Numerical Analysis of Magneto-Hydrodynamic Flow of Non-Newtonian Fluid Past Over a Sharp Wedge in Presence of Thermal Boundary Layer

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Abstract:

In this present study we analyzed the magneto-hydrodynamic flow of non-Newtonian fluid past over a sharp wedge in presence of thermal boundary layer. We have solved the non-linear differential equation with the help of ode45 solver by MATLAB software. we draw the graphs between velocity components of fluid and heats flow against dimensionless variable with different parameters such as magnetic parameter M, power law index parameter n, Reynolds number Re, Prandtl number Pr and wedge parameter m. The various results have been obtained graphically.

Key Words: - *Magnetic Parameter M, Wedge parameter m, Prandtl number Pr, Reynold number Re, m & Porous law index n.*

I. INTRODUCTION

Boundary layer flow problem of non-Newtonian fluid over sharp wedge in the presence of Magnetic fluid has generated considerable interest for its numerous engineering and industrial applications such as the boundary layer along a liquid film, polymer processing and chemical engineering processes viscosity on flow and heat transfer along a symmetric wedge. Heated fluid moving between feeding rolls and wind up rolls, cooling of polymer materials by using a continuous sheet, glass and fiber productions and manufacturing of polymeric sheets are some examples, in which involves flow of a viscoelastic fluid over a stretching sheet. A theoretical analysis of laminar natural convection heat transfer to non-Newtonian fluid has studied by Acrivas (1960). Chamka (1977) has investigated similarity solution for thermal boundary layer on a stretched surface of a non-Newtonian fluid.

Andersson et al. (1996) have been studied flow of a power-law fluid film on an unsteady stretching surface. Hassanien et al. (1998) have investigated flow and heat transfer in a power-law fluid over a non-isothermal stretching sheet. Magyari and Keller (1999) have analyzed heat transfer characteristics of the separation boundary flow induced by a continuous stretching surface. Abd-el-Malek et al. (2002) have been solution of the Rayleigh problem for a power law non-Newtonian conducting fluid via group method. Zhang et al. (2008) have investigated an analysis of the characteristics of the thermal boundary layer in power law fluid. Mukhopadhyay (2009) has study the effect of radiation and variable fluid viscosity on flow and heat transfer along a symmetric wedge. Postelnicu and Pop (2011) have been studied Falkner-Skan boundary layer flow of a power-law fluid past a stretching wedge.

Mahanta (2012) has analyzed the numerical study on heat transfer of non-Newtonian fluid flow over stretching surface with variable viscosity in uniform magnetic field. Manju Bisht and Anirudh Gupta (2014) have been studied the investigation of thermal boundary layer of non-Newtonian fluid past over a wedge. Ramesh Yadav et al. (2016) have been studied investigation of laminar flow of fluid with one porous bounding wall. They have been obtained the effects of slip coefficient of fluid, Reynolds Number Re on fluid velocity. In another paper Ramesh Yadav et al. (2016) have been also investigated Numerical analysis of magneto-hydrodynamic flow of viscous fluid between parallel porous bounding walls. They have been obtained the effects of Hartmann number H, Reynolds number Re and width of the channel on velocity components of fluids. Ramesh Yadav and Vivek Joseph (2016) have been studied numerical analysis of magneto-hydrodynamic flow of fluid with one porous bounding wall. They have been obtained effects of magnetic parameter M, Reynolds number Re and slip coefficient on velocity component of fluids in a channel flow.

In this paper we study numerical analysis of magneto-hydrodynamic flow of non-Newtonian fluid past over a sharp wedge in presence of thermal boundary layer. The numerical results of the resulting coupled ordinary differential equation are obtained using under MATLAB software with the help of ode 45 solver. Results are given the velocity of fluid and temperature distributions for various values of power law index, Prandtl number Pr and Reynolds number Re and Magnetic parameter M.

II. MATHEMATICAL FORMULATION

Let us assume that two dimensional steady, laminar incompressible non-Newtonian fluid obeying the power-law model, flowing over a porous wedge with constant wall temperature, T_w in a stationary coordinate system. The governing equations of boundary layer flow are

The continuity equation

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0, \qquad (1).$$

Momentum and Energy equations are

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U\frac{dU}{dx}+\frac{K}{\rho}\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)^n-\frac{\sigma_e B_0^2}{\rho}\,u, \qquad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\,\frac{\partial^2 T}{\partial x^2}\,, \qquad (3)$$

where σ_e is electrical conductivity of the fluid, B_0 is Magnetic field, ρ is the density of the fluid, α is the thermal diffusivity of the fluid.

Here the boundary conditions

At $y = 0;\ u = v = 0\ and\ T = T_w$, (4)

At $y \to \infty; u \to U(x) = cx^m\ \ and\ T = T_\infty$, (5)

At $x = 0;\ u = U_\infty\ \ and\ T = T_\infty$. (6)

Here u is the velocity of fluid in x-direction, v is the velocity of the fluid in $y-$ directions, ν is the kinematic viscosity of the fluid and U the reference velocity at the edge of boundary layer and is a function of x, $m = \frac{\beta}{2\pi-\beta}$ is the porous wedge parameter and β is the wedge angle, ρ is the density of fluid and T is the temperature in the vicinity of the porous wedge.

We use the following transformations, utilized to facilitate the solution of the governing equations is

$$\psi = \left(\frac{Kx}{\rho}\right)^{\frac{1}{n+1}} [U(x)]^{\left(\frac{2n-1}{n+1}\right)} f(\lambda) \tag{7}$$

$$\lambda = \left[\frac{\rho\{U(x)\}^{(2-n)}}{Kx}\right]^{\frac{1}{n+1}} y \tag{8}$$

$$\theta = \frac{T-T_\infty}{T_w-T_\infty} \tag{9}$$

Stream functions are

$$u = \frac{\partial\psi}{\partial y}\quad \&\quad v = -\frac{\partial\psi}{\partial x}, \tag{10}$$

Using the stream function in the equation (2),

$$\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x\partial y} - \frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2} = U\frac{dU}{dx} + \frac{K}{\rho}\frac{\partial}{\partial y}\left(\frac{\partial^2\psi}{\partial y^2}\right)^n - \frac{\sigma_e B_0^2}{\rho}\frac{\partial\psi}{\partial y}\ , \tag{11}$$

From equation (7) and (8), we get

$$\frac{\partial\psi}{\partial y} = U(x)f'(\lambda) = Cx^m\ f'(\lambda) \tag{12}$$

$$\frac{\partial^2\psi}{\partial x\partial y} = m\,C\,x^{m-1}\ f'(\lambda) \tag{13}$$

$$\frac{\partial^2\psi}{\partial y^2} = C^{\frac{3}{n+1}}\left(\frac{\rho}{K}\right)^{\frac{1}{n+1}}\ x^{\frac{3m-1}{n+1}}\ f''(\lambda) \tag{14}$$

$$\frac{\partial\psi}{\partial x} = \left(\frac{K}{\rho}\right)^{\frac{1}{n+1}}\frac{C^{\frac{2n-1}{n+1}}(2mn-m+1)}{n+1}\ x^{\frac{2mn-m-n}{n+1}}\ f(\lambda) \tag{15}$$

Putting these values from equation (12), (13), (14) & (15) in equation (11), we get

$$f'''(\lambda) + \frac{2mn-m+1}{n(n+1)}\ f(\lambda)[f''(\lambda)]^{(2-n)} + \frac{m}{n}[1-\{f'(\lambda)\}^2][f''(\lambda)]^{(1-n)} - \frac{\sigma_e B_0^2}{\rho Cn}x^{1-m}\ [f''(\lambda)]^{(1-n)}f'(\lambda) = 0, \tag{16}$$

Or

$$f'''(\lambda) + \frac{2mn-m+1}{n(n+1)}\ f(\lambda)[f''(\lambda)]^{(2-n)} + \frac{m}{n}[1-\{f'(\lambda)\}^2][f''(\lambda)]^{(1-n)} - M^2\ [f''(\lambda)]^{(1-n)}f'(\lambda) = 0, \tag{17}$$

$$\theta'' + \eta f PrRe\theta' = 0, \tag{18}$$

Where $M^2 = \frac{\sigma_e B_0^2}{\rho C n} x^{1-m}$, λ is the similarity variable and $f(\lambda)$ and $\theta(\lambda)$ are similarity dependent variables and $\eta = \frac{2mn - m + 1}{n(n+1)}$, $R = \frac{Re}{Re_{(n,x)}^{\frac{2}{n+1}}}$, $Re_{(n,x)} = \frac{x^n U^{2-n}}{\upsilon}$ is the generalized Reynolds number for non-Newtonian fluids and $Re = \frac{xU}{\upsilon}$ and $Pr = \frac{\rho \upsilon C_p}{K}$ are Reynolds number and Prandtl number, respectively.

The associated boundary conditions are:

At $\lambda = 0;\quad f = 0,\quad f' = 0,\quad \theta = 1,$ (19)

At $\lambda = \infty;\quad f' = 1,\ \theta = 0,$ (20)

where prime denote the differentiation with respect to λ.

For Newtonian fluid index of power law fluid $n = 1$, equation (17) and (18) Reduced

$$f''' + \frac{1}{2}(m+1)\, f + [1 - (f')^2] - M^2 f' = 0, \tag{21}$$

$$\theta'' + \frac{1}{2}\Pr(m+1)\, Ref\theta' = 0, \tag{22}$$

The important physical quantity of interest is the Nusselt number which can be defined as

$$Nu_x = \frac{q_w\, x}{(T_0 - T_\infty)K} = -\theta'(0)\, R_{(n,x)}^{\frac{2}{n+1}} \ . \tag{23}$$

Solving the above differential equation (20), using the boundary condition (16) & (17), we get

$$\theta = e^{-\frac{1}{2}\Pr(m+1)Re\ F\ \lambda} \tag{24}$$

III. METHOD OF SOLUTION

In this paper we have solved the above differential equation (17) & (21) numerically using MATLAB software. In this we used ode45 solver for solving set of differential equation with described boundary conditions which is given in equation (19) and (20). For the purposed the time interval (0, 1) with initial condition vector (0, 0, 1) has been taken for convergence criteria has been chosen ('RelTol',1e-4,'AbsTol',[1e-4 1e-4 1e-5]). Different set of parameter has been chosen to investigate the results. The range of dimensionless variable λ $(0 \le \lambda \le 1)$, the value magnetic parameter M has been taken (1, 2, 3, 4), Porous law index n has been taken {1, 3, 5, 8 and 1, 4, 6, 8). The graphs of heat transfer against dimensionless variable $\lambda\ in\ the\ range$ $(0 \le \lambda \le 5)$ a the value Magnetic Parameter M (1, 2, 3, 4), Reynold number Re (1, 2, 3, 4) and Prandtl number Pr (1, 2, 3, 4, 5) has been taken. Various graphs have been plotted with described set of parameters and discussed in detail in the next section.

IV. RESULTS AND DISCUSSION

The non-linear differential equation (17), (21), (22) and (24) subject to (19) and (20) must in be integrated by numerical procedure to use ode45 solver and find the results. The numerical results are obtained to study of effects the various values of the Reynolds number Re, Magnetic parameter M, Power law index n and Prandtl number Pr on dimensionless velocity of Newtonian and non-Newtonian fluids and dimensionless temperature profiles. Velocity profiles of non-Newtonian and Newtonian fluids are given in the figure 1- 7, for prescribed values of

Magnetic Parameter M, Power law index n. Figures 1-3, represents the graphs between velocity component of fluids $f(\lambda)$ against dimensionless variables λ for prescribed values of $m = 2/9, n = 1$; it seen that velocity profiles of fluids increases with enhancement of magnetic parameter M. Figure 4 & 5 represents the axial $f(\lambda)$ and radial velocity $f'(\lambda)$ components against dimensionless variables λ for prescribed values of M = 2, m = 2/9; it is found that axial and radial velocity components of fluids decreases with increase of power law index number n. Figure 6 & 7 represents the graphs between axial $f(\lambda)$ and radial $f'(\lambda)$ velocity components of fluids against dimensionless variable λ for non-Newtonian fluids (n = 2) at constant m = 2/9; it is seen that axial and radial velocity components of fluids increases with increase of Magnetic parameter M. Figures 8, 9 & 10 represents the graph between temperature profile $\theta(\lambda)$ of fluids against dimensionless variable λ at constant values of n = 1, m = 2/9 and other different constant; it is seen that the temperature profile of fluids decreases sharply with increase of following parameter such as Magnetic parameter M, Reynolds Number Re and Prandtl Number Pr.

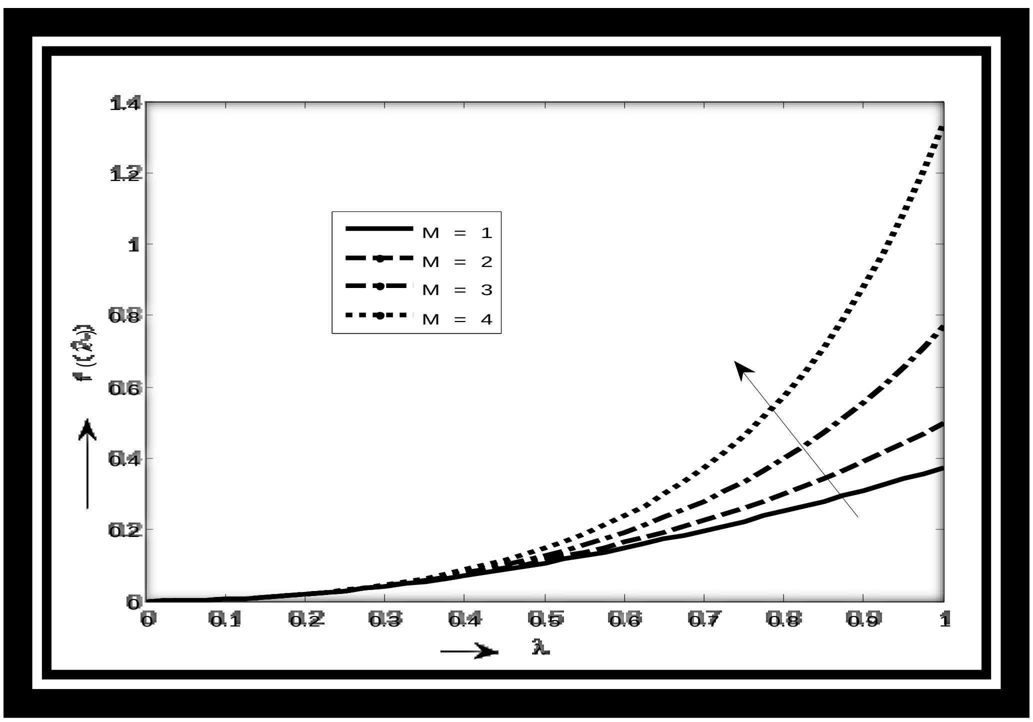

Fig 1. Graph between axial velocity of fluid $f(\lambda)$and dimensionless variable λ with variation of Magnetic Parameter M (Hartmann Number0 at constant variables n = 1, m = 2/9.

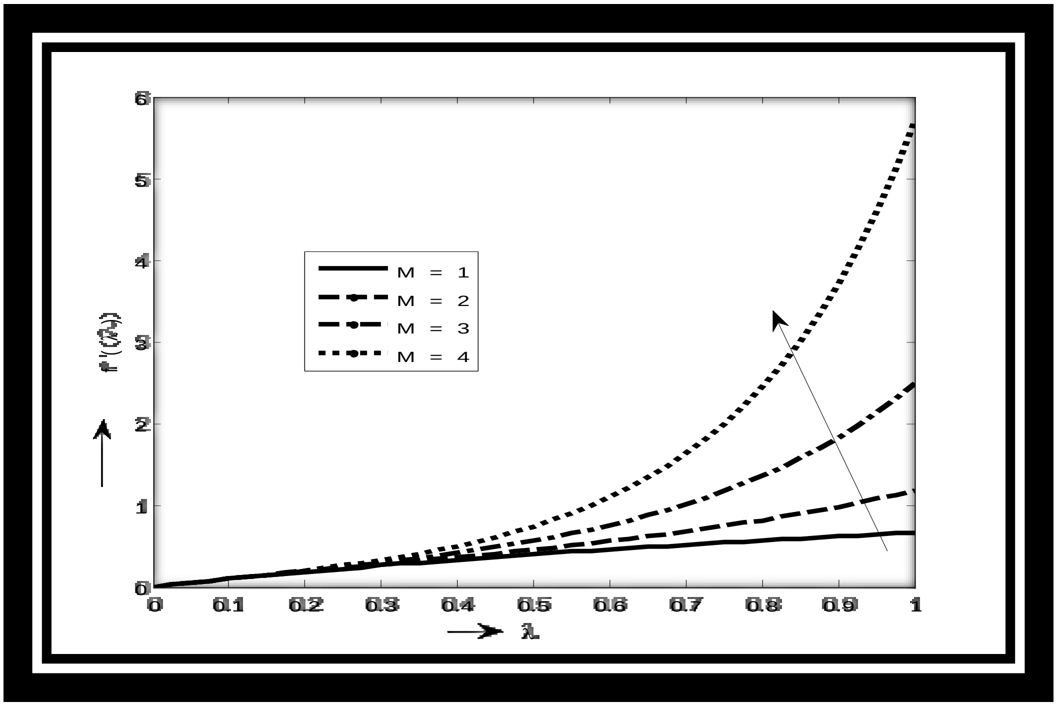

Fig 2. Graph between radial velocity of fluid $f'(\lambda)$ and dimensionless variable λ with variation Prandtl Number Pr at constant variables n = 1, m = 2/9.

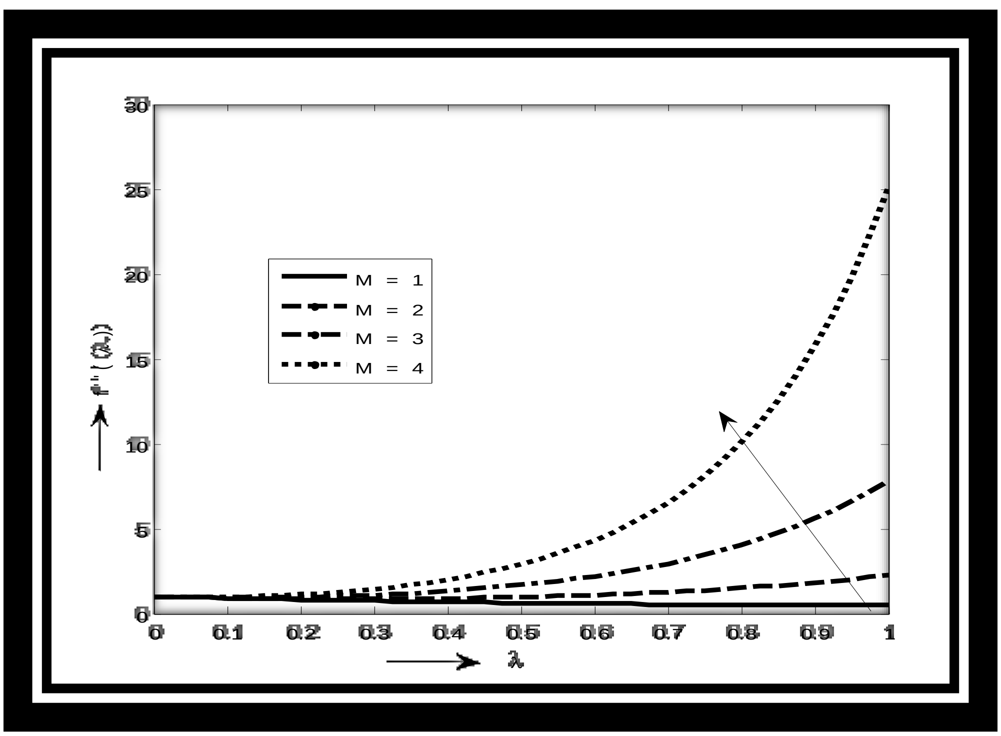

Fig 3. Graph between velocity profiles of fluids $f''(\lambda)$ and dimensionless variable λ with variation of Magnetic Parameter M (Hartmann Number) at constant variables n = 1, m = 2/9.

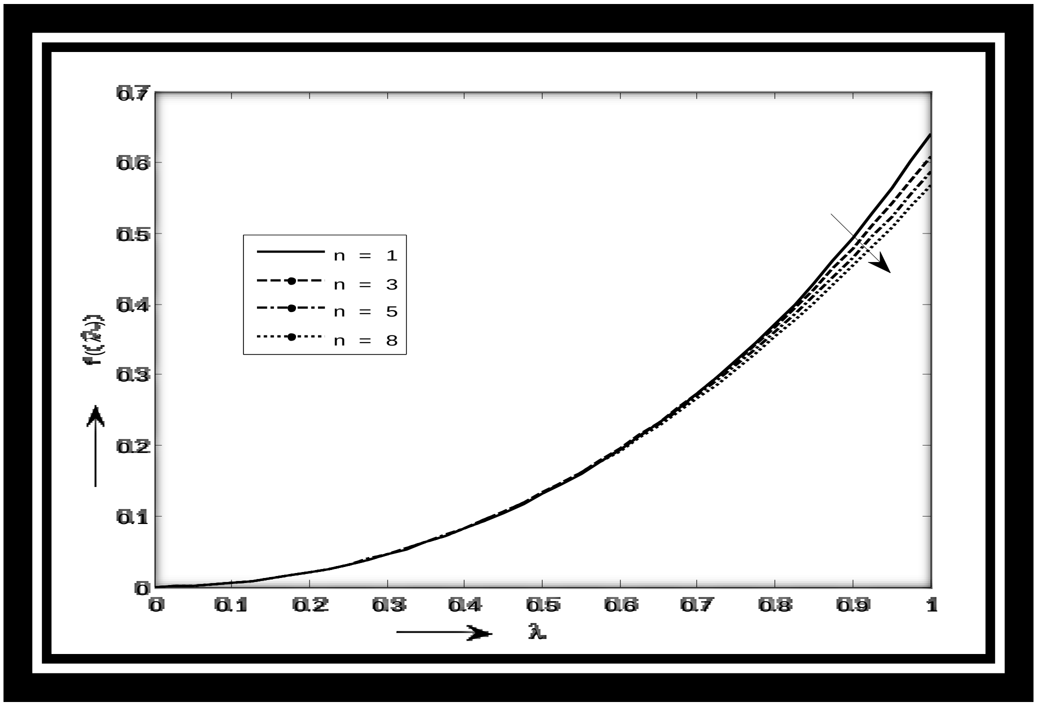

Fig 4. Graph between axial velocity of fluids $f''(\lambda)$and dimensionless variable λ with variation of power law index number (n) at constant variables m = 2/9, M = 2.

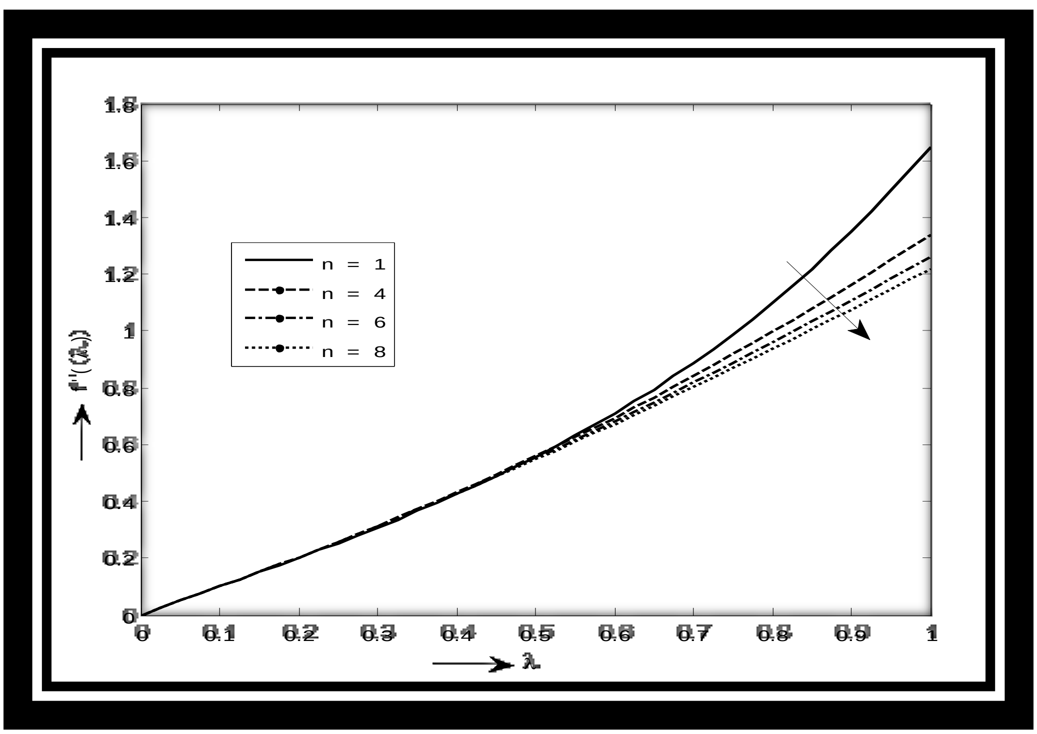

Fig 5. Graph between radial velocity of fluid $f'(\lambda)$and dimensionless variable λ with variation of power law index number (n) at constant variables m = 2/9, M = 2.

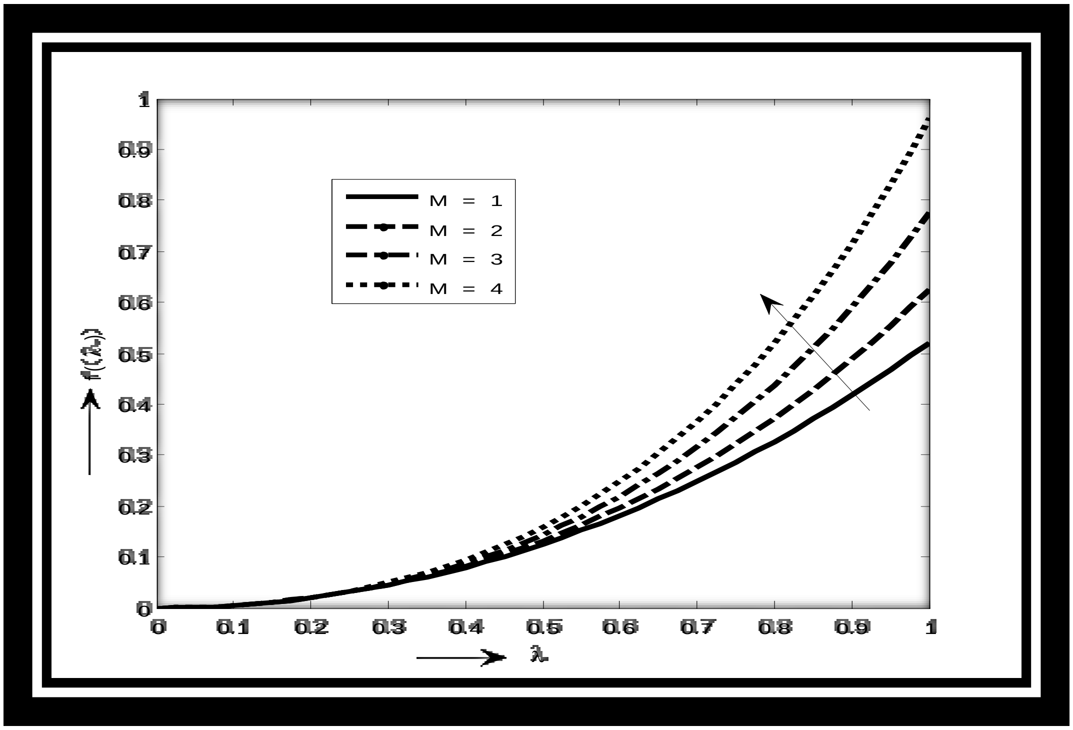

Fig 6. Graph between axial velocity of fluid $f\ (\lambda)$and dimensionless variable λ with variation of Magnetic parameter M at constant variables m = 2/9, n = 2.

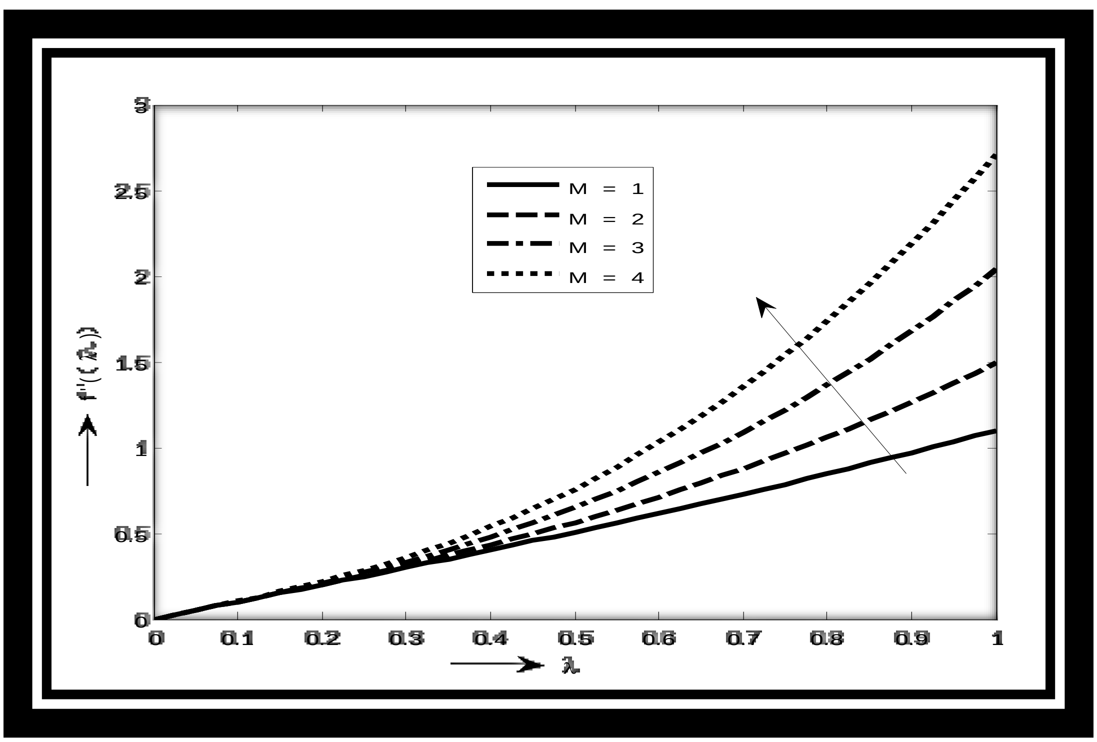

Fig 7. Graph between radial velocity of fluid $f\ '(\lambda)$and dimensionless variable λ with variation of Magnetic parameter M at constant variables m = 2/9, n = 2.

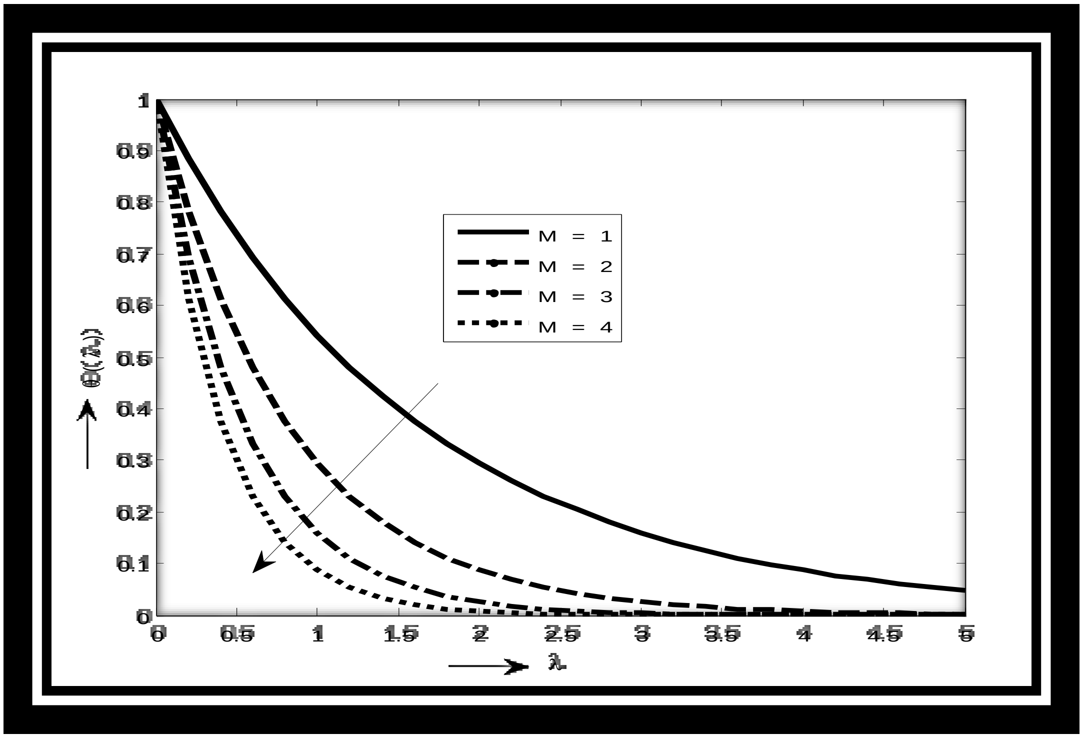

Fig 8. Graph between Temperature profile of fluid $\theta\ (\lambda)$and dimensionless variable λ with variation of Magnetic parameter M at constant variables m = 2/9, n = 1, Pr = 2, Re = 0.6.

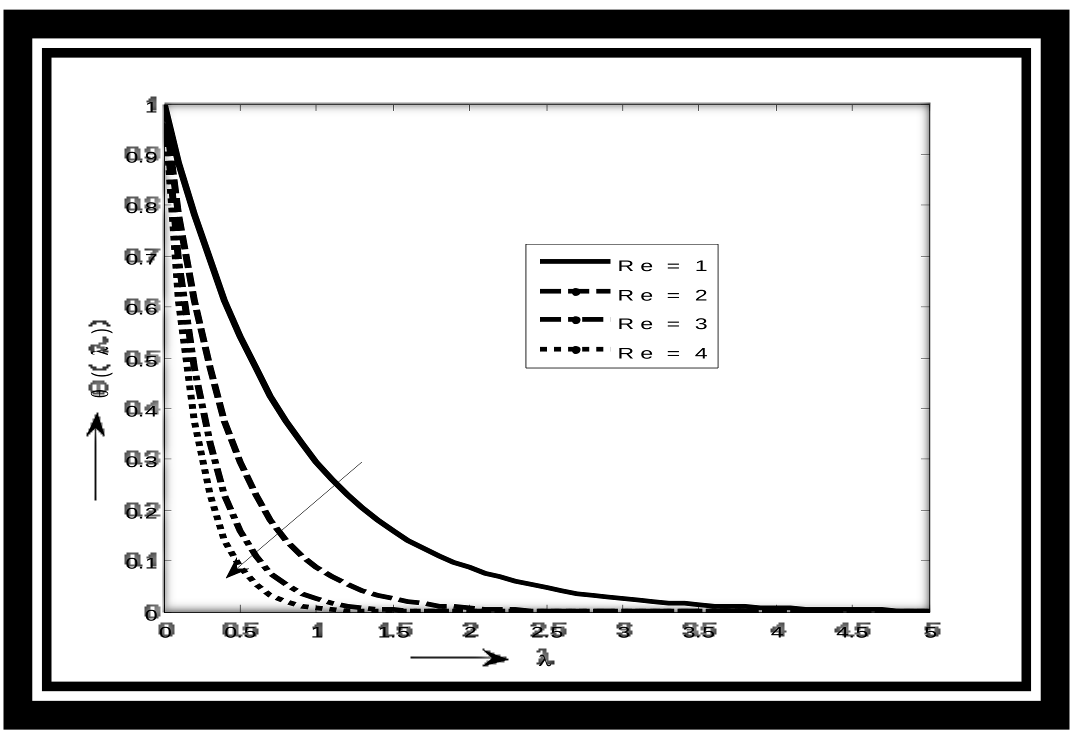

Fig 9. Graph between Temperature profile of fluid $\theta\ (\lambda)$and dimensionless variable λ with variation of Reynolds Number Re at constant variables m = 2/9, n = 1, Pr = 2, M = 1.

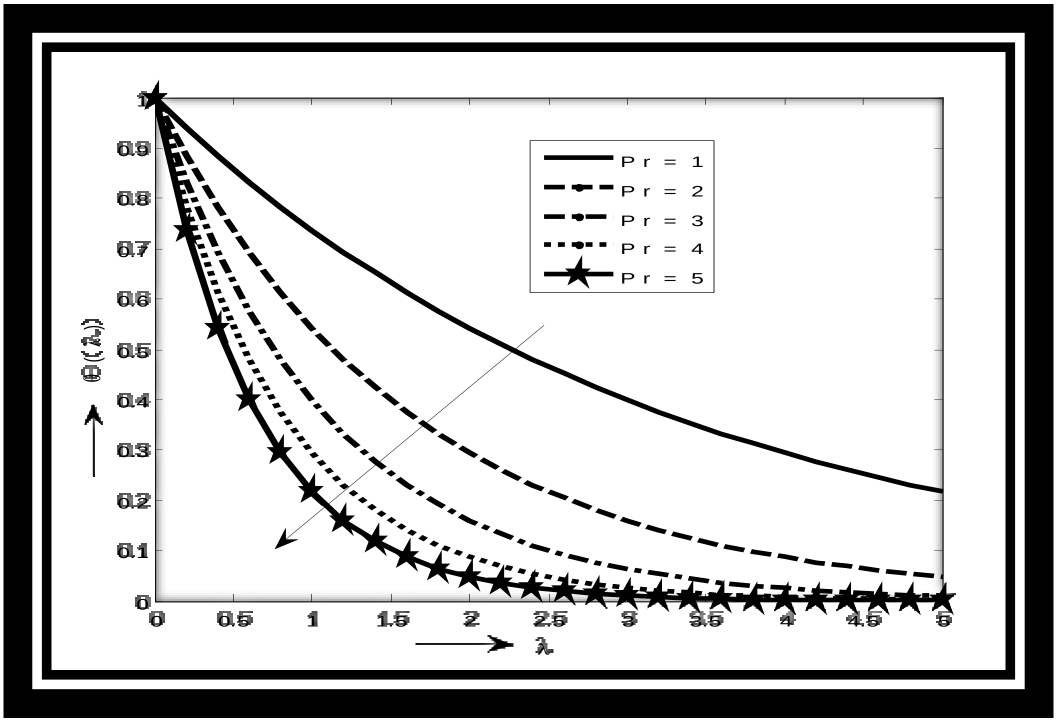

Fig 10. Graph between Temperature profile of fluid $\theta\ (\lambda)$ and dimensionless variable λ with variation of Magnetic parameter M at constant variables m = 2/9, n = 1, Re = 0.5, M = 1.

V. CONCLUSIONS

In this paper, we have presented numerical analysis of magneto-hydrodynamic flow of non-Newtonian fluid past over a sharp wedge in presence of thermal boundary layer in reaction occurring in this case which will have application in chemical coating of metals and removal of particles. The main objective this study to analysis the effects of magnetic parameter M on fluids velocity and heat flow of Newtonian and non-Newtonian fluids simultaneously the effects of Reynolds and Prandtl number of heat flow. In this it is found that the axial and radial velocity component of Newtonian and Non-Newtonian fluid increases sharply with enhancement of magnetic parameter M and reciprocal effect with increase of power law index number n. it is also seen the heat flow of fluids sharply decreases with increase of Magnetic parameter M, Reynolds Number Re and Prandtl number Pr.

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