Fuzzy Semi-Open Sets and Fuzzy Pre-Open Sets in Fuzzy Tri Topological Space

Ranu Sharma[1,a],Bhagyashri A. Deole[2,b]and Smita Verma[3,c]
[1,3]*Department of Applied Mathematics and Computational Science, SGSITS, Indore(M.P.), India,*

Abstract
The main aim of this paper is to introduce two new types of fuzzy open sets namely fuzzy tri semi-open sets and fuzzy tri pre-open sets in fuzzy tri topological space along with their several properties and characterizations. We introduce fuzzy tri continuous function , *fuzzy tri semi*-continuous*function and fuzzy tri pre*-continuous function and obtain some of their basic properties.

Keywords: *fuzzy tri* open sets, *fuzzy tri semi*-open sets, *fuzzy* tri pre-open sets, *fuzzy* tri *continuity*, *fuzzy* tri semi continuity and fuzzy tri pre continuity.

AMS *Mathematics Subject Classification (2010):* 54A40

1. Introduction

Levine N. [7] introduced the idea of semi-open sets and semi-continuity in topological space and Mashhour et. al [8] introduced the concept of pre-open sets and pre continuity in a topological space. Maheshwari S.N. and Prasad [9] introduced semi-open sets in bitopological spaces. Jelic M. [4] generalized the idea of pre-open sets and pre continuity in bitopological space.

The study of tri-topological space was first initiated by Martin M. Kovar [6] .S. Palaniammal [10] studied tri topological space and introduced semi-open and pre-open sets in tri topological space and he also introduced fuzzy tri topological space. N.F. Hameed and Moh. Yahya Abid [3] gives the definition of 123 open set in tri topological spaces. We [14] studied properties of tri semi-open sets and tri pre-open sets in tri topological space.

In 1965, Zadeh L.A. [15] introduced the concept of fuzzy sets. In 1968 Change C.L. [2] introduced the concept of fuzzy topological spaces. Kandil A. [5] introduced fuzzy bitopological spaces in 1991, Fuzzy semi-open sets and fuzzy semi continuous mappings in fuzzy topological spaces was studied by Azad K. K. [1]. Sahana A. S. Bin [12] defined the concept of pre-open sets in fuzzy topological space. Thakur S.S and Malviya R. [13] introduced semi-open sets, semi continuity in fuzzy bitopological spaces. Sampath Kumar S. [11] defined a (τ_i, τ_j) fuzzy pre-open set and characterized a fuzzy pair wise pre continuous mappings on a fuzzy bitopological space.

The purpose of the present paper is to introduce fuzzy tri semi-open sets and fuzzy tri pre-open sets, fuzzy tri continuous function, fuzzy tri semi-continuous function and fuzzy tri pre-continuous functions and their basic properties.

2. Preliminaries

Definition 2.1[10]: Let X be a nonempty set and T_1, T_2 andT_3 are three topologies on X .The set X together with three topologies is called a tri topological space and is denoted by (X, T_1, T_2, T_3).

Definition 2.2[10]: Let(X, T_1, T_2, T_3) be a tri topological space. $A \subset X$ is called semi(1,2,3) open if $A \subset T_{1,2,3}clT_{1,2,3}\,\mathrm{int}A$.

Definition 2.3[10]: Let(X, T_1, T_2, T_3) be a tri topological space. $A \subset X$ is called pre(1,2,3) open if $A \subset T_{1,2,3}\,\mathrm{int}\,T_{1,2,3}cl\,A$.

Definition 2.4[14]: A function f from a tri topological space (X, P_1, P_2, P_3) into another tri topological space (Y, W_1, W_2, W_3) is called tri semi continuous if $f^{-1}(V)$ is tri semi open set in X for each tri open set V in Y .

Definition 2.5[14]: A function f defined from a tri topological space (X, P_1, P_2, P_3) into another tri topological space (Y, W_1, W_2, W_3) is called tri pre continuous function if $f^{-1}(V)$ is tri pre-open set in X for each tri open set V in Y.

Definition 2.6[13]: Let μ be a fuzzy set on a fuzzy bitopological space X. Then μ is called (τ_i, τ_j)-fuzzy semi open set on X if $\mu \le \tau_i - cl(\tau_j - \mathrm{int}(\mu))$.

Definition 2.7 [11]: Let μ be a fuzzy set on a fuzzy bitopological space X. Then μ is called (τ_i, τ_j)-fuzzy pre-open set on X if $\mu \le \tau_i - \mathrm{int}(\tau_j - cl(\mu))$.

Definition 2.8[13]: Let $f : (X, \tau_1, \tau_2) \to (X^*, \tau^*_1, \tau_2{}^*)$ be a mapping .Then f is called fuzzy pairwise semi-continuous mapping if $f^{-1}(\mu)$ is a $(\tau_i, \tau_j) - \mathrm{fso}$ set on X for each $\tau_i{}^* - fo$ set μ on Y.

Definition 2.9[11]: Let $f : (X, \tau_1, \tau_2) \to (X^*, \tau^*_1, \tau_2{}^*)$ be a mapping .Then f is called fuzzy pairwise pre-continuous mapping if $f^{-1}(\mu)$ is a $(\tau_i, \tau_j) - \mathrm{fpo}$ set on X for each $\tau_i{}^* - fo$ set μ on Y.

3. Fuzzy Continuous Function in Fuzzy Tri Topological Spaces

Definition 3.1: Consider two fuzzy tri topological spaces $(X, \tau_1, \tau_2, \tau_3)$, $(Y, \tau'_1, \tau'_2, \tau'_3)$. A fuzzy function $f : I^X \to I^Y$ is called a fuzzy tri continuous function if χ_λ is fuzzy tri open in X, for every tri open set χ_λ in Y.

Example 3.2: Let $X = \{a, b, c\}$ be a non-empty fuzzy set.

Consider three fuzzy topologies $\tau_1 = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{a\}}\}$, $\tau_2 = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{a,b\}}\}$, $\tau_3 = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{b\}}\}$.

Fuzzy open sets in fuzzy tri topological spaces are union of all three topologies.

Fuzzy tri open sets of $X = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{a\}}, \chi_{\{a,b\}}, \chi_{\{b\}}\}$.

Let $Y = \{d, e, f\}$, $\tau'_1 = \{\tilde{1}_Y, \tilde{0}_Y\}$, $\tau'_2 = \{\tilde{1}_Y, \tilde{0}_Y, \chi_{\{e\}}\}$, $\tau'_3 = \{\tilde{1}_Y, \tilde{0}_Y, \chi_{\{d\}}, \chi_{\{d,e\}}\}$

Fuzzy tri open sets of Y are $\{\tilde{1}_Y, \tilde{0}_Y, \chi_{\{d\}}, \chi_{\{e\}}, \chi_{\{d,e\}}\}$.

Consider the function $f : I^X \to I^Y$ is defined as

$$f^{-1}\chi_{\{d\}} = \chi_{\{b\}}, \; f^{-1}\chi_{\{e\}} = \chi_{\{a\}},$$
$$f^{-1}\chi_{\{d,e\}} = \chi_{\{a,b\}}, \; f^{-1}\{\tilde{0}_Y\} = \{\tilde{0}_X\}, \; f^{-1}\{\tilde{1}_Y\} = \{\tilde{1}_X\}.$$

Here the inverse image of each fuzzy tri open set in Y under f is fuzzy tri open set in X. Hence f is fuzzy tri continuous function.

Theorem 3.3: A function $f : (X, \tau_1, \tau_2, \tau_3) \to (Y, \tau'_1, \tau'_2, \tau'_3)$ is fuzzy tri continuous if and only if the inverse image of every fuzzy tri open set in Y is fuzzy tri open in X.

Proof: (Necessary): Let $f : I^X \to I^Y$ be a fuzzy tri continuous function and χ_δ be any fuzzy tri open set in Y .Then $\tilde{1}_Y - \chi_\delta$ is fuzzy tri closed in Y. Since f is fuzzy tri continuous function, $f^{-1}(\tilde{1}_Y - \chi_\delta) = \tilde{1}_X - f^{-1}(\chi_\delta)$ is fuzzy tri closed in X and hence $f^{-1}(\chi_\delta)$ is fuzzy tri open in X .

(Sufficiency): Assume that $f^{-1}(\chi_\lambda)$ is fuzzy tri open in X for each fuzzy open set χ_λ in Y . Let χ_λ be a fuzzy closed set in Y .Then $\tilde{1}_Y - \chi_\lambda$ is fuzzy tri open in Y .By assumption $f^{-1}(\tilde{1}_Y - \chi_\lambda) = \tilde{1}_X - f^{-1}(\chi_\lambda)$ is fuzzy tri open in X which implies that $f^{-1}(\chi_\lambda)$ is fuzzy tri closed in $(X, \tau_1, \tau_2, \tau_3)$. Hence f is fuzzy tri continuous function.

Theorem 3.4: Let $(X, \tau_1, \tau_2, \tau_3)$ and $(Y, \tau'_1, \tau'_2, \tau'_3)$ be two fuzzy tri topological spaces. Then, $f : I^X \to I^Y$ is fuzzy tri continuous function if and only if $f^{-1}(\chi_\lambda)$ is fuzzy tri closed in X whenever χ_λ is fuzzy tri closed in Y .

Theorem 3.5: Let $(X, \tau_1, \tau_2, \tau_3)$ and $(Y, \tau'_1, \tau'_2, \tau'_3)$ be two fuzzy tri topological spaces. Then $f : I^X \to I^Y$ is fuzzy tri continuous function if and only if $f(cl(\chi_\lambda)) \prec cl(f(\chi_\lambda))\ \forall\ \chi_\lambda \prec \tilde{1}_X$

Proof: Suppose $f : I^X \to I^Y$ is a fuzzy tri continuous function. Since $tri-cl[f(\chi_\lambda)]$ is fuzzy tri closed in Y .Then
by theorem (3.4) $f^{-1}[\mathrm{tri}-cl(f(\chi_\lambda))]$ is fuzzy tri closed in X ,

$tri-cl(f^{-1}(\mathrm{tri}-clf(\chi_\lambda))) = f^{-1}(\mathrm{tri}-clf(\chi_\lambda))$ ----(1).

Now : $f(\chi_\lambda) \prec tri-cl(f(\chi_\lambda))$, $\chi_\lambda \prec f^{-1}(f(\chi_\lambda)) \prec f^{-1}(\mathrm{tri}-clf(\chi_\lambda))$.

Then $tri-cl(\chi_\lambda) \prec tri-cl(f^{-1}(\mathrm{tri}-clf(\chi_\lambda))) = f^{-1}(\mathrm{tri}-clf(\chi_\lambda))$ by (1).

Then $f(\mathrm{tri}-cl(\chi_\lambda)) \prec tri-cl(f(\chi_\lambda))$.

Conversely, Let $f(\mathrm{tri}-cl(\chi_\lambda)) \prec tri-cl(f(\chi_\lambda))\ \ \forall\ \chi_\lambda \prec \tilde{1}_X$.

Let χ_δ be fuzzy tri closed set in Y , So that $tri-cl(\chi_\delta) = \chi_\delta$. Now $f^{-1}(\chi_\delta) \prec \tilde{1}_X$, by hypothesis,

$f(\mathrm{tri}-cl(f^{-1}(\chi_\delta))) \prec tri-cl(f(f^{-1}(\chi_\delta)) \prec tri-cl(\chi_\delta) = \chi_\delta$.

Therefore, $tri-cl(f^{-1}(\chi_\delta) \prec f^{-1}(\chi_\delta)$ .But $f^{-1}(\chi_\delta) \prec cl(f^{-1}(\chi_\delta)$ always.

Hence $tri-cl(f^{-1}(\chi_\delta) = f^{-1}(\chi_\delta)$ and so $f^{-1}(\chi_\delta)$ is fuzzy tri closed in X .

Hence by theorem (3.4) f is fuzzy tri continuous function.

Theorem 3.6: Let $(X, \tau_1, \tau_2, \tau_3)$ and $(Y, \tau'_1, \tau'_2, \tau'_3)$ be two fuzzy tri topological spaces. Then, $f : I^X \to I^Y$ is fuzzy tri continuous function if and only if $tri-cl(f^{-1}(\chi_\lambda)) \prec f^{-1}(\mathrm{tri}-cl(\chi_\lambda)), \forall\ \chi_\lambda \prec \tilde{1}_Y$.

Proof: Suppose $f : I^X \to I^Y$ is fuzzy tri continuous function. Since $tri-cl(\chi_\lambda)$ is fuzzy tri closed in Y

Then by theorem (3.4) $f^{-1}(\mathrm{tri}-cl(\chi_\lambda))$ is fuzzy tri closed in X and therefore,

$tri-cl(f^{-1}(\mathrm{tri}-cl(\chi_\lambda))) = f^{-1}(\mathrm{tri}-cl(\chi_\lambda))$.........(1)

Now, $\chi_\lambda \prec tri-cl(\chi_\lambda)$

$\Rightarrow f^{-1}(\chi_\lambda) \prec f^{-1}(\mathrm{tri}-cl(\chi_\lambda))$

Then $tri-cl(f^{-1}(\chi_\lambda)) \prec tri-cl(f^{-1}(\mathrm{tri}-cl(\chi_\lambda))) = f^{-1}(\mathrm{tri}-cl(\chi_\lambda))$ by (1)

Conversely: Let the condition hold and let χ_δ be any fuzzy tri closed set in Y so that $tri-cl(\chi_\delta)=\chi_\delta$

By hypothesis,

$tri-cl(f^{-1}(\chi_\delta)) \prec f^{-1}(\mathrm{tri}-cl(\chi_\delta)) = f^{-1}(\chi_\delta)$.But $f^{-1}(\chi_\delta) \prec tri-cl(f^{-1}(\chi_\delta))$ always.

Hence $tri-cl(f^{-1}(\chi_\delta)) = f^{-1}(\chi_\delta)$ and so $f^{-1}(\chi_\delta)$ is fuzzy tri closed in X. It follows from theorem (3.4) that f is fuzzy tri continuous function.

Theorem 3.7: Let (X,τ_1,τ_2,τ_3) and $(Y,\tau'_1,\tau'_2,\tau'_3)$ be two fuzzy tri topological spaces. Then $f: I^X \to I^Y$ is fuzzy tri continuous function if and only if $f^{-1}(\mathrm{tri}-\mathrm{int}(\chi_\lambda)) \prec tri-\mathrm{int}(f^{-1}(\chi_\lambda)) \quad \forall\, \chi_\lambda \prec \tilde{1}_Y$.

Proof: Let $f: I^X \to I^Y$ be a fuzzy tri continuous function. Since $\mathrm{int}(\chi_\lambda)$ is fuzzy tri open in Y, then by theorem (3.3) $f^{-1}(\mathrm{tri}-\mathrm{int}(\chi_\lambda))$ is fuzzy tri open in X and therefore

$\mathrm{tri}-\mathrm{int}(f^{-1}(\mathrm{tri}-\mathrm{int}(\chi_\lambda))) = f^{-1}(\mathrm{tri}-\mathrm{int}(\chi_\lambda))$.........(1)

Now $\mathrm{tri}-\mathrm{int}(\chi_\lambda) \prec \chi_\lambda$, then

$f^{-1}(\mathrm{tri}-\mathrm{int}(\chi_\lambda)) \prec f^{-1}(\chi_\lambda)$ Then

$\mathrm{tri}-\mathrm{int}(f^{-1}(\mathrm{tri}-\mathrm{int}(\chi_\lambda))) \prec tri-\mathrm{int}(f^{-1}(\chi_\lambda))$ By (1)

Conversely: Let the condition hold and let χ_δ be any fuzzy tri open set in Y so that

$\mathrm{tri}-\mathrm{int}(\chi_\delta) = \chi_\delta$.

By hypothesis

$f^{-1}(\mathrm{tri}-\mathrm{int}(\chi_\delta)) \prec tri-\mathrm{int}\, f^{-1}(\chi_\delta)$

Since $f^{-1}(\mathrm{tri}-\mathrm{int}(\chi_\delta)) = f^{-1}(\chi_\delta)$ then

$f^{-1}(\chi_\delta) \prec tri-\mathrm{int}(f^{-1}(\chi_\delta))$

But $\mathrm{int}(f^{-1}(\chi_\delta)) \prec f^{-1}(\chi_\delta)$ always

So, $\mathrm{tri}-\mathrm{int}(f^{-1}(\chi_\delta)) = f^{-1}(\chi_\delta)$.

Therefore $f^{-1}(\chi_\delta)$ is fuzzy tri open in X. Consequently by theorem (3.3) f is fuzzy tri continuous function.

4. Fuzzy Tri Semi-Open Sets and Fuzzy Tri Pre-Open Sets

Definition 4.1: Let (X,τ_1,τ_2,τ_3) be a fuzzy tri topological space then a fuzzy subset χ_λ of X is said to be fuzzy tri semi-open set if $\chi_\lambda \leq cl(int\chi_\lambda)$ and complement of fuzzy tri semi-open set is fuzzy tri semi-closed. The collection of all fuzzy tri semi-open sets of X is denoted by $FSO(X)$

Example 4.2 Let $X=\{a,b,c\}$ be a nonempty fuzzy set.

$\tau_1 = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{a\}}\}$, $\tau_2 = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{a,b\}}\}$, $\tau_3 = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{a,c\}}\}$

Fuzzy open sets in fuzzy tri topological spaces are union of all three topologies.

Fuzzy tri open sets of $X = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{a\}}, \chi_{\{a,b\}}, \chi_{\{a,c\}}\}$

Fuzzy tri semi-open sets of X are denoted by

$tri-FSO(X) = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{a\}}, \chi_{\{a,b\}}, \chi_{\{a,c\}}\}$.

Definition 4.3: Let $(X, \tau_1, \tau_2, \tau_3)$ be a fuzzy tri topological space then a fuzzy subset χ_λ of X is said to be fuzzy tri pre-open set if $\chi_\lambda \leq tri-int(tri-cl\chi_\lambda)$ and complement of fuzzy tri pre-open set is fuzzy tri pre-closed. The collection of all fuzzy tri semi-open sets of X is denoted by $tri-FPO(X)$.

Example 4.4: Let $X = \{a, b, c\}$ be a nonempty fuzzy set.

$\tau_1 = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{a\}}\}$ $\tau_2 = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{a,b\}}\}$ $\tau_3 = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{b,c\}}\}$

Fuzzy open sets in fuzzy tri topological space are union of all three topologies.

Then fuzzy tri open sets of $X = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{a\}}, \chi_{\{a,b\}}, \chi_{\{b,c\}}\}$

Fuzzy pre-open sets of X denoted by $tri-FPO(X) = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{a\}}, \chi_{\{a,b\}}, \chi_{\{b,c\}}\}$.

Definition 4.5: Let $(X, \tau_1, \tau_2, \tau_3)$ be a fuzzy tri topological space. Let $\chi_\lambda \prec \tilde{1}_X$. An element $\chi_{\{x\}} \leq \chi_\lambda$ is called fuzzy tri semi interior point of χ_λ, if there exist a fuzzy tri semi-open set χ_δ such that $\chi_{\{x\}} \leq \chi_\delta \prec \chi_\lambda$. The set of all fuzzy tri semi interior points of χ_λ is called the fuzzy tri semi interior of χ_λ and is denoted by $tri-s\,\mathrm{int}(\chi_\lambda)$.

Definition 4.6: Let $(X, \tau_1, \tau_2, \tau_3)$ be a fuzzy tri topological space. Let $\chi_\lambda \prec \tilde{1}_X$. A fuzzy element $\chi_{\{x\}} \leq \chi_\lambda$ is called fuzzy tri pre interior point of χ_λ, if there exist a fuzzy tri pre-open set χ_δ such that $\chi_{\{x\}} \leq \chi_\delta \prec \chi_\lambda$. The set of all fuzzy tri pre interior points of χ_λ is called the fuzzy tri pre interior of χ_λ and is denoted by $p\,\mathrm{int}(\chi_\lambda)$.

Theorem 4.7: Let $\chi_\lambda \prec \tilde{1}_X$ be a fuzzy tri topological space. $tri-s\,\mathrm{int}(\chi_\lambda)$ is equal to the union of all fuzzy tri semi-open sets contained in χ_λ.

Note 4.8: 1. $tri-p\,\mathrm{int}(\chi_\lambda) \prec \chi_\lambda$.

2. $tri-p\,\mathrm{int}(\chi_\lambda)$ is fuzzy semi-open sets.

Theorem 4.9: $tri-s\,\mathrm{int}(\chi_\lambda)$ is the largest fuzzy tri semi-open sets contained in χ_λ.

Theorem 4.10: χ_λ is fuzzy tri semi-open if and only if $\chi_\lambda = tri-s\,\mathrm{int}(\chi_\lambda)$

Theorem 4.11: $tri-s\,\mathrm{int}(\chi_\lambda \vee \chi_\delta) \succ tri-s\,\mathrm{int}\,\chi_\lambda \vee tri-s\,\mathrm{int}\,\chi_\delta$.

Definition 4.12: Let $(X, \tau_1, \tau_2, \tau_3)$ be a fuzzy tri topological space. Let $\chi_\lambda \prec \tilde{1}_X$. The intersection of all fuzzy tri semi-closed sets containing χ_λ is called a fuzzy tri semi closure of χ_λ and is denoted as $tri-scl(\chi_\lambda)$.

Note 4.13: Intersection of fuzzy tri semi-closed sets is fuzzy tri semi-closed set, $tri-scl(\chi_\lambda)$ is a fuzzy tri semi-closed set.

Note 4.14: $tri-scl(\chi_\lambda)$ is the smallest fuzzy tri semi-closed set containing χ_λ.

Theorem 4.15: χ_λ is fuzzy tri semi-closed set if and only if $\chi_\lambda = tri-scl(\chi_\lambda)$.

Theorem 4.16: Let χ_λ and χ_δ be fuzzy subsets of $(X, \tau_1, \tau_2, \tau_3)$ and $\chi_{\{x\}} \leq \tilde{1}_X$

a) χ_λ is fuzzy tri pre-closed if and only if $\chi_\lambda = tri-pcl(\chi_\lambda)$

b) If $\chi_\lambda \prec \chi_\delta$, then $tri-pcl(\chi_\lambda) \prec tri-pcl(\chi_\delta)$.

c) $\chi_{\{x\}} \le tri-pcl(\chi_\lambda)$ If and only if $\chi_\lambda \wedge \chi_\delta \ne \tilde{0}_X$ for every fuzzy tri pre-open set χ_δ containing $\chi_{\{x\}}$.

Theorem 4.17: Let χ_λ be a fuzzy subsets of (X,τ_1,τ_2,τ_3) **,** if there exist a fuzzy tri pre-open set χ_δ such that $\chi_\lambda \prec \chi_\delta \prec tri-cl(\chi_\lambda)$, then χ_λ is fuzzy tri pre-open.

Theorem 4.18: In a fuzzy tri topological space (X,τ_1,τ_2,τ_3), the union of any two fuzzy tri semi-open sets is always a fuzzy tri semi-open set.

Proof: Let χ_λ and χ_δ be any two fuzzy semi-open sets in X.

Now $\chi_\lambda \vee \chi_\delta \le tri-cl\left(\text{tri}-\text{int}\,\chi_\lambda\right) \vee tri-cl\left(\text{tri}-\text{int}\left(\chi_\delta\right)\right)$

$\Rightarrow \chi_\lambda \vee \chi_\delta \le tri-cl\left(\text{tri}-\text{int}\left(\chi_\lambda \vee \chi_\delta\right)\right)$. Hence $\chi_\lambda \vee \chi_\delta$ fuzzy tri semi-open sets.

Remark 4.19: The intersection of any two fuzzy tri semi-open sets may not be a fuzzy tri semi-open sets as show in the following example.

Example 4.20: Let $X = \{a,b,c\}$ be a nonempty fuzzy set.

$\tau_1 = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{a\}}\}$ $\tau_2 = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{a,b\}}\}$ $\tau_3 = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{b,c\}}\}$

Fuzzy open sets in fuzzy tri topological space are union of all three fuzzy topologies.

Then fuzzy tri open sets of $X = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{a\}}, \chi_{\{a,b\}}, \chi_{\{b,c\}}\}$

Fuzzy tri semi-open set of X is denoted by $tri-FSO(X) = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{a\}}, \chi_{\{a,b\}}, \chi_{\{b,c\}}\}$ **.**

Here $\chi_{\{a,b\}} \wedge \chi_{\{b,c\}} = \chi_{\{b\}} \succ tri-FSO(X)$ **.**

Theorem 4.21: if χ_λ is a fuzzy tri open set then χ_λ is a fuzzy tri semi-open set.

Proof: Let χ_λ is a fuzzy tri open set.

Therefore, $\chi_\lambda = tri-\text{int}(\chi_\lambda)$.

Now, $\chi_\lambda \prec tri-cl(\chi_\lambda) = tri-cl\left(tri-int\left(\chi_\lambda\right)\right)$.Hence χ_λ isa fuzzy tri semi-open set.

Theorem 4.22: Let χ_λ and χ_δ be fuzzy subsets of X such that $\chi_\delta \le \chi_\lambda \le tri-cl(\chi_\delta)$ .if χ_δ is fuzzy tri semi-open set then χ_λ is also fuzzy tri semi-open set.

Proof: Given χ_δ is fuzzy tri semi-open set. So, we have $\chi_\delta \le tri-cl\left(tri-int\,\chi_\delta\right) \le tri-cl\left(tri-int\left(\chi_\lambda\right)\right)$. Thus

$tri-cl(\chi_\delta) \le tri-cl\left(tri-int\left(\chi_\lambda\right)\right)$. Hence χ_λ is also a fuzzy tri semi-open set.

5. Fuzzy Tri Semi Continuity and Fuzzy Tri Pre Continuity in Fuzzy Tri topological space

Definition 5.1:A fuzzy function f from a fuzzy tri topological space (X,τ_1,τ_2,τ_3) into another fuzzy tri topological space $(Y,\tau'_1,\tau'_2,\tau'_3)$ is called fuzzy tri semi continuous if $f^{-1}\left(\chi_\lambda\right)$ is fuzzy tri semi-open set in X for each fuzzy tri open set χ_λ in Y.

Example 5.2: Let $X = \{a,b,c\}$ be a nonempty fuzzy set.

$\tau_1 = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{a\}}\}$ $\tau_2 = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{a,b\}}\}$ $\tau_3 = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{b,c\}}\}$

Fuzzy open sets in fuzzy tri topological spaces are union of all three fuzzy topologies.

Then fuzzy tri open sets of $X = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{a\}}, \chi_{\{a,b\}}, \chi_{\{b,c\}}\}$

$tri-FSO(X)$ Sets of $X = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{a\}}, \chi_{\{a,b\}}, \chi_{\{b,c\}}\}$.

Let $Y = \{1,2,3\}$ be a nonempty fuzzy set.

$\tau'_1 = \{\tilde{1}_Y, \tilde{0}_Y, \chi_{\{1,3\}}\}$, $\tau'_2 = \{\tilde{1}_Y, \tilde{0}_Y, \chi_{\{2\}}\}$, $\tau'_3 = \{\tilde{1}_Y, \tilde{0}_Y, \chi_{\{2\}}, \chi_{\{1,2\}}\}$

Fuzzy tri open sets of $Y = \{\tilde{1}_Y, \tilde{0}_Y, \chi_{\{2\}}, \chi_{\{1,2\}}, \chi_{\{1,3\}}\}$.

$tri-FSO(Y)$ Sets of $Y = \{\tilde{1}_Y, \tilde{0}_Y, \chi_{\{2\}}, \chi_{\{1,2\}}, \chi_{\{1,3\}}\}$.

Consider the fuzzy function $f : I^X \to I^Y$ is defined as

$$f^{-1}(\chi_{\{2\}}) = \chi_{\{a\}},\ f^{-1}(\chi_{\{1,2\}}) = \chi_{\{a,b\}},\ f^{-1}(\chi_{\{1,3\}}) = \chi_{\{b,c\}},\ f^{-1}(\tilde{0}_Y) = \tilde{0}_X,\ \ f^{-1}(\tilde{1}_Y) = \tilde{1}_X .$$

Since the inverse image of each fuzzy tri open set in Y under f is fuzzy tri semi-open set in X . Hence f is fuzzy tri semi continuous function.

Definition 5.3: A fuzzy function f defined from a fuzzy tri topological space $(X, \tau_1, \tau_2, \tau_3)$ into another fuzzy tri topological space $(Y, \tau'_1, \tau'_2, \tau'_3)$ is called fuzzy tri pre continuous function if $f^{-1}(\chi_\lambda)$ is fuzzy tri pre-open set in X for each fuzzy tri open set χ_λ in Y .

Example 5.4: Let $X = \{a,b,c\}$ be a fuzzy set with three fuzzy topologies

$\tau_1 = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{a\}}\}$, $\tau_2 = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{b\}}\}$, $\tau_3 = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{b,c\}}\}$

Fuzzy open sets in fuzzy tri topological space are union of all three fuzzy topologies.

Fuzzy tri open sets of $X = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{a\}}, \chi_{\{b\}}, \chi_{\{b,c\}}\}$.

$tri-FPO(X)$ Sets of $X = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{a\}}, \chi_{\{b\}}, \chi_{\{b,c\}}\}$.

Let $Y = \{1,2,3\}$ be a fuzzy set with three fuzzy topologies

$\tau'_1 = \{\tilde{1}_Y, \tilde{0}_Y, \chi_{\{2\}}\}$, $\tau'_2 = \{\tilde{1}_Y, \tilde{0}_Y, \chi_{\{3\}}\}$, $\tau'_3 = \{\tilde{1}_Y, \tilde{0}_Y, \chi_{\{1,3\}}\}$

Fuzzy tri open sets of $Y = \{\tilde{1}_Y, \tilde{0}_Y, \chi_{\{2\}}, \chi_{\{3\}}, \chi_{\{1,3\}}\}$.

$tri-FSO(Y)$ Sets of $Y = \{\tilde{1}_Y, \tilde{0}_Y, \chi_{\{2\}}, \chi_{\{3\}}, \chi_{\{1,3\}}\}$

Consider the function $f : I^X \to I^Y$ is defined as

$$f^{-1}(\chi_{\{2\}}) = \chi_{\{a\}},\ f^{-1}\chi_{\{3\}} = \chi_{\{b\}},\ f^{-1}\chi_{\{1,3\}} = \chi_{\{b,c\}}\ f^{-1}(\tilde{0}_Y) = \tilde{0}_X,\ \ f^{-1}(\tilde{1}_Y) = \tilde{1}_X .$$

Since the inverse image of each fuzzy tri open set in Y under f is fuzzy tri pre-open set in X. Hence f is fuzzy tri pre continuous function.

Theorem 5.5: Let $f:(X,\tau_1,\tau_2,\tau_3)\to(Y,\tau'_1,\tau'_2,\tau'_3)$ be a fuzzy tri pre continuous open function .If χ_λ is a fuzzy tri pre-open set of X, then $f(\chi_\lambda)$ is fuzzy tri pre-open in Y.

Proof : First ,let χ_λ be fuzzy tri pre-open set in X .There exist an fuzzy tri open set χ_δ in X such that $\chi_\lambda \prec \chi_\delta \prec tri-cl(\chi_\lambda)$ .since f is fuzzy tri open function then $f(\chi_\delta)$ is fuzzy tri open in Y .Since f is fuzzy tri continuous function, we have $f(\chi_\lambda)\prec f(\chi_\delta)\prec f(\text{tri}-cl(\chi_\lambda))\prec tri-cl(f(\chi_\lambda))$ .This shows that $f(\chi_\lambda)$ is fuzzy tri pre-open in Y. Let χ_λ be fuzzy tri pre-open in X .There exist a fuzzy tri pre-open set χ_δ such that $\chi_\delta \prec \chi_\lambda \prec (\text{tri}-cl(\chi_\delta))$ .Since f is fuzzy tri continuous function, we have $f(\chi_\delta)\prec f(\chi_\lambda)\prec f(\text{tri}-cl(\chi_\delta))\prec tri-cl(f(\chi_\delta))$ by the proof of first part $f(\chi_\delta)$ is fuzzy tri pre-open in X .Therefore, $f(\chi_\lambda)$ is fuzzy tri pre-open in Y.

Theorem 5.6: Let $f:(X,\tau_1,\tau_2,\tau_3)\to(Y,\tau'_1,\tau'_2,\tau'_3)$ be a fuzzy tri pre continuous open function .If χ_λ is an fuzzy tri pre-open set of Y, then $f^{-1}(\chi_\lambda)$ is fuzzy tri pre-open in X.

Proof: First, let χ_λ be a fuzzy tri pre-open set of Y .There exist a fuzzy tri open set χ_δ in Y .such that $\chi_\lambda \prec \chi_\delta \prec tri-cl(\chi_\lambda)$ .Since f is fuzzy tri open set.

We have $f^{-1}(\chi_\lambda)\prec f^{-1}(\chi_\delta)\prec f^{-1}(\text{tri}-cl(\chi_\lambda))\prec tri-cl(f^{-1}(\chi_\lambda))$ .since f is fuzzy tri pre continuous, $f^{-1}(\chi_\delta)$ is fuzzy tri pre-open set in X .By theorem 4.18, $f^{-1}(\chi_\lambda)$ is fuzzy tri pre-open set in X .The proof of the second part is shown by using the fact of first part.

Theorem5.7: The following are equivalent for a fuzzy function $f:(X,\tau_1,\tau_2,\tau_3)\to(Y,\tau'_1,\tau'_2,\tau'_3)$

a) f is fuzzy tri pre-continuous function ;
b) the inverse image of each fuzzy tri closed set of Y is fuzzy tri pre closed in X ;
c) For each $\chi_{\{x\}}\le \tilde{1}_X$ and each fuzzy tri open set χ_λ in χ_δ containing $f(\chi_{\{x\}})$,there exist an fuzzy tri pre-open set χ_α of X containing $\chi_{\{x\}}$ such that $f(\chi_\alpha)\prec\chi_\lambda$;
d) $tri-pcl(f^{-1}(\chi_\lambda))\prec f^{-1}(\text{tri}-cl(\chi_\lambda))$ For every fuzzy subset χ_λ of Y.
e) $f(tri-pcl(\chi_\delta))\prec tri-cl(f(\chi_\delta))$ For every subset χ_δ of X.

Theorem5.8: If $f:(X,\tau_1,\tau_2,\tau_3)\to(Y,\tau'_1,\tau'_2,\tau'_3)$ and $g:(Y,\tau'_1,\tau'_2,\tau'_3)\to(Z,\tau''_1,\tau''_2,\tau''_3)$ be two fuzzy tri semi continuous functions then $gof:(X,\tau_1,\tau_2,\tau_3)\to(Z,\tau''_1,\tau''_2,\tau''_3)$ may not be fuzzy tri semi continuous function.

Theorem 5.9: Every fuzzy tri continuous function is fuzzy tri semi continuous function.

Theorem 5.10: Let $f^{-1}:(X,\tau_1,\tau_2,\tau_3)\to(Y,\tau'_1,\tau'_2,\tau'_3)$ be bijective .Then the following conditions are equivalent:

i) f is a fuzzy tri semi open continuous function.
ii) f is fuzzy tri semi closed continuous function and
iii) f^{-1} is fuzzy tri semi continuous function.

Proof:(i)$\rightarrow$ (ii) Suppose χ_λ is a fuzzy tri closed set in X .Then $\tilde{1}_X - \chi_\lambda$ is a fuzzy tri open set in X .Now by (i) $f(\tilde{1}_X - \chi_\lambda)$ is a fuzzy tri semi open set in Y .Now since f^{-1} is bijective so $f(\tilde{1}_X - \chi_\lambda) = \tilde{1}_Y - f(\chi_\lambda)$ .Hence $f(\chi_\lambda)$ is a fuzzy tri semi closed set in Y .Therefore f is a fuzzy tri semi closed continuous function.

(ii)$\rightarrow$ (iii) Let f is a fuzzy tri semi closed function and χ_λ be a fuzzy tri closed set of X .Since f^{-1} is bijective so $(f^{-1})^{-1}(\chi_\lambda)$ which is a fuzzy tri semi closed set in Y . Hence f^{-1} is fuzzy tri semi continuous function.

(iii)$\rightarrow$ (i) Let χ_λ be a fuzzy tri open set in X .Since f^{-1} is a fuzzy tri semi continuous function so $(f^{-1})^{-1}(\chi_\lambda) = f(\chi_\lambda)$ is a fuzzy tri semi-open set in Y . Hence f is fuzzy tri semi-open continuous function.

Theorem5.11: Let X and Y are two tri topological spaces. Then $f:(X,\tau_1,\tau_2,\tau_3) \rightarrow (Y,\tau'_1,\tau'_2,\tau'_3)$ is fuzzy tri semi continuous function if one of the followings holds:

i) $f^{-1}(\text{tri}-s\,\text{int}(\chi_\lambda)) \le tri-s\,\text{int}(f^{-1}(\chi_\lambda))$, for every fuzzy tri open set χ_λ in Y .

ii) $tri-scl(f^{-1}(\chi_\lambda)) \le f^{-1}(\text{tri}-scl(\chi_\lambda))$, for every fuzzy tri open set χ_λ in Y .

Proof: Let χ_λ be any fuzzy tri open set in Y and if condition (i) is satisfied then $f^{-1}(\text{tri}-s\,\text{int}(\chi_\lambda)) \le tri-s\,\text{int}(f^{-1}(\chi_\lambda))$ .We get $f^{-1}(\chi_\lambda) \le tri-s\,\text{int}(f^{-1}(\chi_\lambda))$ .Therefore $f^{-1}(\chi_\lambda)$ is a fuzzy tri semi open set in X .Hence f is a fuzzy tri semi continuous function. Similarly we can prove (ii).

Theorem 5.12: A fuzzy function $f:(X,\tau_1,\tau_2,\tau_3) \rightarrow (Y,\tau'_1,\tau'_2,\tau'_3)$ is called fuzzy tri semi open continuous function if and only if $f(\text{tri}-s\,\text{int}(\chi_\lambda)) \le tri-s\,\text{int}(f(\chi_\lambda))$, for every fuzzy tri open set χ_λ in X .

Proof: Suppose that f is a fuzzy tri semi open continuous function.

Since $tri-s\,\text{int}(\chi_\lambda) \le \chi_\lambda$ so $f(\text{tri}-s\,\text{int}(\chi_\lambda)) \le f(\chi_\lambda)$.

By hypothesis $f(\text{tri}-s\,\text{int}(\chi_\lambda))$ is a fuzzy tri semi open set and $tri-s\,\text{int}(f(\chi_\lambda))$ is largest fuzzy tri semi open set contained in $f(\chi_\lambda)$ so $f(\text{tri}-s\,\text{int}(\chi_\lambda)) \le tri-s\,\text{int}(f(\chi_\lambda))$.

Conversely, suppose χ_λ is a fuzzy tri open set in X .So $f(\text{tri}-s\,\text{int}(\chi_\lambda)) \le tri-s\,\text{int}(f(\chi_\lambda))$.

Now since $\chi_\lambda = tri-s\,\text{int}(\chi_\lambda)$ so $f(\chi_\lambda) \le tri-s\,\text{int}(f(\chi_\lambda))$ .Therefore $f(\chi_\lambda)$ is a fuzzy tri semi open set in Y and f is fuzzy tri semi open continuous function.

Theorem 5.13: A fuzzy function $f:(X,\tau_1,\tau_2,\tau_3) \rightarrow (Y,\tau'_1,\tau'_2,\tau'_3)$ is called fuzzy tri semi closed continuous function if and only if $tri-scl(f(\chi_\lambda)) \le f(\text{tri}-scl(\chi_\lambda))$, for every tri closed set χ_λ in X .

Proof: Suppose that f is a fuzzy tri semi closed continuous function. Since $\chi_\lambda \le tri-scl(\chi_\lambda)$ so $f(\chi_\lambda) \le f(\text{tri}-scl(\chi_\lambda))$ .By hypothesis, $f(\text{tri}-scl(\chi_\lambda))$ is a fuzzy tri semi closed set and

$tri-scl(f(\chi_\lambda))$ is fuzzy smallest tri semi closed set containing $f(\chi_\lambda)$ so $tri-scl(f(\chi_\lambda)) \le f(\text{tri}-scl(\chi_\lambda))$.

Conversely, suppose χ_λ is a fuzzy tri closed set in X .So $tri-scl(f(\chi_\lambda)) \le f(\text{tri}-scl(\chi_\lambda))$.

Since $\chi_\lambda = tri-scl(\chi_\lambda)$ so $tri-scl(f(\chi_\lambda)) \le f(\chi_\lambda)$ .Therefore $f(\chi_\lambda)$ is a fuzzy tri semi closed set in Y and f is a fuzzy tri semi closed continuous function.

Theorem 5.14: Every fuzzy tri semi continuous function is fuzzy tri continuous function.

CONCLUSION:

We studied new form of fuzzy semi-open set and fuzzy pre-open set in fuzzy tri topological space. We also studied fuzzy tri continuous function, fuzzy tri semi continuous function and fuzzy tri pre continuous function in fuzzy tri topological space. It is established that composition of any two fuzzy tri semi-open sets and two fuzzy pre-open sets in fuzzy tri topological space is again a fuzzy tri semi-open set and a fuzzy tri pre-open set respectively in fuzzy tri topological space .In fuzzy tri continuity, fuzzy tri semi continuity and fuzzy tri pre continuity, inverse image of every fuzzy tri open set, fuzzy tri semi-open set and fuzzy tri pre-open set is again fuzzy tri open set, fuzzy tri semi open and fuzzy tri pre-open set respectively.

REFERENCES

[1] Azad K.K., On fuzzy semi-continuity, fuzzy almost continuity and fuzzy weakly continuity, J. Math. ,Anal. Appl. 82(1981),14-32.

[2] Change C.L., Fuzzy topological spaces,(1968) J. Math. Anal. Appl. 24,182-190.

[3] Hammed N.F. & Mohammed Yahya Abid (2011).Certain types of separation axioms in tri topological spaces. Iraqi journal of science., Vol 52,(2),212-217.

[4] Jelic M., A decomposition of pairwise continuity. J. Inst. Math. Comput. Sci. Math.Ser, 1990, 3, 25-29.

[5] Kandil A., Biproximities and fuzzy bitopological spaces, Simon Steven 63(1989) ,45-66.

[6]Kovar, M.,(2000).On 3-Topological version of Thet-Reularity. Internat.J.Matj,Sci. ,23(6),393-398.

[7] Levine N.(1963). semi open sets and semi –continuity in topological spaces, Amer. Math., 70,36-41.

[8] Mashhour A.S.,Abd El-Monsef, M.E. and El. Deep. S. N.(1981a), On pre continuous and weak Pre continuous mappings, Proc. Math. Phys. Soc. Egyp , vol 53,47-53.

[9] Maheshwari S.N. & Prasad R. (1977). Semi-open sets and semi-continuous functions in bitopological spaces. *Math. Notae*. 26, 29-37.

[10] Palaniammal S. **,**(2011).Study of Tri topological spaces, Ph.D Thesis.

[11] Sampath Kumar (1994), on fuzzy pair wise α -continuous and fuzzy pair wise pre-continuity , Fuzzy sets and Systems,vol 62,issue 231-238.

[12] Sahana A. S. Bin (1991) ,on fuzzy strongly semi-continuity and fuzzy pre continuity, Fuzzy sets and Systems, vol . 44 ,303-308.

[13] Thakur S.S and Malviya R., Semi-open sets, Semi continuity and semi open mapping in fuzzy bitopological Spaces, Fuzzy sets and Systems ,(1994),64,421-426.

[14] Tapi U.D. and Sharma R .and Deole B.A., Semi-open sets and Pre open sets in Tri Topological space, i-manager's Journal on Mathematics, Vol. 5 No. 3 July-September 2016.

[15] Zadeh L.A. ,Fuzzy sets ,information and control (1965) ,8,338-353.