Porous Curved Pivoted Slider Bearings Lubricated with Micropolar Fluids

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Abstract

In this paper, the performance of the curved pivoted porous slider bearings is studied. The general modified Reynolds-type equation is derived on the basis of micro continuum theory. Microscopic effects generated by micromotions of particles in suspension in a viscous fluid drastically change the character of the flow between the solid walls. The closed form of expressions for the mean film pressure and load carrying capacity, frictional force is obtained. Numerical computations show that the performance of the slider bearings is prepared by the uses of lubricants with additives (micropolar fluids) as compared to the lubricants. Further it is observed that the micropolarity parameters 1 and N increase the pressure, load carrying capacity, frictional force, coefficient of friction and centre of pressure. The effect of porous parameters causes the reduction of pressure and load carrying capacity.

Key words: pivoted slider bearing, micropolar fluids, porous.

1. Introduction

The study of lubrication with micropolar fluids has been a fascinating subject of practical life in recent years. The presence of fluid film greatly reduces the sliding friction between two solid objects. The enormous practical importance of this effect has stimulated a great deal of research both theoretical and experimental. More than twelve decades have passed since the publication of Osborne Reynolds' paper on the theory of hydrodynamic lubrication [1]. Since then, the application of hydrodynamic lubrication theory has played a major role in the development of mechanical systems that are so important for the functioning of modern society. Self lubricated porous bearings, in which pores are impregnated with oil, are extensively used in industrial applications where their low cost makes them economically viable as well as periodic lubrication is restricted or impractical. Porous bearings are usually made from compressed and sintered metal powders(bronze, iron and stainless steel). The sintering process produces a porous structure that can absorb lubricating oil. With the rotation of the shaft, the oil in the pores comes out to lubricate the bearings and return into pores when it stops.

The classical continuum theory could not justify the whole description of flow behaviour of fluids with additives which lead to the development of microcontinuum theory for fluids. "micropolar fluids" is the one such theory attributed to Eringen[2]. The main characteristic of these theory micropolar fluids is the presence of suspended rigid micro structured particles. Two independent kinematic vector fields are introduced in this theory, namely the vector field representing the translation velocity of the fluid particles and secondly the vector field representing the angular velocities of the particles i.e. the micro rotation vector. This theory leads to provide a model for lubricants containing suspended additive particles.

This theory was fully utilized recently to different lubrication problems [3], in which considered lubricant was contaminated by dirt or metal particles or when additives are used. After continuous usage, this theory could yield excellent results showing an increase in the load carrying capacity and a reduction in the coefficient of friction [3]. In recent years there have been numerous analytical studies of porous bearings which include work by Cameron *et.al* [4], Wu [5], Murti [6] and Bhatt [7], Verma *et.al* [8] studied about the porous inclined slider bearings lubricated with micropolar fluid and found that the load capacity of slider bearings is greater for the micropolar fluid as compared to Newtonian fluid. The main objective of this paper is to study the effect of lubrication of micropolar fluids on the hydrodynamic lubrication of curved pivoted porous slider bearings which has not been studied so far.

2. MATHEMATICAL FORMULATION OF THE PROBLEM

The basic equations governing the flow of thin films are micropolar lubricants under the usual assumptions of the lubrication theory for

$$\left(\mu + \frac{\chi}{2}\right)\frac{\partial^2 u}{\partial y^2} + \chi \frac{\partial v_3}{\partial y} - \frac{\partial p}{\partial x} = 0$$
(1)

$$\frac{\partial p}{\partial y} = 0 \tag{2}$$

$$\left(\mu + \frac{\chi}{2}\right)\frac{\partial^2 w}{\partial y^2} - \chi \frac{\partial v_1}{\partial y} - \frac{\partial p}{\partial z} = 0$$
(3)

$$\gamma \frac{\partial^2 v_1}{\partial y^2} - 2 \chi v_1 + \chi \frac{\partial w}{\partial y} = 0$$
(4)

$$\gamma \frac{\partial^2 v_3}{\partial y^2} - 2\gamma v_3 - \chi \frac{\partial u}{\partial y} = 0$$
(5)

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$
(6)

where (u, v, w) are the components of the lubricant in x, y, z directions respectively and (v_1, v_2, v_3) are microrotational velocity components, χ is the spin velocity and γ is the material coefficient.

The physical configuration of the pivoted slider bearing under study is shown in the Figure 1. It consists of two surfaces separated by a lubricant film. The x-axis is taken to lie along the length of the lower plane, while the y-axis is taken to lie across the lubricant film. The upper curved pivoted surface of the bearing is in relative motion with lower porous plane.

The relevant boundary conditions for the velocity and microrotational velocity components are i) at the upper surface (y=h)

$$u=U, w=0, v=0$$
 (7a)

$$v_1 = v_3 = 0$$
 (7b)
ii) at the lower surface (y=0)
 $u=0, v=v^*, w=0$ (8a)
 $v_1 = v_3 = 0$ (8b)

3. SOLUTION OF THE PROBLEM

The solution of the equations (1), (3), (4) and (5) subject to the corresponding boundary conditions given in the equations (7a), (7b), (8a) and (8b) is obtained as

$$u = \frac{1}{\mu} \left(\frac{y^2}{2} \frac{\partial p}{\partial x} + A_{11} y \right) - \frac{2N^2}{m} \left[A_{21} \sinh(my) + A_{31} \cosh(my) \right] + A_{41}$$
(9)

$$w = \frac{1}{\mu} \left(\frac{y^2}{2} \frac{\partial p}{\partial z} + A_{12} y \right) - \frac{2N^2}{m} \left[A_{22} \sinh(my) + A_{32} \cosh(my) \right] + A_{42}$$
(10)

$$v_{1} = \frac{1}{\mu} \left[y \frac{\partial p}{\partial z} + A_{12} \right] + \left[A_{22} \cosh\left(m y\right) + A_{32} \sinh\left(m y\right) \right]$$
(11)

$$v_{3} = \left[A_{21}\cos\left(my\right) + A_{31}\sinh\left(my\right)\right] - \frac{1}{2\mu}\left(y\frac{\partial p}{\partial x} + A_{11}\right)$$
(12)

where

$$A_{11} = 2 \mu A_{21}$$

$$A_{21} = \frac{A_{31} \sinh (mh) - \frac{h}{2 \mu} \frac{\partial p}{\partial x}}{1 - \cos (mh)}$$

$$A_{31} = \frac{1}{\mu} \left\{ -\frac{U}{2} [\cos (mh) - 1] + \frac{h}{2 \mu} \frac{\partial p}{\partial x} \left(\frac{h}{2} [\cosh (mh) - 1] + h - \frac{N^2}{m} \sinh (mh) \right) \right\} \times \frac{1}{A_5}$$

$$A_{41} = \frac{2 N^2}{m} A_{31}$$

$$A_5 = \frac{h}{\mu} \left\{ \sinh (mh) - \frac{2 N^2}{mh} [\cosh (mh) - 1] \right\}$$

in which

$$m = \frac{N}{l}, N = \left(\frac{\chi}{\chi + 2\mu}\right)^{\frac{1}{2}}, l = \left(\frac{\gamma}{4\mu}\right)^{\frac{1}{2}},$$

where *N* is non-dimensional parameter called coupling number because it characterizes the coupling of linear and angular momentum equations. When *N* is identically zero, the equations of linear and angular momentum are decoupled and the equation of linear momentum reduces to the classical Navier-Stokes equations. The parameter *l* is called the characteristic length because it characterizes the interaction between micropolar fluid and the film gap. The dimension of parameter *l* is length and can be identified as a size of microstructure additives present in the lubricant. In the limiting case of $l \Box 0$, the effect of microstructure becomes negligible.

Integrating the equation of continuity (6) with respect to y over the film thickness h, gives

$$\int_{y=0}^{h} \frac{\partial v}{\partial y} dy = -\int_{y=0}^{h} \left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}\right) dy$$
(13)

By replacing the velocity components u and v with their expressions given in equation (9) & (10) and also using the corresponding boundary conditions given in the equations (7a), (7b), (8a) &(8b),equation (13) gives the Reynolds type equation for micropolar fluid in the form

$$\frac{\partial}{\partial x} \left[f\left(N,l,h\right) \frac{\partial p}{dx} \right] + \frac{\partial}{\partial z} \left[f\left(N,l,h\right) \frac{\partial p}{\partial z} \right] = -12 \ \mu \left(v^*\right)_{y=0} + 6 \ \mu U \ \frac{\partial h}{\partial x}$$
(14)

where

$$f(N, l, h) = h^{3} + 12 l^{2} - 6 N^{2} l h^{2} \coth\left(\frac{Nh}{2l}\right)$$
(15)

The flow of micropolar lubricants in the porous matrix is governed by the modified Darcy's law [8]

$$\vec{q} = -\frac{\phi}{\mu + \chi} \nabla p^* \tag{16}$$

Where $\vec{q} = (u^*, v^*, w^*)$ is the modified Darcy's velocity vector, ϕ is the permeability of the porous matrix and p^* is the pressure in the porous matrix. Due to continuity of the fluid in the porous matrix, the pressure p^* satisfies the Laplace's equation

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2} = 0$$
(17)

Integrating with respect to y over the porous layer thickness δ and using the boundary condition of the

solid backing $\left(\frac{\partial p^*}{\partial y} = 0\right)$ at $y = -\delta$, we obtain

$$\left(\frac{\partial p^*}{\partial y}\right)_{y=0} = -\int_{y=-\delta}^0 \left(\frac{\partial^2 p^*}{\partial x^2} + \frac{\partial^2 p^*}{\partial z^2}\right) dy$$
(18)

Assuming that , the porous layer thickness δ to be small and using the continuity condition of pressure $(p=p^*)$ at the interface (y=0) of porous matrix. Equation (18) reduces to

$$\left(\frac{\partial p^{*}}{\partial y}\right)_{y=0} = -\delta \left(\frac{\partial^{2} p}{\partial x^{2}} + \frac{\partial^{2} p}{\partial z^{2}}\right)$$
(19)

then the velocity component of the modified Darcy's velocity v^* at the interface (y=0) is given

$$\left(v^{*}\right)_{y=0} = \frac{\phi \,\delta}{\left(\mu + \chi\right)} \left[\frac{\partial^{2} p}{\partial x^{2}} + \frac{\partial^{2} p}{\partial z^{2}}\right]$$
(20)

Substituting this in equation (14), the dynamic Reynolds equation is obtained in the form

$$\frac{\partial}{\partial x} \left[\left(f\left(N,l,h\right) + \frac{12\,\mu\,\phi\,\delta}{\left(\mu + \chi\right)} \right) \frac{\partial p}{\partial x} \right] + \frac{\partial}{\partial z} \left[\left(f\left(N,l,h\right) + \frac{12\,\mu\,\phi\,\delta}{\left(\mu + \chi\right)} \right) \frac{\partial p}{\partial z} \right] = 6\,\mu U \frac{\partial h}{\partial x} \quad (21) \text{ where}$$

$$f\left(N,l,h\right) = h^{3} + 12\,l^{2}h - 6\,N^{2}lh^{2} \coth\left[\frac{Nh}{2l}\right]$$

Using the following non-dimensional scheme into equation (21)

$$\bar{x} = \frac{x}{L}, \ \bar{h} = \frac{h}{h_0}, \ \bar{l} = \frac{h_0}{l}, \ \bar{p} = \frac{ph_0^2}{\mu UL}, \ \psi = \frac{k\delta}{h_0^3}$$
 (22)

The modified Reynolds equation (21) can be written in a non-dimensional form as

$$\frac{\partial}{\partial x} \left\{ \left(f\left(N, \overline{l}, \overline{h}\right) + 12\psi \left[\frac{1-N^2}{1+N^2} \right] \frac{\partial \overline{p}}{\partial \overline{x}} \right) \right\} = 6 \frac{\partial \overline{h}}{\partial \overline{x}}$$
(23)

where

$$f\left(N,\overline{l},\overline{h}\right) = \overline{h^{3}} + 12\overline{hl^{2}} - 6N\overline{l}\overline{h} \operatorname{coth}\left(\frac{N\overline{h}}{2\overline{l}}\right)$$
(24)

The relevant boundary conditions for the pressure are

$$\overline{p} = 0 \quad \text{at} \quad \overline{x} = 0,1 \tag{25}$$

The solution of equation (23) subject to boundary conditions given in equation (25)

is obtained in the form

$$\overline{p} = \int_{0}^{1} \left(6\,\overline{h} + A \right) \left[\overline{h^{3}} + 12\,\overline{l^{2}}\,\overline{h} - 6\,N.\overline{l.h.} \operatorname{coth} \left(\frac{N\,\overline{h}}{2\,\overline{l}} \right) + 12\,\psi \left\{ \frac{1-N^{2}}{1+N^{2}} \right\} \right]^{-1} d\overline{x}$$
(26)

where

$$A = -\frac{\int_{0}^{1} 6\bar{h} \left[\bar{h}^{3} + 12\bar{l}^{2}h - 6N\bar{l}.\bar{h}^{2} \coth\left(\frac{N\bar{h}}{2l}\right) + 12\psi\left\{\frac{1-N^{2}}{1+N^{2}}\right\}\right]d\bar{x}}{\int_{0}^{1} \left[\bar{h}^{3} + 12\bar{l}^{2}\bar{h} - 6N\bar{l}.\bar{h}^{2} \coth\left(\frac{N\bar{h}}{2l}\right) + 12\psi\left\{\frac{1-N^{2}}{1+N^{2}}\right\}\right]d\bar{x}}$$

The non-dimensional load carrying capacity, \overline{w} is given by

$$\overline{w} = \frac{wh_0}{\mu UL^2} = \int_0^1 \overline{p} . d\overline{x}$$
(27)

The frictional force, \overline{F} per unit width on the sliding pivoted surface at y=H is defined by

$$F = \int_{0}^{1} \left(\tau_{yx} \right)_{y=H}$$
(28)

The non-dimensional frictional force $\overline{F} = \frac{Fh_0}{\mu UL}$

$$\overline{F} = \int_{0}^{1} \left(1 + \frac{\chi}{2\mu} \right) \overline{h} \cdot \left(6h + A \right) \left[\overline{h^{3}} + 12l^{2}\overline{h} - 6l.\overline{h} \coth\left(\frac{N\overline{h}}{2l}\right) + 12\psi\left(\frac{1-N^{2}}{1+N^{2}}\right) \right]^{-1} \times A dx$$
(29)

where

$$A = \left\{ \left[\frac{\left(1 - N^2\right) \sinh\left(mh\right) \cdot \left\langle\cosh\left(mh\right) - 1\right\rangle + h - \frac{N^2}{m} \sinh\left(mh\right)}{\left(1 - \cosh\left(mh\right)\right) \cdot h \cdot \left[\sinh\left(mh\right) - \frac{2N^2}{mh} \left\langle\cosh\left(mh\right) - 1\right\rangle\right]} \right] + N^2 \cosh\left(mh\right) \right\}$$

The coefficient of friction is

$$C = \frac{F}{W}$$
(30)

The locus of the centre of pressure where the resultant force acts is

$$\overline{X} = \frac{X}{L} = \int_{0}^{1} \overline{p} \, \overline{x} \, d\overline{x} \tag{31}$$

4. RESULTS AND DISCUSSION

The performance of curved pivoted porous slider bearings lubricated with micropolar fluids is studied

on the basis of various non-dimensional parameters such as the coupling number, $N \left[= \left(\frac{\chi}{\chi + 2\mu} \right)^{\frac{1}{2}} \right]$, which

characterizes the coupling between Newtonian and microrotational viscosities, the parameter l in which l has the dimension of length and may be considered as chain length of microstructure additives. The effect of

permeability is observed through the non-dimensional permeability parameter, $\psi \left(= \frac{kH}{h_0^3} \right)$ and it is to be

noted that as $\psi \to 0$, the problem reduces to the corresponding solid case and as $\overline{l}, N \to 0$, it reduces to the corresponding Newtonian case.

4.1 PRESSURE

The effect of micropolar lubricants on the variation of the non-dimensional pressure \overline{p} with \overline{x} is depicted in figure 2 for different values of \overline{l} with parameter value N = 0.6, permeability parameter $\psi = 0.01$ and profile parameter $\alpha = 3.0$. It is observed that \overline{p} increases significantly for increasing values of \overline{l} . The variation of non-dimensional pressure \overline{p} with \overline{x} for different values of N is shown in the figure 3 with the parameter values $\overline{l} = 0.02$, $\beta = 0.6$, $\psi = 0.01$ and $\alpha = 3.0$. It is observed that \overline{p} increases significantly for increases of N. The effect of permeability parameter, ψ on the variation of non-dimensional of \overline{p} with \overline{x} is shown in figure 4. It is observed that the increasing values of permeability parameter ψ decreases \overline{p} .

4.2 LOAD CARRYING CAPACITY

The variation of non-dimensional load carrying capacity, \overline{W} with β as a function \overline{l} with the parameters N = 0.6, $\psi = 0.1$ and $\alpha = 3$ is shown in the figure 5. It is observed that \overline{W} increases for increasing values of \overline{l} . Figure 6 predicts the variation of non-dimensional load carrying capacity \overline{W} with $\overline{\beta}$ as a function N with the parameters $\overline{l} = 0.2$, $\psi = 0.01$ and $\alpha = 3$. It is observed that \overline{W} increases for increasing values of N. Figure 7 predicts that the variation of non-dimensional load carrying capacity \overline{W} with β as a permeability parameter ψ with parameters N = 0.6, $\overline{l} = 0.2$ and $\alpha = 3.0$, it is observed that increasing values of permeability parameter ψ decreases, the load carrying capacity \overline{W} as compared to the Newtonian case.

4.3 FRICTIONAL FORCE

Figure 8 shows the variation of non-dimensional frictional force \overline{F} on pivoted slider with the curvature parameter β for different values of \overline{l} . It is observed that frictional force increases with increasing values of \overline{l} . Figure 9 shows the variation of non-dimensional frictional force \overline{F} on pivoted slider with the parameter N for different values of \overline{l} . It is observed that, the frictional force increases with increasing values of Ν.

4.4 COEFFICIENT OF FRICTION

The variation coefficient of friction \overline{C} with the curvature parameter β is depicted in figure 10. It is observed that the coefficient of friction increases for increasing values of \overline{l} . The variation coefficient of friction \overline{C} with the curvature parameter β is depicted in figure 11. It is observed that the coefficient of friction increases for increasing values of N.

4.5 CENTRE OF PRESSURE

The variation of centre of pressure \overline{X} with the curvature parameter, β for different values of \overline{l} is depicted in figure 12. It is observed that the centre of pressure shifts towards the outlet edge for micropolar fluid. The variation of centre of pressure, X with the curvature parameter, β for different values of N is depicted in figure 13. It is observed that the centre of pressure shifts towards the outlet edge for micropolar fluid.

5. CONCLUSIONS

On the basis of Eringen's micropolar fluid theory, this article predicts the effect of micropolar fluid on the curved pivoted porous slider bearings. On the basis of the results computed, the following conclusions would be drawn

- 1. The use of classical lubricants blended with microstructure additives provides an increasing load carrying capacity and decreases the coefficient of friction, hence improves the performance of the curved pivoted slider bearings as compared to the corresponding Newtonian case.
- 2. The presence of porous facing on the bearing surface affects the performance of the bearing.
- 3. The presence of microstructure additives shifts the centre of pressure towards the exit region.

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NOMENCLATURE

a	film thickness ratio $\left(= \frac{h_1}{h_0} \right)$
С	non-dimensional coefficient of friction $\left(= \frac{FL}{Wh_0} \right)$
F	frictional force per unit width
\overline{F}	non-dimensional frictional force $\left(= \frac{Fh_0}{\mu UL} \right)$
h	film thickness
h_0	outlet film thickness
H_0	porous layer thickness
h 1	inlet film thickness
\overline{h}	dimensionless film thickness
1	characteristic length of the polar suspension $\left[= \left(\frac{\gamma}{4 \mu} \right)^{\frac{\gamma}{2}} \right]$
ī	non-dimensional form of $1\left(=\frac{l}{c}\right)$
L	bearing length
Ν	coupling number $\left[= \left(\frac{\gamma}{\gamma + 2\mu} \right)^{\frac{1}{2}} \right]$
р	lubricant film pressure
\overline{p}	non-dimensional fluid film pressure $\left(\frac{ph_0^2}{\mu UL}\right)$
U	sliding velocity
u, v, w	components of fluid velocity in the x, y, z directions, respectively
v_1, v_2, v_3	microrotational velocity components in the x, y and z directions respectively
W	load carrying capacity per unit width
W	non-dimensional load carrying capacity $\left(= \frac{Wh_0^2}{\mu UL^2} \right)$
\overline{X}	non-dimensional centre of pressure
x, y, z	Cartesian co-ordinates
β	pad curvature $\left(\frac{H_c}{h_0}\right)$
γ	viscosity coefficient for micropolar fluids
μ	viscosity coefficient
τ	dimensionless response time
χ	spin viscosity
δ	porous layer thickness
Ψ	permeability parameter $\left(= \frac{kH}{h_o^3} \right)$



Figure 1. Curved Pivoted porous slider bearing with convex pad surface.



values of \overline{l} , with N = 0.6, $\beta = 0.6$, $\psi = 0.01$ and $\alpha = 3.0$.























