Effect of Imperfect Interface Between Fluid Saturated Porous Solid and Micropolar Elastic Solid on Wave Propagation

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Abstract

The present investigation deals with the reflection and refraction phenomenon at the imperfect interface between fluid saturated porous solid half space and micropolar elastic solid half space. P-wave or SV-wave is considered to be incident on the plane interface through fluid saturated porous solid half space. Incident wave impinge obliquely at the interface. Amplitudes ratios of various reflected and refracted waves to that of incident wave are derived and deduced for normal force stiffness, transverse force stiffness and for welded contact. After obtaining the amplitudes ratios, they have been computed numerically for a specific model and results obtained graphically with angle of incidence of incident wave. It is found that these amplitude ratios depend on angle of incidence of incident wave and material properties of the medium and these are affected by the stiffness also.

Keywords: *Amplitude ratio, reflection, refraction, micropolar elastic solid, porous solid.*

1. Introduction

The micropolar theory of elasticity constructed by Eringen and his co-workers intended to be applied on such materials and for problems where the ordinary theory of elasticity fails because of microstructure in the materials. Micropolar elastic materials, roughly speaking, are the classical elastic materials with extra independent degree of freedom for the local rotations. These materials respond to spin inertia and body and surface couples and as a consequence they exhibit certain new static and dynamic effects, e.g. new types of waves and couples stresses.

From a continuum mechanical point of view, micropolar elastic solids may be characterized by a set of constitutive equations which define the elastic properties of such materials. A linear theory as a special case of the nonlinear theory of micro-elastic solids was first constructed by Eringen and Suhubi (1964a, b). Later, Eringen (1965) and (1966) recognized and extended this theory.

Eringen (1966a, 1990) developed the theories of 'micropolar continua' and 'microstructures continua' which are special cases of the theory of 'micromorphic continua' earlier developed by Eringen and his coworkers (1964). Thus,theEringen's '3M' theories (Micromorphic, Microstretch, Micropolar) are the generalization the classical theory of elasticity. In classical continuum, each particle of a continuum is represented by a geometrical point and can have three degree of freedom of translation during the process of deformation.

Eringen's theory of micropolar elasticity keeps importance because of its applications in many physical substance for example material particles having rigid directors, chopped fibres composites, platelet composites, aluminium epoxy, liquid crystal with side chains, a large class of substance like liquid crystal with rigid molecules, rigid suspensions, animal blood with rigid cells, foams, porous materials, bones, magnetic fields, clouds with dust, concrete with sand and muddy fluids are example of micropolar materials.

Soft soils such as sand and clay consist of small particles, and open the pore space between the particles is filled with water. In soil mechanics this is denoted as a saturated or partially saturated porous medium. The deformations of such porous media depend upon the stiffness of the porous material, and upon the behaviour of the fluid in the pores.

Imperfect interface considered in this problem means that the stress components are continuous and small displacement field is not. The small vector difference in the displacements assumed to depend linearly on the traction vector. More precisely jumps in the displacement components are assumed to be proportional (in terms of spring-factor-type interface parameters) to their respective interface components. The infinite values of interface parameters imply vanishing of displacement jumps and therefore correspond to perfect interface conditions. The finite values of the interface parameters define an imperfect interface. The values of the

interface are depends on the material properties o the medium. Recently many authors have used the imperfect conditions at an interface to the various types of the problems (Kumar and Rupender, Kumar and Chawla, Chong and Wei etc.).

Using the theory of de Boer and Ehlers (1990) for the fluid saturated porous medium and Eringen (1968) for micropolar elastic solid medium, the reflection and refracted phenomenon of longitudinal wave at a loosely bonded interface between fluid saturated porous solid half space and micropolar elastic solid half space is studied. Such problem of reflection and refraction where aluminium-epoxy composites as micropolar elastic solid is in loosely bonded contact with the crust as the fluid saturated porous solid. As such model may be found in the earth's crust, so the results of our problem can be applicable to the earth's crust, to a water-mud-rock boundary, or to some other specific problems in engineering or seismology.

2. Formulation of the problem

Consider a two dimensional problem by taking the z-axis pointing into the lower half-space and the plane interface z=0 separating the uniform fluid saturated porous solid half space M_1 [z > 0] and micropolar elastic solid half space M_2 [z<0]. Consider a longitudinal wave (P-wave) or transverse wave (SV-wave) propagating through a medium M_1 and incident at the plane z=0 and making an angle θ_0 with normal to the surface. Corresponding to incident longitudinal wave, we get two reflected waves in the medium M_1 and three refracted waves in medium M_2. See fig. 1.

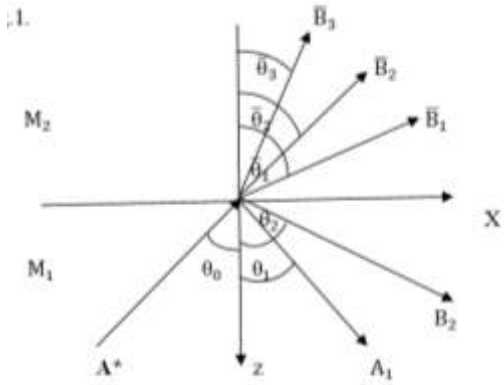

Fig.1 Geometry of the problem

3. Basics Equations and Constitutive Relations

3.1. For medium M_1(fluid saturated porous half space)

Following de Boer and Ehlers (1990b), the governing equations in a fluid-saturated incompressible porous medium are

$$\mathrm{div}(\eta^S \dot{\mathbf{x}}_S + \eta^F \dot{\mathbf{x}}_F) = 0. \tag{1}$$

$$\mathrm{div}\mathbf{T}_E^S - \eta^S \,\mathrm{grad}\, p + \rho^S(\mathbf{b} - \ddot{\mathbf{x}}_s) - \mathbf{P}_E^F = 0 \tag{2}$$

$$\mathrm{div}\mathbf{T}_E^F - \eta^F \,\mathrm{grad}\, p + \rho^F(\mathbf{b} - \ddot{\mathbf{x}}_F) + \mathbf{P}_E^F = 0 \tag{3}$$

where $\dot{\mathbf{x}}_i$ and $\ddot{\mathbf{x}}_i (i = S, F)$ denote the velocities and accelerations, respectively of solid (S) and fluid (F) phases of the porous aggregate and p is the effective pore pressure of the incompressible pore fluid. ρ^S and ρ^F are the densities of the solid and fluid phases respectively and **b** is the body force per unit volume. $\mathbf{T}_E^S$ and $\mathbf{T}_E^F$ are the effective stress in the solid and fluid phases respectively, $\mathbf{P}_E^F$ is the effective quantity of momentum supply and η^S and η^F are the volume fractions satisfying

$$\eta^S + \eta^F = 1. \tag{4}$$

If $\mathbf{u}_S$ and $\mathbf{u}_F$ are the displacement vectors for solid and fluid phases, then

$$\dot{x}_S = \dot{\mathbf{u}}_S, \quad \ddot{\mathbf{x}}_s = \ddot{\mathbf{u}}_s, \quad \dot{\mathbf{x}}_F = \dot{\mathbf{u}}_F, \quad \ddot{\mathbf{x}}_F = \ddot{\mathbf{u}}_F. \tag{5}$$

The constitutive equations for linear isotropic, elastic incompressible porous medium are given by de Boer, Ehlers and Liu (1993) as

$$\mathbf{T}_E^S = 2\mu^S \mathbf{E}_S + \lambda^S (E_S.\mathbf{I})\mathbf{I}, \tag{6}$$

$$\mathbf{T}_E^F = 0, \tag{7}$$

$$\mathbf{P}_E^F = -\mathbf{S}_v(\dot{\mathbf{u}}_F - \dot{\mathbf{u}}_S) \tag{8}$$

where λ^S and μ^S are the macroscopic Lame's parameters of the porous solid and $\mathbf{E}_S$ is the linearized Langrangian strain tensor defined as

$$\mathbf{E}_S = \frac{1}{2}(\mathrm{grad}\,\mathbf{u}_S + \mathrm{grad}^T \mathbf{u}_S), \tag{9}$$

In the case of isotropic permeability, the tensor $\mathbf{S}_v$ describing the coupled interaction between the solid and fluid is given by de Boer and Ehlers (1990b) as

$$\mathbf{S}_v = \frac{(\eta^F)^2 \gamma^{FR}}{K^F}\mathbf{I}, \tag{10}$$

where γ^{FR} is the specific weight of the fluid and K^F is the Darcy's permeability coefficient of the porous medium.

Making the use of (5) in equations (1)-(3), and with the help of (6)-(9), we obtain

$$\mathrm{div}(\eta^S \dot{\mathbf{u}}_S + \eta^F \dot{\mathbf{u}}_F) = 0, \tag{11}$$

$$(\lambda^S+\mu^S)\text{grad div}\,\mathbf{u}_S+\mu^S\text{div grad}\,\mathbf{u}_S-\eta^S\text{grad p}+\rho^S(\mathbf{b}-\ddot{\mathbf{u}}_s)+S_v(\dot{\mathbf{u}}_F-\dot{\mathbf{u}}_S)=0 \quad (12)$$

$$-\eta^F\text{grad p}+\rho^F(\mathbf{b}-\ddot{\mathbf{u}}_F)-S_v(\dot{\mathbf{u}}_F-\dot{\mathbf{u}}_S)=0 \quad (13)$$

For the two dimensional problem, we assume the displacement vector $\mathbf{u}_i(i=F,S)$ as

$$\mathbf{u}_i=(u^i,0,w^i)\text{where}\, i=F,S. \quad (14)$$

Equations (11) - (13) with the help of eq. (14) in absence of body forces take the form

$$\eta^S\left[\frac{\partial^2 u^S}{\partial x\,\partial t}+\frac{\partial^2 w^S}{\partial z\,\partial t}\right]+\eta^F\left[\frac{\partial^2 u^F}{\partial x\,\partial t}+\frac{\partial^2 w^F}{\partial z\,\partial t}\right]=0, \quad (15)$$

$$\eta^F\frac{\partial p}{\partial x}+\rho^F\frac{\partial^2 u^F}{\partial t^2}+S_v\left[\frac{\partial u^F}{\partial t}-\frac{\partial u^S}{\partial t}\right]=0, \quad (16)$$

$$\eta^F\frac{\partial p}{\partial z}+\rho^F\frac{\partial^2 w^F}{\partial t^2}+S_v\left[\frac{\partial w^F}{\partial t}-\frac{\partial w^S}{\partial t}\right]=0, \quad (17)$$

$$(\lambda^S+\mu^S)\frac{\partial\theta^S}{\partial x}+\mu^S\nabla^2 u^S-\eta^S\frac{\partial p}{\partial x}-\rho^S\frac{\partial^2 u^S}{\partial t^2}+S_v\left[\frac{\partial u^F}{\partial t}-\frac{\partial u^S}{\partial t}\right]=0, \quad (18)$$

$$(\lambda^S+\mu^S)\frac{\partial\theta^S}{\partial z}+\mu^S\nabla^2 w^S-\eta^S\frac{\partial p}{\partial z}-\rho^S\frac{\partial^2 w^S}{\partial t^2}+S_v\left[\frac{\partial w^F}{\partial t}-\frac{\partial w^S}{\partial t}\right]=0, \quad (19)$$

where

$$\theta^S=\frac{\partial(u^S)}{\partial x}+\frac{\partial(w^S)}{\partial z} \quad (20)$$

and

$$\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2} \quad (21)$$

Also, t_{zz}^S and t_{zx}^S the normal and tangential stresses in the solid phase are as under

$$t_{zz}^S=\lambda^S\left(\frac{\partial u^S}{\partial x}+\frac{\partial w^S}{\partial z}\right)+2\mu^S\frac{\partial w^S}{\partial z} \quad (22)$$

$$t_{zx}^S=\mu^S\left(\frac{\partial u^S}{\partial z}+\frac{\partial w^S}{\partial x}\right). \quad (23)$$

The displacement components u^j and w^j are related to the dimensional potential ϕ^j and ψ^j as

$$u^j=\frac{\partial\phi^j}{\partial x}+\frac{\partial\psi^j}{\partial z};\ w^j=\frac{\partial\phi^j}{\partial z}-\frac{\partial\psi^j}{\partial x};\ j=S,F. \quad (24)$$

Using equation (24) in equations (15)-(19), we obtain the following equations determining ϕ^S, ϕ^F, ψ^S, ψ^F and p as:

$$\nabla^2\phi^S-\frac{1}{{C_1}^2}\frac{\partial^2\phi^S}{\partial t^2}-\frac{S_v}{(\lambda^S+2\mu^S)(\eta^F)^2}\frac{\partial\phi^S}{\partial t}=0 \quad (25)$$

$$\phi^F=-\frac{\eta^S}{\eta^F}\phi^S \quad (26)$$

$$\mu^S\nabla^2\psi^S-\rho^S\frac{\partial^2\psi^S}{\partial t^2}+S_v\left[\frac{\partial\psi^F}{\partial t}-\frac{\partial\psi^S}{\partial t}\right]=0 \quad (27)$$

$$\rho^F\frac{\partial^2\psi^F}{\partial t^2}+S_v\left[\frac{\partial\psi^F}{\partial t}-\frac{\partial\psi^S}{\partial t}\right]=0, \quad (28)$$

$$(\eta^F)^2 p-\eta^S\rho^F\frac{\partial^2\phi^S}{\partial t^2}-S_v\frac{\partial\phi^S}{\partial t}=0, \quad (29)$$

where

$$C_1=\sqrt{\frac{(\eta^F)^2(\lambda^S+2\mu^S)}{(\eta^F)^2\rho^S+(\eta^S)^2\rho^F}}. \quad (30)$$

Assuming the solution of the system of equations (25) - (29) in the form

$$(\phi^S,\phi^F,\psi^S,\psi^F,p)=({\phi_1}^S,{\phi_1}^F,{\psi_1}^S,{\psi_1}^F,p_1)\exp(i\omega t), \quad (31)$$

where ω is the complex circular frequency.

Making the use of (31) in equations (25)-(29), we obtain

$$\left[\nabla^2+\frac{\omega^2}{{C_1}^2}-\frac{i\omega S_v}{(\lambda^S+2\mu^S)(\eta^F)^2}\right]{\phi_1}^S=0, \quad (32)$$

$$[\mu^S\nabla^2+\rho^S\omega^2-i\omega S_v]{\psi_1}^S=-i\omega S_v{\psi_1}^F \quad (33)$$

$$[-\omega^2\rho^F+i\omega S_v]{\psi_1}^F-i\omega S_v{\psi_1}^S=0, \quad (34)$$

$$(\eta^F)^2 p_1+\eta^S\rho^F\omega^2{\phi_1}^S-i\omega S_v{\phi_1}^S=0, \quad (35)$$

$${\phi_1}^F=-\frac{\eta^S}{\eta^F}{\phi_1}^S. \quad (36)$$

Equation (32) corresponds to longitudinal wave propagating with velocity V_1, given by

$$V_1{}^2 = \frac{1}{G_1} \tag{37}$$

$$\text{where} G_1 = \left[\frac{1}{C_1{}^2} - \frac{iS_v}{\omega(\lambda^S+2\mu^S)(\eta^F)^2}\right]. \tag{38}$$

From equation (33) and (34), we obtain

$$\left[\nabla^2 + \frac{\omega^2}{V_2{}^2}\right]\psi_1{}^S = 0, \tag{39}$$

Equation (39) corresponds to transverse wave propagating with velocityV_2, given by

$$V_2{}^2 = \frac{1}{G_2}\text{where}$$

$$G_2 = \left\{\frac{\rho^S}{\mu^S} - \frac{iS_v}{\mu^S\omega} - \frac{S_v{}^2}{\mu^S(-\rho^S\omega^2 + i\omega S_v)}\right\}, \tag{40}$$

3.2. For medium M_2(micropolar elastic solid)

The equation of motion in micropolar elastic medium are given by Eringen (1968) as

$$(c_1{}^2 + c_3{}^2)\nabla^2\phi = \frac{\partial^2\phi}{\partial t^2} \tag{41}$$

$$(c_2{}^2 + c_3{}^2)\nabla^2\vec{U} + c_3{}^2\nabla\times\vec{\Phi} = \frac{\partial^2\vec{U}}{\partial t^2} \tag{42}$$

$$(c_4{}^2\nabla^2 - 2\omega_0{}^2)\vec{\Phi} + \omega_0{}^2\nabla\times\vec{U} = \frac{\partial^2\vec{\Phi}}{\partial t^2} \tag{43}$$

where

$$c_1{}^2 = \frac{\lambda+2\mu}{\rho} c_2{}^2 = \frac{\mu}{\rho}; \quad c_3{}^2 = \frac{\kappa}{\rho}$$

$$c_4{}^2 = \frac{\gamma}{\rho j}; \quad \omega_0{}^2 = \frac{\kappa}{\rho j} \tag{44}$$

Parfitt and Eringen (1969) have shown that equation (41) corresponds to longitudinal wave propagating with velocityV_{11}, given by $V_{11}{}^2 = c_1{}^2 + c_3{}^2$, and equations. (42)-(43) are coupled equations in vector potentials $\vec{U}$ and $\vec{\Phi}$ and these correspond to coupled transverse and micro-rotation waves. If $\frac{\omega^2}{\omega_0{}^2} > 2$, there exist two sets of coupled-wave propagating with velocities $1/\lambda_1$ and $1/\lambda_2$; where

$$\lambda_1{}^2 = \frac{1}{2}\left[B - \sqrt{B^2 - 4C}\right],$$

$$\lambda_2{}^2 = \frac{1}{2}\left[B + \sqrt{B^2 - 4C}\right], \tag{45}$$

where

$$B = \frac{q(p-2)}{\omega^2} + \frac{1}{(c_2{}^2 + c_3{}^2)} + \frac{1}{c_4{}^2}$$

$$C = \left(\frac{1}{c_4{}^2} - \frac{2q}{\omega^2}\right)\frac{1}{(c_2{}^2 + c_3{}^2)}$$

$$p = \frac{\kappa}{\mu+\kappa}; \quad q = \frac{\kappa}{\gamma} \tag{46}$$

We consider a two dimensional problem by taking the following components of displacement and micro rotation as

$$\vec{u} = (u, 0, w), \quad \vec{\Phi} = (0, \Phi_2, 0), \tag{47}$$

where

$$u = \frac{\partial\phi}{\partial x} - \frac{\partial\psi}{\partial z}; \quad w = \frac{\partial\phi}{\partial z} + \frac{\partial\psi}{\partial x} \tag{48}$$

and components of stresses are as

$$t_{zz} = (\lambda + 2\mu + \kappa)\frac{\partial^2\phi}{\partial z^2} + \lambda\frac{\partial^2\phi}{\partial x^2} + (2\mu+\kappa)\frac{\partial^2\psi}{\partial x\partial z} \tag{49}$$

$$t_{zx} = (2\mu+\kappa)\frac{\partial^2\phi}{\partial x\partial z} - (\mu+\kappa)\frac{\partial^2\psi}{\partial z^2} + \mu\frac{\partial^2\psi}{\partial x^2} - \kappa\Phi_2, \tag{50}$$

$$m_{zy} = \gamma\frac{\partial\Phi_2}{\partial z} \tag{51}$$

In medium M_1

$$\{\phi^S, \phi^F, p\} = \{1, m_1, m_2\}[A_{01}\exp\{ik_1(x\sin\theta_0 - z\cos\theta_0) + i\omega_1 t\} + A_1\exp\{ik_1(x\sin\theta_1 + z\cos\theta_1) + i\omega_1 t\}] \tag{52}$$

$$\{\psi^S, \psi^F\} = \{1, m_3\}[B_{01}\exp\{ik_2(x\sin\theta_0 - z\cos\theta_0) + i\omega_2 t\} + B_1\exp\{ik_2(x\sin\theta_2 + z\cos\theta_2) + i\omega_2 t\}] \tag{53}$$

$$\text{Where} m_1 = -\frac{\eta^S}{\eta^F}; \quad m_2 = -\left[\frac{\eta^S\omega_1{}^2\rho^F - i\omega_1 S_v}{(\eta^F)^2}\right],$$

$$m_3 = \frac{i\omega_2 S_v}{i\omega_2 S_v - \omega_2{}^2 \rho^F} \quad (54)$$

where A_{01} and B_{01} are amplitudes of incident P-wave and SV-wave respectively, and k_1 and k_2 are the wave numbers and A_1 and B_1 are amplitudes of the reflected P-wave and SV-wave respectively, to be determined from boundary conditions.

In medium M_2

$$\phi = \bar{B}_1 \exp\{i\bar{k}_0(x \sin\bar{\theta}_1 - z\cos\bar{\theta}_1) + i\omega_1 t\}, \quad (55)$$

$$\psi = \bar{B}_2 \exp\{i\bar{\delta}_1(x \sin\bar{\theta}_2 - z\cos\bar{\theta}_2) + i\omega_2 t\} + \bar{B}_3 \exp\{i\bar{\delta}_2(x \sin\bar{\theta}_3 - z\cos\bar{\theta}_3) + i\omega_3 t\}, \quad (56)$$

$$\Phi_2 = E\bar{B}_2 \exp\{i\bar{\delta}_1(x \sin\bar{\theta}_2 - z\cos\bar{\theta}_2) + i\omega_2 t\} + F\bar{B}_3 \exp\{i\bar{\delta}_2(x \sin\bar{\theta}_3 - z\cos\bar{\theta}_3) + i\omega_3 t\}, \quad (57)$$

$$\text{Where} E = \frac{\bar{\delta}_1{}^2\left(\bar{\delta}_1{}^2 - \frac{\omega^2}{(c_2{}^2 + c_3{}^2)} + pq\right)}{\text{deno}.} \quad (58)$$

$$F = \frac{\bar{\delta}_2{}^2\left(\bar{\delta}_2{}^2 - \frac{\omega^2}{(c_2{}^2 + c_3{}^2)} + pq\right)}{\text{deno}.} \quad (59)$$

and

$$\text{deno.} = p\left(2q - \frac{\omega^2}{c_4{}^2}\right), \quad \bar{\delta}_1{}^2 = \lambda_1{}^2\omega^2, \quad \bar{\delta}_2{}^2 = \lambda_2{}^2\omega^2 \quad (60)$$

where $\bar{k}_0$, $\bar{\delta}_1$ and $\bar{\delta}_2$ are the wave numbers and $\bar{B}_1$, $\bar{B}_2$ and $\bar{B}_3$ are amplitudes of refracted wave, refracted coupled transverse and refracted micro-rotation P-wave or SV-wave, respectively and to be determined from boundary conditions.

4. Boundary Conditions

The appropriate boundary conditions are the continuity of displacement, micro rotation and stresses at the interface separating media M_1 and M_2. Mathematically, these boundary conditions at z=0 can be expressed as:

$$t_{zz} = t_{zz}^S - p; \; t_{zx} = t_{zx}^S; \; m_{zy} = 0;$$

$$t_{zx} = k_t(u^S - u); \; t_{zz}^S = k_n(w^S - w). \quad (61)$$

In order to satisfy the boundary conditions, the extension of the Snell's law will be

$$\frac{\sin\theta_0}{V_0} = \frac{\sin\theta_1}{V_1} = \frac{\sin\theta_2}{V_2} = \frac{\sin\bar{\theta}_1}{V_{11}} = \frac{\sin\bar{\theta}_2}{\lambda_1^{-1}} = \frac{\sin\bar{\theta}_3}{\lambda_2^{-1}} \quad (62)$$

For P-wave,

$$V_0 = V_1, \quad \theta_0 = \theta_1, \quad (63)$$

For SV-wave,

$$V_0 = V_2, \quad \theta_0 = \theta_2, \quad (64)$$

Also

$$k_1 V_1 = k_2 V_2 = \bar{k}_0 V_{11} = \bar{\delta}_1 \lambda_1^{-1} = \bar{\delta}_2 \lambda_2^{-1} = \omega, \quad \text{at } z = 0 \quad (65)$$

For the incident longitudinal wave at the interface z=0, putting $B_{01} = 0$ in the equation (53) and for the incident transverse wave putting $A_{01} = 0$ in the equation (52). Using equations (52)-(53) and (55)-(57) in equations (22) - (24) and (48)-(51) and with the help of (61)-(65), we get a system of five non homogeneous equations which can be written as

$$\sum_{j=1}^{5} a_{ij} Z_j = Y_i, \quad (i = 1,2,3,4,5) \quad (66)$$

where

$$Z_1 = \frac{A_1}{A^*}; \; Z_2 = \frac{B_1}{A^*}; \; Z_3 = \frac{\bar{B}_1}{A^*}; \; Z_4 = \frac{\bar{B}_2}{A^*}; \quad Z_5 = \frac{\bar{B}_3}{A^*} \quad (67)$$

where Z_1 to Z_5 are the amplitudes ratios of reflected P-wave and reflected SV-wave, refracted longitudinal wave, refracted coupled-wave at an angle $\bar{\theta}_2$, refracted coupled-wave at an angle $\bar{\theta}_3$. Also a_{ij} and Y_i in non-dimensional form are as

$$a_{11} = -\left\{\frac{\lambda^s}{\mu^s} + 2\cos^2\theta_1 + \frac{m_2}{\mu^s k_1^2}\right\},$$

$$a_{12} = -\frac{2k_2^2}{k_1^2}\sin\theta_2\cos\theta_2,$$

$$a_{13} = \frac{\bar{k}_0^2}{k_1^2}\left(\frac{\lambda}{\mu^s} + \frac{(2\mu + \kappa)}{\mu^s}\cos^2\bar{\theta}_1\right),$$

$$a_{14} = \frac{(2\mu + \kappa)}{\mu^s}\frac{\bar{\delta}_1^2}{k_1^2}\sin\bar{\theta}_2\cos\bar{\theta}_2,$$

$$a_{15} = \frac{(2\mu + \kappa)}{\mu^s}\frac{\bar{\delta}_2^2}{k_1^2}\sin\bar{\theta}_3\cos\bar{\theta}_3;$$

$$a_{21} = 2\sin\theta_1\cos\theta_1,$$

$$a_{22} = \frac{k_2^2}{k_1^2}(\cos^2\theta_2 - \sin^2\theta_2),$$

$$a_{23} = \frac{(2\mu+\kappa)}{\mu^s}\frac{\bar{k}_0^2}{k_1^2}\sin\bar{\theta}_1\cos\bar{\theta}_1,$$

$$a_{24} = \frac{\bar{\delta}_1^2}{\mu^s}\left\{\mu\cos2\bar{\theta}_2 + k\cos^2\bar{\theta}_2 - \frac{kE}{\bar{\delta}_1^2}\right\},$$

$$a_{25} = \frac{\bar{\delta}_2^2}{\mu^s}\left(\mu\cos2\bar{\theta}_3 + k\cos^2\bar{\theta}_3 - \frac{kF}{\bar{\delta}_2^2}\right);$$

$$a_{31} = a_{32} = a_{33} = 0,\ a_{34} = \cos\bar{\theta}_2,$$

$$a_{35} = \frac{\bar{\delta}_2 F}{k_1 E}\cos\bar{\theta}_3;\ a_{41} = i\frac{\sin\theta_0}{\mu^s},$$

$$a_{42} = i\frac{k_2}{k_1\mu^s}\cos\theta_2,$$

$$a_{43} = -\frac{1}{k_1\mu^s}\left\{\frac{(2\mu+k)\bar{k}_0^2}{k_t}\sin\bar{\theta}_1\cos\bar{\theta}_1 + i\bar{k}_0\sin\bar{\theta}_1\right\},$$

$$a_{44} = -\frac{1}{\mu^s}\left\{\frac{\mu\bar{\delta}_1^2}{k_1k_t}(\cos^2\bar{\theta}_2 - \sin^2\bar{\theta}_2) + \frac{k\bar{\delta}_1^2}{k_1k_t}\cos^2\bar{\theta}_2 + i\frac{\bar{\delta}_1}{k_1}\cos\bar{\theta}_2 + \frac{kE}{k_t}\right\},$$

$$a_{45} = -\frac{1}{\mu^s}\left\{\frac{\mu\bar{\delta}_2^2}{k_1k_t}(\cos^2\bar{\theta}_3 - \sin^2\bar{\theta}_3) + \frac{k\bar{\delta}_2^2}{k_1k_t}\cos^2\bar{\theta}_3 + i\frac{\bar{\delta}_2}{k_1}\cos\bar{\theta}_3 + \frac{kF}{k_t}\right\};$$

$$a_{51} = i\frac{\cos\theta_1}{\mu^s};\ a_{52} = -i\frac{k_2}{k_1\mu^s}\sin\theta_2,$$

$$a_{53} = \frac{\bar{k}_0^2}{\mu^s k_1 k_n}(\lambda + (2\mu+k)\cos^2\bar{\theta}_1) + i\frac{\bar{k}_0}{k_1\mu^s}\cos\bar{\theta}_1,$$

$$a_{54} = -\frac{1}{k_1\mu^s}\left\{\frac{(2\mu+k)\bar{\delta}_1^2}{k_n}\sin\bar{\theta}_2\cos\bar{\theta}_2 + i\bar{\delta}_1\sin\bar{\theta}_2\right\},$$

$$a_{55} = -\frac{1}{k_1\mu^s}\left\{\frac{(2\mu+k)\bar{\delta}_2^2}{k_n}\sin\bar{\theta}_3\cos\bar{\theta}_3 + i\bar{\delta}_2\sin\bar{\theta}_3\right\}; \quad (68)$$

For incident longitudinal P-wave:

$$A^* = A_{01}, B_{01} = 0, Y_1 = -a_{11}, Y_2 = a_{21}, Y_3 = a_{31}, Y_4 = -a_{41}, Y_5 = a_{51}. \quad (69)$$

For incident transverse SV- wave:

$$A^* = B_{01}, A_{01} = 0, Y_1 = a_{12}, Y_2 = -a_{22},$$

$$Y_3 = a_{32}, Y_4 = a_{42}, Y_5 = -a_{52}. \quad (70)$$

5. Particular Case

Case I: Normal force stiffness ($k_n \neq 0, k_t \to \infty$)

In this case, we obtained a system of five non-homogeneous equations as those given by equation (66) with the change a_{ij} as

$$a_{43} = -\frac{1}{k_1\mu^s}\ i\bar{k}_0\sin\bar{\theta}_1, \qquad a_{44} = -\frac{1}{k_1\mu^s}i\bar{\delta}_1\cos\bar{\theta}_2,$$

$$a_{45} = -\frac{1}{k_1\mu^s}i\bar{\delta}_2\cos\bar{\theta}_3. \quad (71)$$

Case II: Transverse force stiffness ($k_n \to \infty, k_t \neq 0$)

In this case also, a system of five non-homogeneous equations as those given by equation (66) is obtained with the changed a_{ij} as given below

$$a_{53} = \frac{1}{k_1\mu^s}\ i\bar{k}_0\cos\bar{\theta}_1,\ \ a_{54} = -\frac{1}{k_1\mu^s}i\bar{\delta}_1\sin\bar{\theta}_2,$$

$$a_{54} = -\frac{1}{k_1\mu^s}i\bar{\delta}_2\sin\bar{\theta}_3 \quad (72)$$

Case III: Welded contact ($k_n \to \infty, k_t \to \infty$)

Again in this case, a system of five non-homogeneous equations is obtained as those given by equation (66) with the changed a_{ij} as

$$a_{43} = -\frac{1}{k_1\mu^s}\ i\bar{k}_0\sin\bar{\theta}_1,\ \ a_{44} = -\frac{1}{k_1\mu^s}i\bar{\delta}_1\cos\bar{\theta}_2,$$

$$a_{45} = -\frac{1}{k_1\mu^s}i\bar{\delta}_2\cos\bar{\theta}_3.\ \ a_{53} = \frac{1}{k_1\mu^s}\ i\bar{k}_0\cos\bar{\theta}_1,$$

$$a_{54} = -\frac{1}{k_1\mu^s}i\bar{\delta}_1\sin\bar{\theta}_2\ ;\ a_{54} = -\frac{1}{k_1\mu^s}i\bar{\delta}_2\sin\bar{\theta}_3 \quad (73)$$

6. Numerical Results and Discussion

The theoretical results obtained above indicate that the amplitudes ratios $Z_i, (i = 1,2,3,4,5)$ depend on the angle of

incidence of incident wave P-wave or SV-Wave. In order to study in more detail the behaviour of various amplitudes ratios, we have computed them numerically for a particular model for which the values of relevant elastic parameters are as follow

In medium M_1, the physical constants for fluid saturated porous medium are taken from de Boer, Ehlers and Liu (1993)as

$$\eta^S = 0.67, \quad \eta^F = 0.33, \quad \rho^S = 1.34\,\frac{Mg}{m^3},$$

$$\rho^F = 0.33\,\frac{Mg}{m^3}, \quad \lambda^S = 5.5833\,\frac{MN}{m^2}, K^F = 0.01\frac{m}{s},$$

$$\gamma^{FR} = 10.00\frac{KN}{m^3}, \quad \mu^S = 8.3750\frac{N}{m^2}, \qquad (74)$$

In medium M_2, the physical constants for micropolar elastic solid are taken from Gauthier (1982) as

$$\lambda = 7.59 \times 10^{10}\,\frac{N}{m^2}, \qquad \mu = 1.89 \times 10^{10}\,\frac{N}{m^2},$$

$$\kappa = 1.49 \times 10^{8}\,\frac{N}{m^2}, \rho = 2.19 \times 10^{3}\,\frac{kg}{m^3},$$

$$\gamma = 2.68 \times 10^{4}\ N; j = 1.96 \times 10^{-6} m^2\ ; \frac{\omega^2}{{\omega_0}^2} = 200. \ (75)$$

A computer programme in MATLAB has been developed to calculate the modulus of amplitudes ratios $|Z_i|, (i = 1,2,3,4,5)$ for various reflected and refracted waves for the particular model and to depict graphically. $|Z_1|$ and $|Z_2|$ represent the modulus of amplitudes ratios for reflected P-wave or reflected SV-wave when P-wave is an incident wave or SV-wave is an incident wave respectively. Also $|Z_3|, |Z_4|$ and $|Z_5|$ represent the modulus of amplitudes ratios for refracted P-wave or refracted SV-wave when P-wave is an incident wave or SV-wave is an incident wave respectively. The variations in all figures are shown for the range $0^o \leq \theta \leq 90^o$.

Incident P-wave

Figures (2)–(6) depicts the variations of modulus of the amplitudes ratios of reflected P-wave and refracted P-wave with angle of incidence of indent P-wave. In all these figures (2)–(6), dashed line represent the general case (GEN) of imperfect boundary, whereas small dashed line represents the normal force stiffness case (NFS). Also bold dashed line represents the transverse force stiffness case (TFS) and solid line depicts the welded contact (WD).

In figure (2), more variations in the values of GEN and WD cases. Also, for NFS and TFS cases the values are almost same. In figure (3), values are different for each case. In figure (4), all values are almost same. In figures (5)-(6), values for NFS are different from three cases which have almost same values.

Incident SV-wave

Figures (7)–(11) depicts the variations of modulus of the amplitudes ratios of reflected P-wave and refracted P-wave with angle of incidence of indent P-wave. In all these figures (7)–(11), dashed line represent the general case (GEN) of imperfect boundary, whereas small dashed line represents the normal force stiffness case (NFS). Also bold dashed line represents the transverse force stiffness case (TFS) and solid line depicts the welded contact (WD).

In figure (7), more variations in the values of GEN and WD cases. Also, for NFS and TFS cases the values are almost same. In figure (8), more variations in the values of GEN and WD cases. Also, for NFS and TFS cases the values are almost same. In figures (9)-(11), values for NFS are different from three cases which have almost same values.

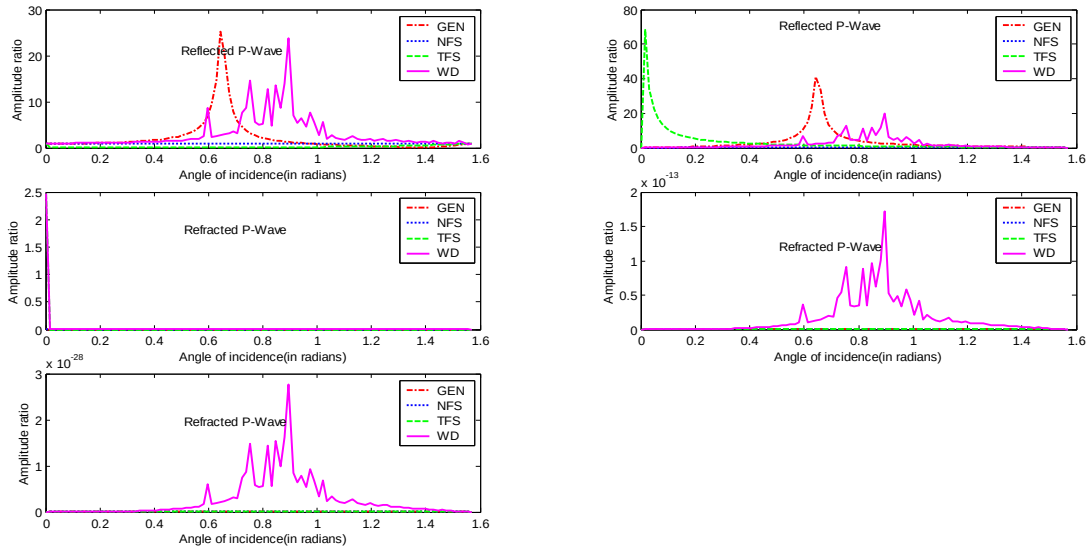

Fig.2-6: Variation of the amplitudes ratios $|Z_i|, i = 1,2,3,4,5.$ with angle of incidence of incident P-Wave.

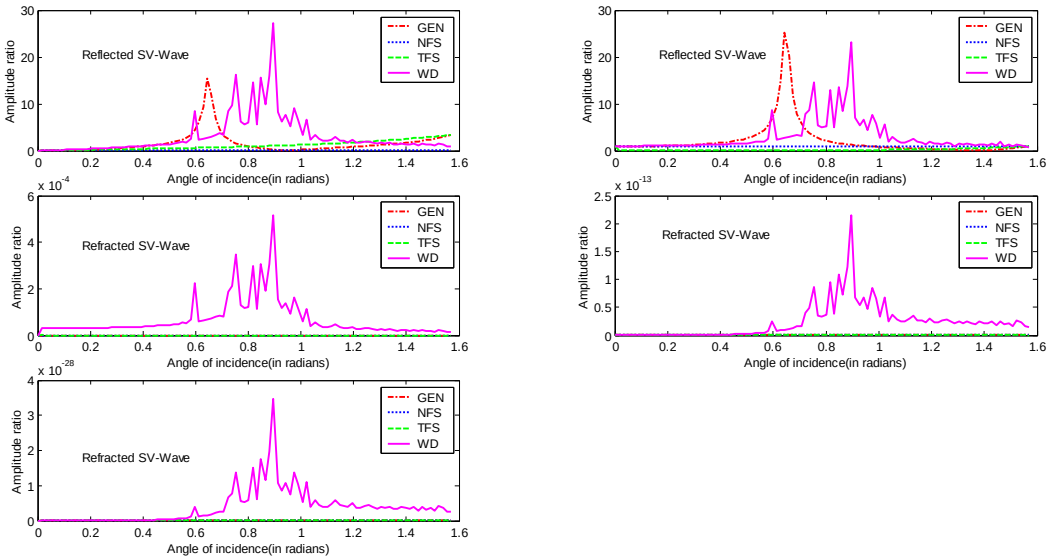

Fig.7-11: Variation of the amplitudes ratios $|Z_i|, i = 1,2,3,4,5.$ with angle of incidence of incident SV-Wave.

7. Conclusion

In conclusion, a mathematical study of reflection and refraction at an imperfect interface between fluid saturated porous solid half space and micropolar elastic solid half space is made when P-wave or SV-wave is incident. It is observed that the amplitudes ratios of various reflected and refracted waves depend on the angle of incidence of the incident wave and material properties of half spaces. Effect of stiffness is observed on amplitudes ratios. The model presented in this paper is one of the more realistic forms of the earth models. It may be of some use in engineering, seismology and geophysics etc.

8. References

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