, data of a natural population from Singh and Chaudhary (1986, page.177) has been considered.

Population Description

X= Area under Wheat in 1971 and Y= Area under Wheat in 1974
The population parameters computed for the above population is given below:

N= 34	**n= 3**	**$\bar{Y}$= 856.4118**
ρ = 0.4491	**M= 767.5**	**$\bar{X}$= 208.8824**
C_x= 0.7205	**β_2=2.9123**	**β_1 = 0.8732**

The variance/mean squared error of the existing and proposed estimators at different values of α_1 and α_2 are given in the following table

Table 1: Mean Squared Errors for different values of α_1 and α_2

Existing Estimators		
SRSWOR Sample mean	$\bar{y}$	**163356.41**
Ratio Estimator	$\hat{\bar{Y}}_R$	**155579.71**

Proposed Estimators			
α_1	α_2	$\hat{\bar{Y}}_{P_1}$	$\hat{\bar{Y}}_{P_2}$
0.1	**0.9**	**140307.40**	**141938.20**
0.2	**0.8**	**129360.83**	**130052.11**
0.3	**0.7**	**119061.45**	**119650.02**
0.4	**0.6**	**110132.20**	**110744.34**
0.5	**0.5**	**101050.76**	**103216.29**
0.6	**0.4**	**97045.99**	**97339.09**
0.7	**0.3**	**92709.2**	**92844.20**
0.8	**0.2**	**89676.25**	**89690.97**
0.9	**0.1**	**88177.00**	**88326.88**

From Table 1, it is observed that the proposed estimators have less mean squared errors than the SRSWOR sample mean and ratio estimator and hence one can easily conclude that the proposed ratio cum median based modified ratio estimators are performing better than the SRSWOR sample mean and ratio estimator for estimating the finite population mean of the study variable.

For the more clarity of assessing the performance ,the percentage relative efficiencies (PRE) of the proposed estimators with respect to the existing estimators are obtained by using the formula
$\mathbf{PRE(e,p)} = \frac{\mathbf{MSE(e)}}{\mathbf{MSE(p)}} * \mathbf{100}$ and are given in the following tables:

Table 2: PRE of proposed estimators with respect to SRSWOR sample mean

α_1	α_2	$\hat{\hat{Y}}_{P_1}$	$\hat{\hat{Y}}_{P_2}$
0.1	0.9	116.43	115.08
0.2	0.8	126.28	125.59
0.3	0.7	137.20	136.50
0.4	0.6	148.33	147.49
0.5	0.5	161.66	158.11
0.6	0.4	168.33	167.79
0.7	0.3	176.20	175.95
0.8	0.2	182.16	181.73
0.9	0.1	185.26	184.84

Table 3: PRE of proposed estimators with respect to Ratio Estimator

α_1	α_2	$\hat{\hat{Y}}_{P_1}$	$\hat{\hat{Y}}_{P_2}$
0.1	0.9	110.88	109.61
0.2	0.8	120.26	119.63
0.3	0.7	130.67	130.03
0.4	0.6	141.27	140.49
0.5	0.5	153.96	150.73
0.6	0.4	160.32	159.83
0.7	0.3	167.81	167.57
0.8	0.2	173.49	173.46
0.9	0.1	176.44	176.14

From Tables 2 and 3 it is observed that the PRE values of the proposed estimators with respect to SRSWOR sample mean and ratio estimator are greater than 100 and hence we conclude that the proposed estimators have greater efficiency.

- In fact the PREs are ranging from
  - 115.08 to 185.26 for the case of SRSWOR sample mean
  - 109.61 to 176.44for the case of ratio estimator

CONCLUSION

The present paper deals with some more ratio cum median based modified ratio estimators with the linear combinations of the known parameters such as skewness and coefficient of variation of the auxiliary variable. The efficiencies of the proposed ratio cum median based modified ratio estimators are assessed algebraically as well as numerically with that of SRSWOR sample mean, and ratio estimator. Further it is shown from the numerical comparison that the PREs of proposed ratio cum median based modified ratio estimators with respect to the existing estimators are more than 100. Hence the proposed ratio cum median based modified ratio estimators may be recommended for the use of practical applications.

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