The Kumar – Laplace Series

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Abstract:
In this article I will be giving a series which provides the Laplace Transform of a real valued function and is expandable by Maclaurin's series. It's easy way to find the Laplace transform of any real valued function which is expandable by Maclaurin's series.

Keywords:
Convergence, The Kumar – Laplace series, Real valued function, Laplace transforms and Maclaurin's series.

I. INTRODUCTION

In mathematics, we know that there are many real valued functions those are converge or diverge to a certain limit. A Laplace transform exists, if the integral of a given function on closed interval [a , b] converge. But "this series exist for any real valued function which are expandable by Maclaurin's series".

Now, to determine the Laplace transform of any real valued function (t) , where $0 \leq t \leq \infty$, we use the following according to the definition of Laplace transform:

$$L\{F(t)\} = \int_0^\infty e^{-st} F(t)\,dt$$

Where s > 0.

To determine the Laplace transform, no need to evaluate the above integration for any real valued function (which is expandable by Maclaurin's series) in The Kumar-Laplace series.

II. SERIES

Now, we will state a series which is equivalent to Laplace transform in terms of 's' where, s>0.

Let $F(t)$ be a real valued function. By Maclaurin's series expansion, we get the following series:

$$L\{F(t)\} = \frac{F(o)}{s} + \frac{F'(0)}{s^2} + \frac{F''(0)}{s^3} + \ldots..$$

Where, $0 \leq t < \infty$.

If the above series converges, then Laplace transform of $F(t)$ exist.

Now, considering nth terms of the above series, we get:

$$L\{F(t)\}_n = \frac{F(o)}{s} + \frac{F'(0)}{s^2} + \ldots.. + \frac{F^{(n-1)}(o)}{s^n}$$

If s>0, then, for any finite value $L\{F(t)\}_n$ converges.

III. DERIVATION

Let $F(t)$ be a real valued function (where $0 \le t < \infty$). Then, the Laplace transform of F(t) is given by,

$$L\{F(t)\} = \int_0^\infty e^{-st} \; F(t)\,dt$$

Where s > 0.

Let $F(t)$ be defined on a closed interval [0 , t] such that

(1) $F^{(n-1)}(t)$ is continuous on closed interval [0 , t].

(2) $F^{(n)}(t)$ exists in an open interval (0 , t).

Then, there exists a real number 'θ' where 0 < θ < 1 such that:

$$F(t) = F(0) + t * F^{'}(0) + \frac{t^2}{2!} F^{''}(0) + \ldots.. + \frac{F^{(n)}(\theta t)}{n!}$$

Let $\frac{F^{(n)}(\theta \quad t)}{n!}$ converge to 'A'.

$$\therefore \; F(t) = F(0) + t * F^{'}(0) + \frac{t^2}{2!} F^{''}(0) + \ldots..$$

Now, multiplying the above equation by e^{-st} on both sides, we get:

$$e^{-st} * F(t) = e^{-st} * F(0) + t * e^{-st} * F^{'}(0) + \frac{t^2}{2!} * e^{-st} * F^{''}(0) + \ldots..$$

Now, on integrating the above equation where $0 \le t < \infty$, we get:

$$\int_0^\infty e^{-st} * F(t)\,dt = \int_0^\infty e^{-st} * F(0)\,dt + \int_0^\infty e^{-st} * t * F'(0)\,dt + \ldots..$$

$$\therefore \int_0^\infty e^{-st} * F(t)\,dt = F(o) * \int_0^\infty e^{-st}\,dt + F'(0) * \int_0^\infty e^{-st} * t\,dt + \ldots..$$

Now, by the definition of Laplace transforms, we have:

$$L\{F(t)\} = F(0) * L\{t^0\} + F'(0) * L\{t\} + ...$$

This gives us:

$$L\{F(t)\} = \frac{F(0)}{s} + \frac{F'(0)}{s^2} + \frac{F''(0)}{s^3} + ...$$

Which is known as "The Kumar – Laplace series".

COMMENT ON RESULT: We can observe that there is no integration to determine the Laplace transform of any given real valued function, which is expandable by Maclaurin's series.

IV. APPLICATION

Some examples for Kumar – Laplace series:

(1) Let $F(t) = t^{1/2}$, $F(0) = 0$

$$F'(t) = \frac{1}{2} * t^{-1/2}, \; F'(0) = \infty .$$

Solution: Here, Maclaurin's expansion does not exist. Hence, we can't find its Laplace transform by

Kumar – Laplace series for $F(t) = t^{1/2}$.

(2) Let

$F(t) = \sin t, F(0) = 0$

$F'(t) = \cos t, F'(0) = 1$

$F''(t) = -\sin t, F''(0) = 0$

$F'''(t) = -\cos t, F'''(0) = -1$

Solution: By Kumar – Laplace series we have the equation:

$$L\{F(t)\} = \frac{F(0)}{s} + \frac{F'(0)}{s^2} + \frac{F''(0)}{s^3} + \frac{F'''(0)}{s^4} + ...$$

On substituting the given values we get:

$$L\{F(t)\} = 0 + \frac{1}{s^2} + 0 - \frac{1}{s^4} + ...$$

$$\Rightarrow L\{F(t)\} = \frac{1}{s^2} - \frac{1}{s^4} + \frac{1}{s^6} - \frac{1}{s^8} + ...$$

$$\Rightarrow L\{F(t)\} = \frac{1}{s^2}\{1 - \frac{1}{s^2} + \frac{1}{s^4} - \frac{1}{s^6} + ...\}$$

$$\Rightarrow L\{F(t)\} = \frac{1}{s^2}\{\frac{1}{1 + \frac{1}{s^2}}\}$$

$$\Rightarrow L\{F(t)\} = \frac{1}{s^2} * \frac{s^2}{s^2 + 1}$$

$$\Rightarrow L\{F(t)\} = \frac{1}{1 + s^2} \text{ where } s > 0.$$

V. REFERENCES

There is no specific reference used for this paper. I used only my third year mathematics B.Sc. class knowledge.