A Mathematical Approach to Eliminate the Deficiency of Minerals in Soils using Fuzzy Linear Programming.

[1]Rama S, [2]Anna Rose K.B and[2]Palliyil Stefy Solomon

[1,2]Department of Mathematics, Mount Carmel College, Bengaluru, Karnataka, India

Abstract:

Project planning is the important task in many areas like construction, resource allocation and many. A sequence of activities has to be performed to complete one task. Each activity has its unique processing time and all together to identify the critical activities which affect the completion of the project. In this project the fertility of the soil is to be improved for a better crop production by removing the deficiency of the minerals from the soils by adding appropriate manures according to the requirements. This paper discuss about the fuzzy linear programming problem. In this paper a fuzzy optimization problem has been modelled, which work as pure fuzzy linear programming problem using trapezoidal fuzzy numbers and solved it with the help of fuzzy version of Big M method. By applying the fuzzy concepts, the infeasibility of the optimal solution has been eliminated and an optimal solution has been obtained. The results are tabulated. For numerical study the data for the soils in the northern and southern part of India has been taken.

Keywords: Fuzzy linear programming, deficiency, trapezoidal fuzzy numbers.

1.Introduction:

Agriculture is a major sector of the county, so efforts need to be made to improve it. About 70% of the total population depends on the agriculture and is one of the sources of livelihood. Most of the Indian agriculture depends upon the qualities and extent of soil. So soil fertility has to be improved, but there is limitation in extensive use of chemical fertilizers due to lack of adequate soil moistures. Due to chemical fertilizers acidification of soil and minerals depletion of the soil may occur.

Any real life problem with specific objective function and system of constraints, several mathematical methods and techniques would be applied. Linear programming is one of the method that is used most commonly. Linear programming is a technique for determining an optimum schedule of interdependent activities with available resources. The main objective of the linear programming problem is to find out the minimum or maximum value of a linear function under some linear constraints. The mathematical formulation of linear programming problem is

$$\text{Min } Z=C\,X$$

$$AX <=> b \text{ and } X \geq 0 \qquad (1)$$

whereC-cost matrix, X-variable matrix,b-column vector, Z -objective function and A-coefficient matrix of coefficient

But in real world situations there exists uncertainties and ambiguity in system of data. These uncertainties occur due to lack of knowledge expression and preference of human judgement. Due to these

uncertainties the fuzzy concept has been introduced. Zimmermann categorized these uncertainties as fuzziness and proposed the linear programming problem formulation. Further fuzzy constraint coefficient in linear programming was considered by Fang and HU. Fuzziness can be categorized into two types namely ambiguity and vagueness. The description of uncertainty which is associated with linguistic information can be done by vagueness. Ambiguity is specified to the situation when the choice between two or more alternatives is left unspecified and the occurrence of the individual alternative is unknown payable to insufficiency in knowledge and tools. Ambiguity can be further classified based on the viewpoint of the ways the ambiguity comes form, that is either by preference based ambiguity or possibility based ambiguity. These ambiguities and uncertainties cannot be solved by traditional mathematical optimization techniques. To solve the problems with such ambiguities and uncertainties Fuzzy Optimization technique has been used. Fuzzy optimization solves the fuzzy model optimally using optimization techniques and tools by formulating fuzzy information in terms of their membership function. In fuzzy linear programming problem the constraint or objective function need not to be specified in precise like crisp terms. In such situations the fuzzy linear programming occurs. In general fuzzy linear programming the cost matrix or the coefficient matrix or the variable matrix or the column vector represents the fuzziness.

Any problem area that involves natural language, fuzzy logic plays an important role. Fuzzy logic has an importance role in artificial intelligence and expert system. It is used in some spell checkers to suggest a list of probable words to replace a misspelled one, also helps to solve various safety problems and facilitate automatic driving, optimal navigation by voice instructions in natural language, automatic parking and a host of other functions. It has vital role in areas involved in health care. One of the complex application areas of fuzzy logic is expert systems for medical diagnosis and treatments planning that are based on approximate reasoning. Other facets includes fuzzy control of various medical devises, comprehensive evaluation of patients abnormal physiological conditions based not only on physiological data, but also on the appearance and behaviour, artificial limbs, navigation system aiding the visually handicapped, robots design to assist patients, and other problem domain. Fuzzy optimization is also applied in genetic algorithm.

This paper is about using the linear programming technique with fuzzy concepts. The main objective is to improve the fertility of the soil for a better crop production by removing the deficiency of the minerals from the soils by adding appropriate manures according to the requirements. Here manures like cow dung manure, chicken manure and pig manure are used for eliminating such deficiency from the soil. This project is objectified for solving the deficiency in soils that are commonly available in northern and southern regions of India. Since manures are applied on different soils the vagueness had been given in the nature of the soil such as the right hand side column vector in the constraints has been treated as fuzzy vector. The problem is solved in such a manner that the consistency of minerals in the soil should be neither below the deficiency level nor above the excess level. The solution of the problem determines the suitable manures which helps in eliminating the deficiency of the minerals in all the soils with minimum cost spent on the manures.

2. Motivational works:

Ebrahimnejad (2011)had made a unique approach to overcome the limitations in the fuzzy primal problem and fuzzy dual simplex method. Amit Kumar, Pushpinder Singh and Jagadeep Kaur (2010)proposed an algorithm which is the direct extension of classical algorithm and is applied to find the fuzzy optimal solution

for a Fuzzy Linear Programming Problem.Hareth and Samarathanga (2015) had explained a fuzzy multi criteria mathematical programming model which is used to find the optimal allocation of land for paddy which satisfies the two objectives namely profit maximization and cost minimization subject to the make use of water constraint and demand constraint.Mansur Hassan (2015)had solved a fuzzy linear programming problem which includes fuzziness in the right hand side column vector using software. MohaddesandMohdGhazaliMohayidean (2008) had explained about the fuzzy multi objective modelling to watershed development plan. Nasseri and Ardil (2009) used linear ranking method to solve the linear programming problem in a crisp environment. Neela Patel, Manish Thaker and Chandrika Chaudhary(2016) applied fuzzy optimization technique in agricultural planning specially for farmers in Patan district, North Gujarat, India. Parvathi and Malathi (2012) had explained about intuitionistic fuzzy numbers and arithmetic operation defined on them. Pattnaik (2013) had introduced Robust's ranking technique to solve the fuzzy decision problems. Poonam Gupta (2017) gave an overview of application of fuzzy logic in our daily life.Rajarajeswari and SahayaSudha (2013) discussed about the ranking Procedure based on Hexagonal Fuzzy numbers, which can be applied to a Multi-objective Linear programming problem (MOLPP) with fuzzy coefficients.Rajarajeswari, SahayaSudha and Karthika (2013)explained a new operation on Hexagonal Fuzzy number in which the methods of addition, subtraction, and multiplication has been modified by some conditions. Ravi Shankar, Ananda Rao, MadhuLatha and Sireesha (2010) describes a new method for optimizing the non- linear objective function with fuzzy constraints and fuzzy coefficients with the help of genetic algorithm**.**Samir Dey and Tapan Kumar Roy (2006) explained to solve the multi objective structural problem using fuzzy logic using computational algorithm. Satheesh Kumar and Nandhini (2017)proposed ranking methodology to solve Fuzzy Linear Programming Problem by converting membership function into a crisp number and further had been solved by simplex method.Shapla Shirin and Kamrunnahar(2014) discussed on the three methods which are used to solve the fuzzy optimization problems namely Bellman-Zadeh's method, Zimmermand's method and fuzzy version of simplex method. Singh and Bharathi (2014)created model for crop planning by using intuitionistic fuzzy optimization technique. Stefan Chanas and Pawel Zielinski (2000) according to Orbvsky's concept the set of non-dominated solution with assumed fuzzy preference relation are considered to be the solution. The fuzziness is applied on the coefficient of the objective function. This paper discusses about solving a fuzzy preference relation to reduce the problem by using sufficient condition to determine un-non dominated solution to determine the optimum solution of a Linear Programming Problem.Yazdani, Zaefarian,Nasseri and Ardil (2005) introduced a new method for solving fuzzy linear programming problem.

3. Mathematical Formulation:

Consider the four different soils available in a region which are represented by A, B, C and D. For the growth of the crop production minerals are the important factors. Due to the deficiency in minerals, crop production decreases. So the level of the minerals in the soil for the crop production should be maintained. Let P, Q and R be the minerals which are commonly obtained in the soil essential for the crop production. It is not necessary that every soil is rich in all the minerals. The fertility of the soil varies according to the climate and region. So, to eliminate the deficiency of such essential minerals, fertilizers should be added to the soil. Since

chemical manures are harmful for the soil as well as to the crop, natural manures are more preferable. Let L, M and N be the three different natural manures. Let a_{ij} be the percentage of the j^{th}mineral available in 100 grams of i^{th}manure, which is tabulated as below,

Table. 1. Proportion of availability of minerals in the natural manures

Manures\Minerals	P	Q	R	Cost of the manures (for 100gms)
L	a_{11}	a_{12}	a_{13}	c_1
M	a_{21}	a_{22}	a_{23}	c_2
N	a_{31}	a_{32}	a_{33}	c_3
(Deficiency, Sufficiency)	(d_1,e_1)	(d_2,e_2)	(d_3,e_3)	

Here four different soils are considered to eliminate the deficiency. Not only deficiency is a problem to the crop production of the soil, also excess of the minerals may also cause damage to the soil that may affect the crop production. Already some amount of the minerals are present in the soil and has been denoted as b_{ij}. As four soils has been considered the vagueness would be given for the soil and hence the fuzzy linear programming problem has been formulated with an objective to minimize the cost spent on manures as

$$\text{Min } z \sim c_1x_1+c_2x_2+c_3x_3$$

$$\text{s.t : } e_1 \geq a_{11}x_1 + a_{21}x_2 + a_{31}x_3 + (b_{11},b_{12},b_{13},b_{14}) \geq d_1$$

$$e_2 \geq a_{12}x_1 + a_{22}x_2 + a_{32}x_3 + (b_{21},b_{22},b_{23},b_{24}) \geq d_2$$

$$e_3 \geq a_{13}x_1 + a_{23}x_2 + a_{33}x_3 + (b_{31},b_{32},b_{33},b_{34}) \geq d_3$$

$$x_1,x_2,x_3 \geq 0 \qquad \ldots\ldots(2)$$

The problem is solved by using linear programming technique manually. As the right hand side column vector has been considered as fuzzy numbers, to apply the optimization techniques a unique crisp representation is needed and which would be given by the fuzzy trapezoidal numbers. This model has been applied for any four soils in the hemisphere of earth.

3. Results and discussions

The above mathematical model has been applied for northern and southern part of the soils in India. The three mineral deficiency in the soil are considered for namely nitrogen, phosphorous and potassium. The three natural manures cowdung, chicken and pig manures are considered to increase the fertility of the soil. The Table.2 gives the amount of minerals present in the manures, deficiency level and the cost of manures.

Table.2 Manures and Minerals

Manures (in 100gm)	**P (in 100 gms)**	**Q (in 100 gms)**	**R (in 100gms)**	**Price(Rs.) (for 100 gm)**
Cow Dung manure	1	0.2	0.9	9.98
Chicken manure	1.6	1.5	0.6	12.05
Pig manure	0.9	0.2	0.4	10.05

(Deficiency, Sufficiency)	(3.5,5)	(0.3,0.5)	(2.5,4)	

3.1. Northern Region soils of India and the elimination of deficiency of the minerals

In northernpartof India the four important soils namely Mountain soil, Desert soil, Forest soil and Alluvial soil are considered. The amount of minerals present in the soil are given in Table.3.

Table.3 The percentage of minerals availability on the northern region soils of India

Soils	Nitrogen	Phosphorous	Potassium
Mountain soil	2.6	0.28	2.41
Desert soil	1	0.001	0.084
Forest soil	3.43	0.2	1.20
Alluvial soil	1	0.22	1.6

Using the proposed mathematical model the given data can be formulated as

$$\text{Minimize } z = 9.98\, x_1 + 10.05\, x_2 + 12.05\, x_3$$

$$5 \geq (2.6,1,3.43,1) + x_1 + 1.6\, x_2 + 0.9\, x_3 \geq 3.5$$

$$0.5 \geq (0.28, 0.001, 0.2, 0.22) + 0.2\, x_1 + 1.5\, x_2 + 0.2\, x_3 \geq 0.3$$

$$4 \geq (2.41,0.84,1.20,1.6) + 0.9\, x_1 + 0.6\, x_2 + 0.4\, x_3 \geq 2.5$$

$$x_1, x_2, x_3 \geq 0 \qquad (3)$$

By adding appropriate slack, surplus and artificial variables in the constraints and the (equation 3) has been reduced into standard form. Applying Charne's penalty method this would be solved and the optimal simplex table is given Table.4. Fuzzy trapezoidal numbers are used to give the unique representation for the fuzzy vector.

Table.4 Optimal simplex table for Northern region soils of India

		C_j	**-9.98**	**-12.05**	**-10.05**	**0**	**0**	**0**	**0**	**0**	**0**
C_B	B	X_B	X_1	X_2	X_3	S_1	S_2	S_3	S_4	S_5	S_6
0	S_3	(0.2,0.2,0.2,0.22)	0	0	0	0	0	1	1	0	0

0	S_2	(1.4,1.49,0.27,1.57)	0	0	0	1	1	0	0	0	0
0	S_5	(0.66,0,0,1.3)	0	0	0.4	-1.04	0	0	-0.71	1	0
-12.05	x_2	(0.17,0.17,0.17,0)	0	1	0.02	0.17	0	0	0.85	0	0
-9.98	x_1	(0.71,2.51,1.25,2.43)	1	0	0.87	-1.27	0	0	-1.36	0	0
0	S_6	(0.82,1.63,1.64,0.2)	0	0	-0.4	1.04	0	0	0.71	0	1
		Z_j-C_j	0	0	1.12	10.6	0	0	3.33	0	0

From the obtained results it has been observed that the problem of deficiency in soils are solved by adding 0.71 gm of cow dung manure and 0.17 gm of chicken manure to mountain soil, 2.51 gm of cow dung manure and 0.17 gm of chicken manure to desert soil, 1.25 gm of cow dung manure and 0.17 gm of chicken manure to forest soil and 2.43 gm of cow dung manure in alluvial soil. Hence the cost spent for the manures are also minimized. Rather by adding this amount of manures would not increase the sufficiency levels of the minerals.

3.2. Southern Region soils of India and the elimination of deficiency of the minerals

In southern part of India the four important soils namely red soil, black soil, marshy soil and laterite soil are considered. The amount of minerals present in the soil is given in Table.5.

Table.5 The percentage of minerals availability on the southern region soils of India

Soils	Nitrogen	Phosphorous	Potassium
Red soil	2.08	0.08	0.24
Black soil	2.28	0.15	2
Laterite soil	2.1	0.09	1.36
Marshy soil	2.1	0.001	0.01

Using the proposed mathematical model the given data can be formulated as

$$\text{Minimize } z = 9.98\, x_1 + 10.05\, x_2 + 12.05\, x_3$$

$$5 \geq (2.08,2.28,2.1,2.1) + x_1 + 1.6\, x_2 + 0.9\, x_3 \geq 3.5$$

$$0.5 \geq (0.08,0.15,0.9,0.001) + 0.2\, x_1 + 1.5\, x_2 + 0.2\, x_3 \geq 0.3$$

$$4 \geq (0.24,2,1.36,0.01) + 0.9\, x_1 + 0.6\, x_2 + 0.4\, x_3 \geq 2.5$$

$$x_1, x_2, x_3 \geq 0 \quad (3)$$

By adding appropriate slack, surplus and artificial variables in the constraints and the (equation 3) has been reduced into standard form. Applying Charne's penalty method this would be solved and the optimal simplex table is given Table.4. Fuzzy trapezoidal numbers are used to give the unique representation for the fuzzy vector.

Table.6 Optimal simplex table for Southern region soils of India

		C_j	**-9.98**	**-12.05**	**-10.05**	**0**	**0**	**0**	**0**	**0**	**0**
C_B	B	X_B	X_1	X_2	X_3	S_1	S_2	S_3	S_4	S_5	S_6
0	S_3	(0.28,0.09,0.06,0.26)	0	-1.37	-0.11	0	0	1	0	-0.22	0
0	S_2	(0.32,1.5,1.49,0.02)	0	0.93	0.46	0	1	0	0	1.11	0
-9.98	x_1	(2.5,1.21,1.37,2.76)	1	0.67	0.44	0	0	0	0	-1.11	0
0	S_4	(0,0.11,0.14,0)	0	1.37	0.11	0	0	0	1	0.22	0
0	s_1	(1.13,0,0,1.3)	0	-0.93	-0.46	1	0	0	0	-1.11	0
0	S_6	(1.5,0.9,0.135,1.43)	0	0	0	0	0	0	0	1	1
		Z_j-C_j	0	5.3	5.6	0	0	0	0	11.07	0

From the results obtained it has been shown that the pig manure and chicken manure would not give any significant effect on the process of removal of mineral deficiency from the soils. By adding cow dung manures in the proportion (0.25,1.21,1.37,2.76) to the soils remove the mineral deficiency from the soils. Also the cost spent for the manure also has been minimized. Rather by adding this amount of manures would not increase the sufficiency levels of the minerals.

4. Conclusion:

The project discuss about the fuzzy linear programming problem. Here an example of fuzzy optimization problem has been modelled, which works as pure fuzzy linear programming problem using trapezoidal fuzzy numbers and solved it with the help of fuzzy version of Charne's penalty method. While solving the above problems separately few problems showed infeasibility in the solution process. But solving by fuzzy linear programming approach by giving vagueness to the soil, the desired optimum solution has been obtained. By fuzzy optimization an optimal solution has been obtained that concludes that cow dung manure and chicken manure is more efficient in eliminating the deficiency of minerals in the soil which are essential for the improvement of crop production. The results obtained by this method are interesting as it satisfies the constraints and achieve the set goals in the optimal way by minimum cost. Many times it may be difficult to obtain an optimal solution analytically, so MATLABhas been used to solve such problem effortlessly.

References:

1) Ali Ebrahimnejad (2011), "A primal dual simplex algorithm for solving linear programming problems with symmetric trapezoidal fuzzy numbers", *Applied Mathematics*, 2, 676-684.
2) Amit Kumar, PushpinderSingh and Jagadeep Kaur (2010), "Generalized simplex algorithm to solve fuzzy linear programming problems with ranking of generalized fuzzy numbers", *Turkish Journal of Fuzzy System*, 1(2), 80-103.
3) H M I U Hareth and Samarathanga (2015), "Multi objective fuzzy linear programming in agriculture production planning", *International Journal of Scientific and Technology Research*,4(10), 242-250.
4) S. K. Jain and D. M. Mehta, *"Operation Research (Theory and Application)"*, Galgotia Publications Private Limited, 2nd edition.
5) S. Kalavathy, "*Operations Research*", Vikas Publishing House Private Limited, 4thedition.
6) Mansur Hassan (2015), "Proposed simplex method for fuzzy linear programming with fuzziness at the right hand side", *IOSR Journal of Mathematics*,11(3), 58-65.
7) S. A. Mohaddes and MohdGhazaliMohayidean (2008), "Application of the Fuzzy Approach for Agricultural Production Planning in a Watershed, a Case Study of the Atrak Watershed, Iran", *American-Eurasian Journal of Agriculture& Environment Sci*ence, 3 (4):636-648.
8) S. H. Nasseri and E. Ardil (2009), "Simplex Method for Fuzzy Variable Linear Programming Problems", *World Academy of Science, Engineering and Technology International Journal of Mathematical and Computational Sciences,*3(10), 884-888.
9) Neela Patel, Manish Thaker and Chandrika Chaudhary(2016), "Study of Some Agricultural Crop Production Planning Condition through Fuzzy Multi-Objective Linear Programming Mathematical Model", *International Journal of Science and Research*,5(4), 1329-1332.
10) R.Parvathi and L.Malathi (2012), "Intuitionistic fuzzy simplex method", *International Journal of Computer Application*, 48(6),39-48.
11) M Pattnaik (2013), "Fuzzy multi objective linear programming problem sensitive analysis", *Journal of Mathematics and Computer Science*, 7, 131-137.
12) Poonam Gupta(2017), "Applications of Fuzzy Logic in Daily life", *International Journal of Advanced Research in Computer Science*, 8(5),1795-1800.
13) P Rajarajeswari and A.SahayaSudha (2013), "Ranking of Hexagonal Fuzzy Numbers for Solving Multi Objective Fuzzy Linear Programming Problem", *International Journal of Computer Applications*, 84(8), 14-19.
14) P. Rajarajeswari, A.SahayaSudha and R.Karthika (2013), "A new operation on Hexagonal Fuzzy Number", *International Journal of Fuzzy Logic Systems*, 3(3),15-26.
15) N. Ravi Shankar, G. Ananda Rao, J MadhuLatha and V. Sireesha (2010), "Solving a fuzzy non- linear optimization problem by genetic algorithm",*International Journalof Contemporary Mathematical Sciences*,5(16),791-803.
16) Samir Dey and Tapan Kumar Roy (2006), "Solving Multi-Objective Structural Design Problem using Fuzzy Optimization Method: A Comparative Study",*International Journal of Innovative Research in Science & Engineering*, 3(4),161-170.
17) B. SatheeshKumar and R Nandhini (2017), "An Optimum Solution for the Fuzzy Linear Programming Problem", *International Journal of Pure and Applied Mathematics*, 117(13), 91-98.
18) Shapla Shirin and Kamrunnahar(2014), "Application of Fuzzy Optimization Problem in Fuzzy Environment", *Dhaka University Journal of Science*, 62(2), 119-125.
19) S. R. Singh and S. K. Bharathi, "Intuitionistic fuzzy optimization technique in agriculture production planning: A small farm holder perspective", *International Journal of Computer Application*, 89(6),17-23.
20) Stefan Chanas and Pawel Zielinski (2000), "The equivalence of two optimization method for fuzzy linear programming problem", *European Journal of Operation Research*,121,56-63.
21) Dr. P. R. Vittal and V. Malini , " *Operations Research*", Margham publications.
22) A Yazdani, R. Zaefarian, S. H. Nasseri and E. Ardil(2005), "Simplex Method for Solving Linear Programming Problems with Fuzzy Numbers", *World Academy of Science, Engineering and Technology*, 10,284-288.