A SEQIR Model for the Control of Spread of Re-Emerging Contagious Infectious Disease

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Abstract — *It is well known that many infectious diseases like influenza, H1N1, and many more are periodic. Such type of diseases reappears in the society in either same or similar manner. Therefore, in this article, we proposed an SEQIR model by introducing the effective contact rate function to predict and control the spread of such types emerging and re-emerging contagious disease. Infectious diseases spread through close contact. Therefore, we formulate an effective contact rate function to control the spread of infectious diseases or an epidemic. Numerical simulation of the model has been performed with the help of fourth order Runge- Kutta method to illustrate the effect of our control strategy.*

Keywords — *Contagious infectious disease; SEQIR model; effective contact rate function; simulation; action time; control of infectious diseases.*

I. INTRODUCTION

Control of emerging contagious infectious disease is a very difficult task for doctors and health agencies, Therefore, in mathematical epidemiology literature, many mathematical model and strategies like SIS [10], SIR [2, 8, 16], SEIR [10, 19], SEIRS [18], SVEIR [1] and many more models (where S, V, E, I and R denotes the population of susceptible, vaccinated, exposed, infected and recovered individuals respectively) have been proposed by various researcher.

In epidemiology literature, many diseases are periodic i.e. they reappear in the society like Chikungunya, Swine flu, SARS and many more [12, 15] in either same or similar manner. In this article, we are trying to control the spread of such type of contagious diseases in the society if this kind of diseases reappears. The whole study of this article is based on prediction and control of such kind periodic diseases. It is also well-known fact that many diseases, such as influenza, measles, whooping cough, etc., exhibit seasonal (periodic) fluctuations, [8, 11, and 17] and spread of the disease depends on the contact rate. Contact rate was considered as a parameter before Juhan Zhang et al. [6] have considered time varying periodic effective contact rate for the control of rabies in China. In this paper, we proposed an SEQIR model, which is a five-compartment model, by introducing effective contact rate as a periodic function of time t, to control the spread of disease. The spread of contagious disease mainly depends on the effective contact between susceptible and infected individuals. Effective contact rate is sometimes known as a transmission rate [9] between susceptible and infectious. In this article, we take effective contact rate as a periodic function of time because the spread of such kind of diseases depends on the periodic contact, for example, contacts between students and teachers in school. We describe the spread control of the infection in the society with the help of action time. Action time or period has been introduced in this manuscript which may be defined as "A time taken by the health agencies to control the spread of infection from infected to susceptible individuals by various means such as by increasing the immunity of infected and susceptible, vaccination of both susceptible and infected, yoga etc."

When analyse a new outbreak, the researcher usually starts with the SIR and SEIR model. Here we proposed SEQIR model for new emerging contagious infectious disease. It is known that the behaviour of nonlinear differential equation model system can be explored by fourth-order Runge- Kutta method [4, 5, and 7]. Therefore, we use Runge-Kutta method to solve the model numerically.

The rest of paper is organised as follows: A SEQIR mathematical model in Section 2. Effective contact rate function is described in Section 3. Basic properties of solutions are given in Section 4. The numerical simulation and discussion are in Section 5. Finally, the conclusion is summarised in Section 6.

II. A SEQIR MODEL FOR INFECTIOUS DISEASE

In this section, we have proposed an SEQIR model for the transmission of emerging contagious diseases. For this, the total population is divided into five compartments: susceptible $S(t)$, exposed $E(t)$, quarantined $Q(t)$,

infective $I(t)$ and recovered $R(t)$.The parameters $p, \beta, \alpha, \delta, \theta, d, \gamma$ and μ are recruitment rate per day, the effective contact rate, rate of development of clinical, rate of quarantine, Quarantine individual's recovery rate, disease induced mortality rate, infected individual's recovery rate and natural mortality rate respectively. We assume that the quarantined population is not suffering severity of the disease and the whole quarantined population will recover. And also, it is assumed that quarantined individuals do not transmit the infection. Our motive is to control the spread of emerging contagious disease using a mathematical model. For this, we have assumed that susceptible individuals can be infected only through contact with infectious individuals. Therefore, instead of considering β as a parameter, we have formulated an effective contact rate which is a function of time t.The progression of infection through different compartments shown with help of block diagram, which is given below.

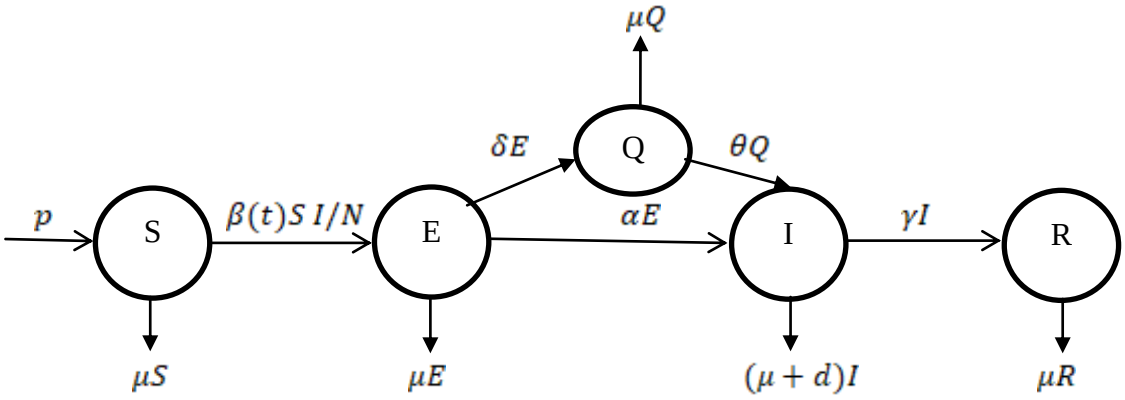

Figure 1: Progression of infection from susceptible (S) through latent (E), quarantined (Q), infected (I) and recovered (R) compartments for the model.

The rate of change of the population in each compartment is given by the following system of nonlinear differential equations

$$\frac{dS}{dt} = p - \mu S - \beta(t) S \frac{I}{N} \quad (1)$$

$$\frac{dE}{dt} = \beta(t) S \frac{I}{N} - \mu E - \alpha E - \delta E \quad (2)$$

$$\frac{dQ}{dt} = \delta E - (\mu + \theta) Q, \quad (3)$$

$$\frac{dI}{dt} = \alpha E - (\mu + \gamma + d) I, \quad (4)$$

$$\frac{dR}{dt} = \gamma I - \mu R + \theta Q. \quad (5)$$

Where $S(0) = S_0, E(0) = E_0, Q(0) = Q_0,\ I(0) = I_0, R(0) = R_0$, $t \geq 0$ and $\beta(t) > 0$ is an effective contact rate function. The total population size is $N(t) = S(t) + E(t) + Q(t) + I(t) + R(t)$.

The explanation of above model parameters is listed in Table 1.

Parameter	Description	Value	Source
p	Recruitment rate	0.0001 per day	Assumption
μ	Natural mortality rate	3.65×10^{-5} per day	[1]
α	Rate of development of clinical symptoms	0.1887 per day	[2]
d	Disease-induced mortality rate	0.008 per day	[1]
γ	Infective Recovery rate	0.1 per day	[2]
c	Action time	2 to 21days	Assumption [13,14]
b	Spread controlling parameter	(c + 1) days	Estimation

δ	Rate of quarantined	0.125 per day	Assumption
θ	Quarantined individual's recovery rate	0.075 per day	Assumption

Table 1: Parameters description and values used in simulation

III. EFFECTIVE CONTACT RATE FUNCTION $\beta(t)$

This section will be used to formulate effective contact rate function. It is understood that the only way of transmission of contagious Infectious disease is close contact between susceptible and infectious individuals and also the probability of getting a disease is not constant at any point of time. Since the occurrence of many contagious infectious diseases like Chikungunya, SARS and many more are seasonal and prevalent during starting of the winters in many countries and its transmissions are very fast, therefore contact rate will be considered as a periodic function of time. Following assumptions have been made to formulate the effective contact rate function:

1). It has been observed from the literature and data, that intensity of the infection of periodic contagious infectious disease goes up till a certain period of time.

2). Prime reason for the spread of disease is contact between infected and susceptible individuals which happens to be in a periodic manner for example in college, office etc. Therefore, effective contact rate should increase with time in a periodic manner.

3). Also, it is considered that effective contact rate cannot be completely zero at any time t.

A force of infection (F) will be considered with periodically ('seasonal') varying contact rate i.e.

$$F = \frac{\beta(t)I}{N}, \beta(t+T) = \beta(t) \tag{5}$$

with period T equal to one year.
Hence effective contact rate function $\beta(t)$ has been modeled as follows

$$\beta(t) = \frac{H(t)+c}{b}, 2 \leq c \leq 21, b = c+1 \; and \; t \geq 0 \tag{6}$$

Where $H(t)$ is also a Periodic function of time with period T.

For the purpose of simulations effective contact rate function $\beta(t)$ has been modeled as follows:

$$\beta(t) = \frac{(\sin t \times \cos t)^2 + c}{b}, 2 \leq c \leq 21, b = c+1 \; and \; t \geq 0 \tag{7}$$

where b represent the spread controlling parameter to minimize the infection of disease on the society and c represents an action time, which is "A time taken by the health agencies to control the spread of infection from infected to susceptible individuals by various means such as by increasing the immunity of infected and susceptible, vaccination of both susceptible and infected, yoga etc." We are taking the values of c greater and equal to 2 days, which is the minimum incubation period of contagious diseases like Ebola, SARS and many more. It has been assumed that minimum action time to control the spread of the disease should not be less than incubation period, and therefore c can take minimum value as 2 days. The spread controlling parameter will always depend on the action time.

IV. BASIC PROPERTIES OF THE MODEL

The model Equations $(1)-(5)$ monitors the human populations, it is assumed that all state variables and parameters of the model are nonnegative i.e. $(S, E, Q, I, R) \in \mathbb{R}^5_+$ and $p, \mu, \alpha, \theta, \gamma, d, \delta \geq 0$.
Theorem 1: The variables of the model equation (1)-(5) with positive real data will remain positive for all time $t > 0$.

Proof: Let $t_* = \sup\{t > 0: S > 0, E > 0, Q > 0, I > 0, R > 0 \in [0,t]\}$. Thus, $t_* > 0$. It follows from the first equation of the model i.e. equation (1) that

$$\frac{dS}{dt} = p - \mu S - \beta(t) S \frac{I}{N}$$

which can be re-written as,

$$\frac{d}{dt}\left[S(t)\, e^{\left\{\mu t + \frac{1}{N}\int_0^t \beta(y) I(y) dy\right\}}\right] = p\, e^{\left\{\mu t + \frac{1}{N}\int_0^t \beta(y) I(y) dy\right\}}.$$

Hence,

$$S(t_*)\, e^{\left\{\mu t_* + \frac{1}{N}\int_0^t \beta(y) I(y) dy\right\}} - S(0) = \int_0^{t_*} p\, e^{\left\{\mu \tau + \frac{1}{N}\int_0^t \beta(y) I(y) dy\right\} d\tau},$$

So that

$$S(t) = S(0) e^{-\left\{\mu t_* + \frac{1}{N}\int_0^t \beta(y) I(y) dy\right\}} + \left[e^{-\left\{\mu t_* + \frac{1}{N}\int_0^t \beta(y) I(y) dy\right\}}\right] \int_0^{t_*} p\, e^{\left\{\mu \tau + \frac{1}{N}\int_0^t \beta(y) I(y) dy\right\}} d\tau > 0.$$

Similarly, $E > 0, Q > 0, I > 0$ and $R > 0$.

Lemma 1: The closed set

$$A = \{(S, E, Q, I, R) \in \mathbb{R}_+^5 : N \leq \frac{p}{\mu}\}$$

is positively- invariant.

Proof: We have $(S, E, Q, I, R) \in \mathbb{R}_+^5$ be any solution with non-negative initial conditions. Then

$$\limsup_{t \to \infty} S(t) \leq \frac{p}{\mu}$$

The rate of change of the total population, obtained by adding Equations $(1) - (5)$, is given by

$$\frac{dN}{dt} = p - \mu N - dI < p - \mu N,$$

with $N = S + E + Q + I + R$. It follows that

$$0 < \limsup_{t \to \infty} N(t) \leq N_0,$$

With $\limsup_{t \to \infty} N(t) = N_0$ if and only if $\limsup_{t \to \infty} I(t) = 0$

Then $N(t) = \frac{p}{\mu} + \left(N(0) - \frac{p}{\mu}\right) e^{-\mu t}$

Thus $0 < N \leq N_0 = \frac{p}{\mu}$, as $t \to \infty$.Therefore all feasible solution of the system (Equations$(1) - (5)$) enter in the region

$$A = \{(S, E, Q, I, R) \in \mathbb{R}_+^5 : N \leq \frac{p}{\mu}\}$$

It can be shown that all solutions of system (Equations$(1) - (5)$) starting in A remain in A for all $t \geq 0$. Hence, A is positively invariant and it is sufficient to consider solutions in A.

The system (Equations$(1) - (5)$) is continuous and its derivative implies that solutions exist and is unique. Since solutions approach lies in A they are bounded and hence exist for $t \geq 0$. Therefore, the model is epidemiologically and mathematically well posed.

V. NUMERICAL SIMULATION AND DISCUSSION

In this section, we have simulated the model numerically to understand the control strategy, which depends on action time and other factors to minimise the effect of the disease. The numerical simulation of the model equations (1) -(5) has been done using Fourth order Runge- Kutta method in MATLAB 2012b. The values of the parameters have been given in table 1[1, 3].

In this article, we used the initial data which were used by Rachah and Torres [3] for the simulation. The total population N is 1, therefore $S_0 = 0.88,\ E_0 = 0.07\ , Q_0 = 0\ , I_0 = 0.05$ and $R_0 = 0$ so that the sum of S, E, Q, I and R will remain equal to the total population.

Since the aim of the present study is to assess the behaviour of periodic contagious disease by assessing the susceptible, exposed and infected population with respect to the action time, therefore graphs for these populations have been drawn by taking different values of action time.

Figure 2, Figure 3, Figure 4 and Figure 5 show the population of susceptible, infected and recovered individuals for $c = 5, 10, 15$ and 20 days respectively.

It can be seen from these figures that the peak values of infected population will be increasing when the values of action time are increased. Also, it can be observed from these graphs that the duration to achieve a decreased peak value will be increased with decreasing value of action time while the total duration for the eradication of the disease is almost same for all values of action time. It shows that if the time taken to take preventive measures is less, then the effect of the disease on the society will be reduced.

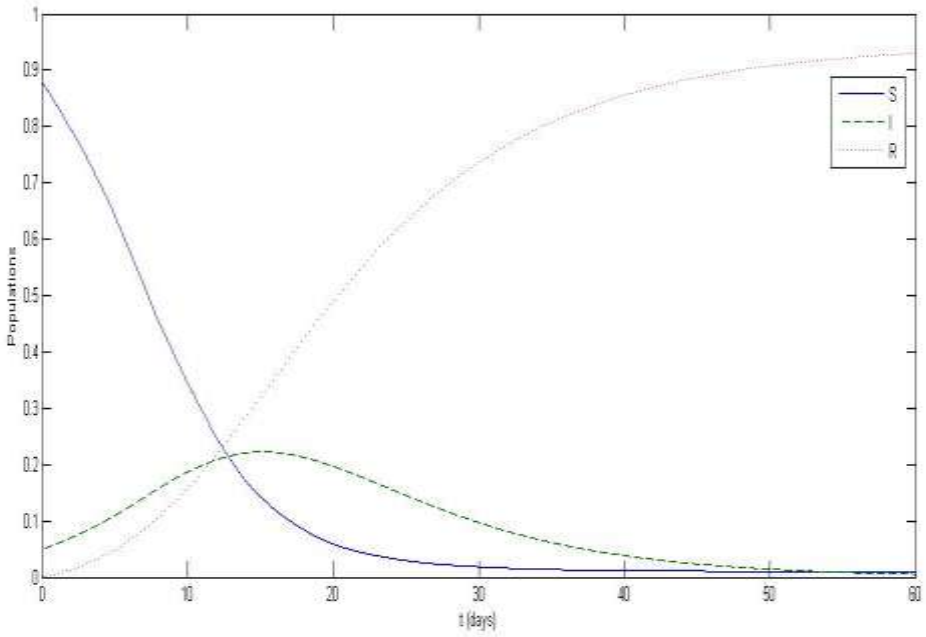

Figure 2: SIR graph at $c = 5$.

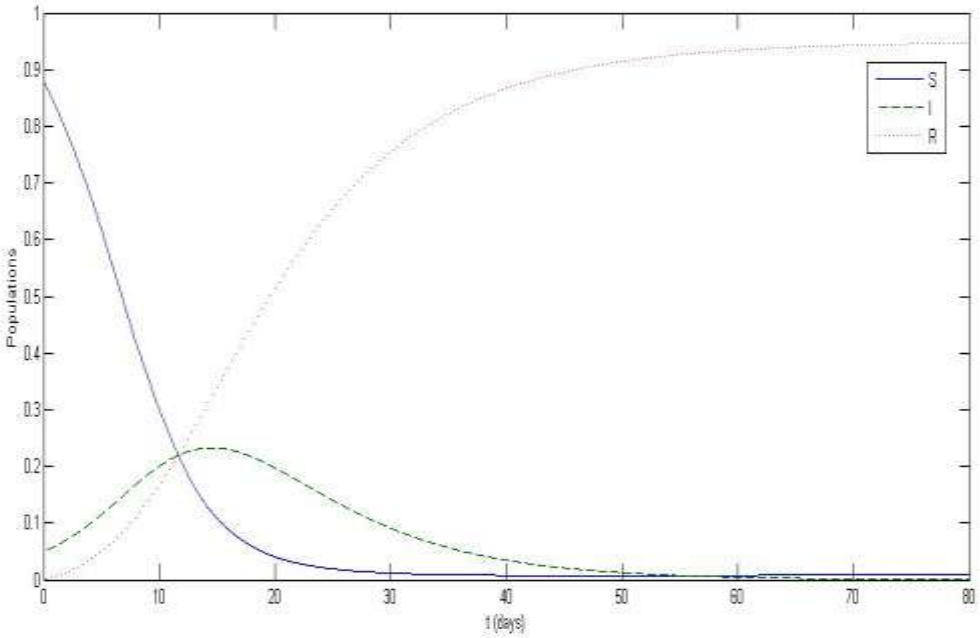

Figure 3: SIR graph at $c = 10$.

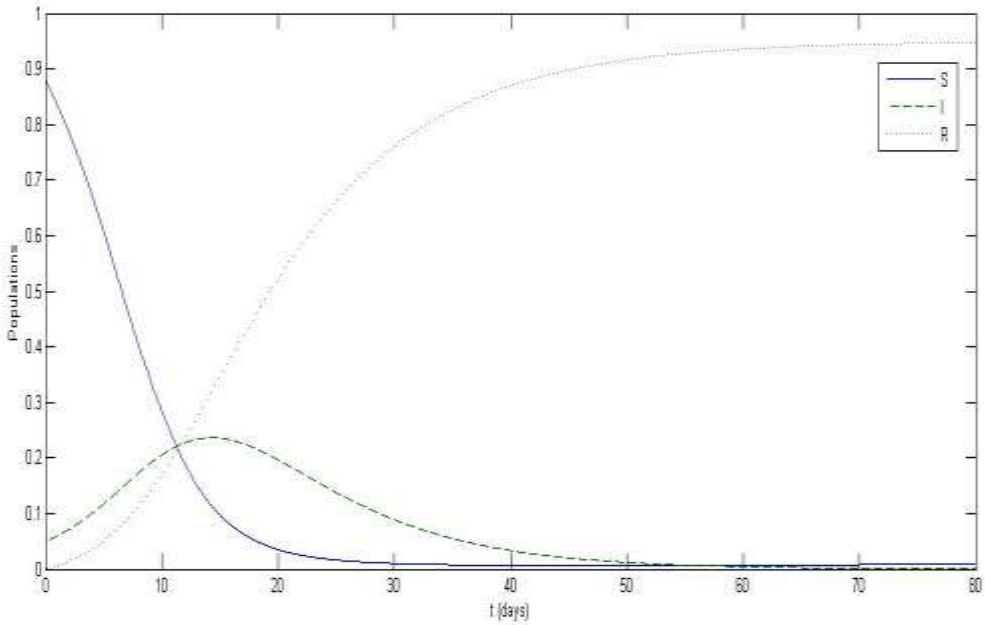

Figure 4: SIR graph at $c = 15$.

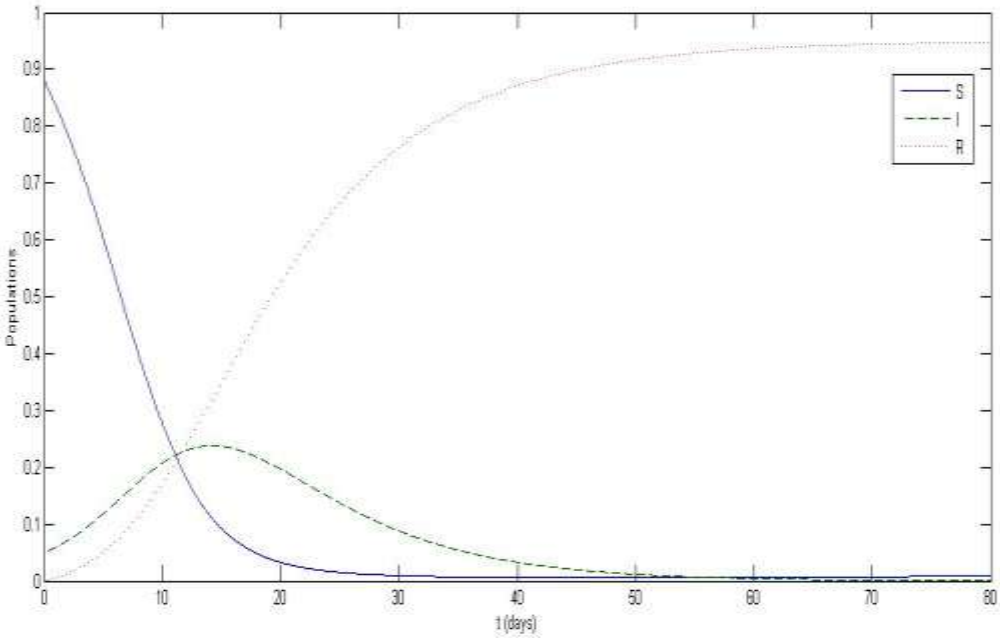

Figure 5: SIR graph at $c = 20$.

Figure 6 shows the population of exposed individuals with four different values of action time. It is evident from the figure that when we take less time to initiate preventive measures, then the population of exposed individuals is less in comparison of greater time taken for initiation of preventive measures.

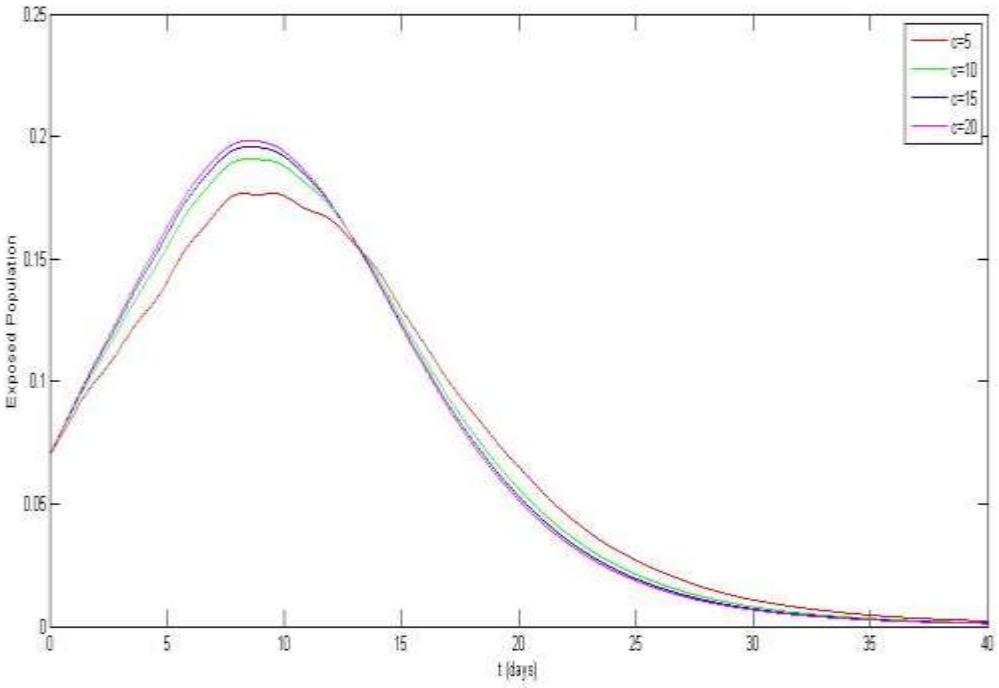

Figure 6: Number of exposed individuals at various values of c.

The population of quarantined individuals at various values of action time $c = 5,10,15$ and 20 days is shown in figure 7. It can be observed from figure 7 that population of quarantined individuals increase with the increment in the value of action time. Also the total number of quarantined population is decreasing with respect to the decreased value of action time. It can be understood that if time to take preventive measures is increased, then more number of people will need to be quarantined in the society. Preventive measures may include various methods such as the decreased contact rate, awareness about the disease and many more.

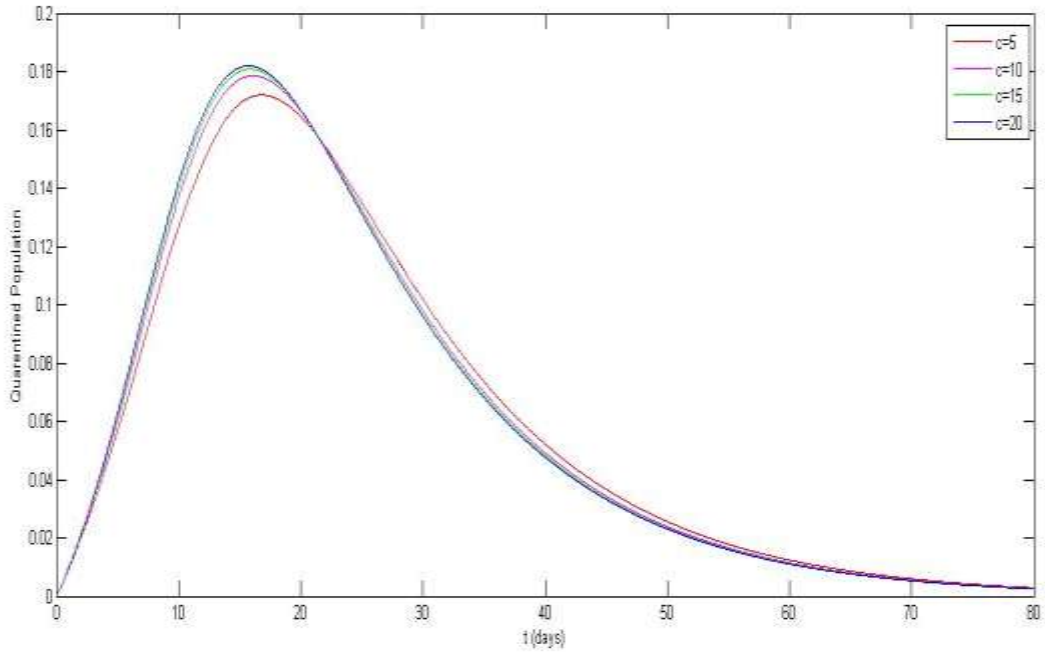

Figure 7: Number of quarantined individuals at various values of c.

The population of infected individuals at various values of action time $c = 5, 10, 15$ and 20 days is shown in figure 8. It can be observed from figure 8 that population of infected individuals increase with the increment in

the value of action time. Also the total number of infected population is decreasing with respect to the decreased value of action time. It can be understood that if time to take preventive measures is increased, then more number of people will be infected in the society.

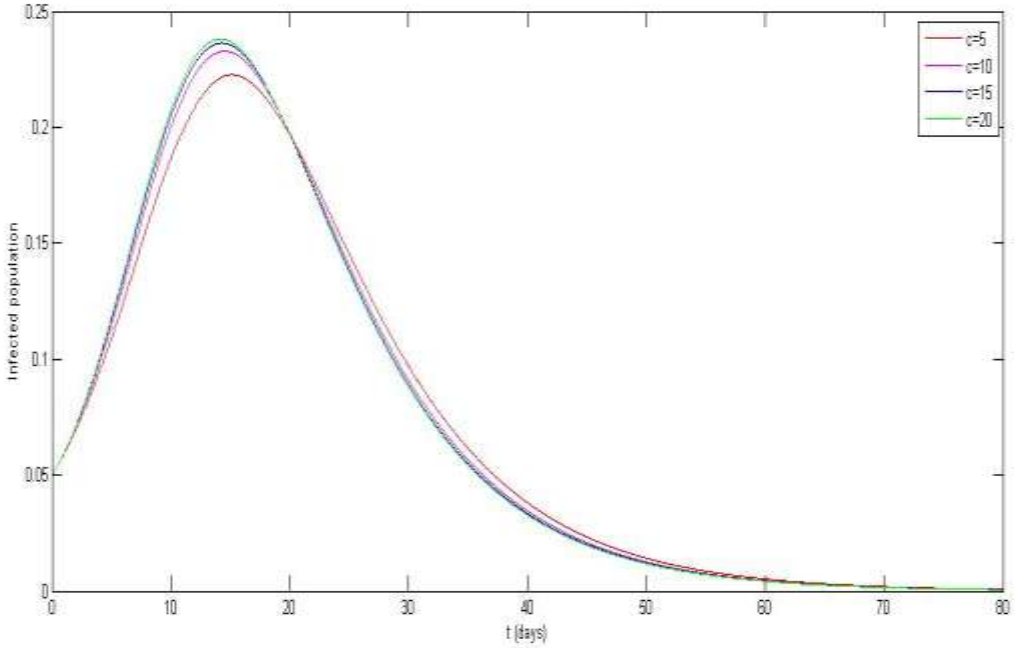

Figure 8: Number of infected individuals at various values of c.

Figure 9 shows effective contact rate of a contagious disease with time at different values of action time. It can be seen that as time increases, the area under the curve for effective contact rate with small action time is low in comparison of greater values of action time. Action time may be utilised to take preventive method to control the disease and also increasing the immunity of the susceptible to avoid them to become infected and of the infected to make a fast recovery.

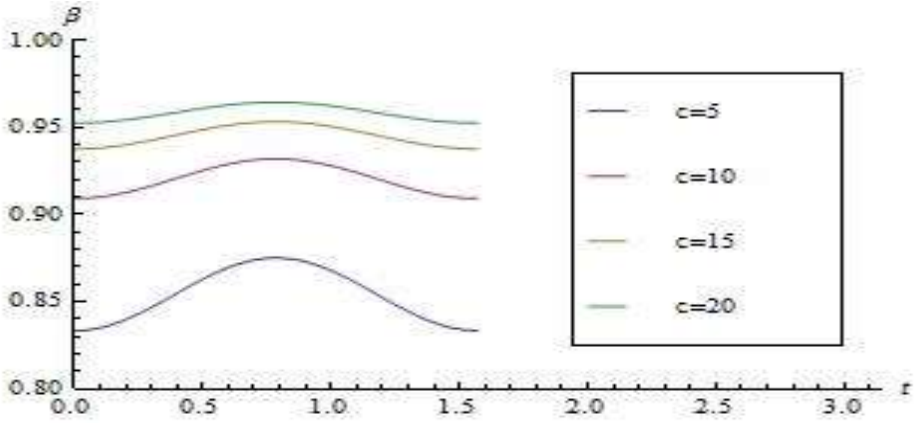

Figure 9: Effective contact rate $\beta(t)$ at different values of c.

Hence, the quarantization of the not critical population before infected individual's compartment saves the time and money spent on the vaccination of the individuals. Also, action time plays an important role in controlling the emerging contagious periodic infectious disease like SARS, EBOLA, Chikungunya and many more.

VI. CONCLUSION

To understand the control strategies of an epidemic, mathematical modelling can be proved more appropriate over statistical and experimental studies because of the limitation of data collection and experiments over

humans. Therefore, in this article we tried to give a new control strategy for an emerging contagious infectious disease with the help of a mathematical model. We have shown that with the help of model analysis, the model is locally asymptotically stable at Z if basic reproduction number is less than equals to one. And disease will become endemic at Z* if basic reproduction number is greater than one. We also conclude that if we take minimum time for the preventive measure to control the spread of disease, we are able to save many human lives.

VII.ACKNOWELDGMENT

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REFERENCES

[1] A.B. Gumel, C. Connell Mccluskey and J.Watmough, An SVEIR model for assessing the potential impact of an imperfect anti-SARS vaccine, Math. Biosci. and Eng. 3 (2006), pp. 485-494.
[2] A. Kumar and Nilam, Stability of a Time Delayed SIR Epidemic Model Along with Nonlinear Incidence Rate and Holling Type-II Treatment Rate, International Journal of Computational Methods, vol. 15 (1) (2018), pp. 1850055_1 -17.
[3] Amira Rachah and Delfim F. M. Torres, Predicting and controlling the Ebola infection, Mathe. Methods in the app. Scie, (2016), DOI: 10.1002/mma.3841.
[4] B. Bradie, A Friendly Introduction to Numerical Analysis. Prentice Hall, New Jersey, (2006).
[5]. J.R. Brannan and W.E. Boyce, Differential Equations with Boundary Value Problems: An Introduction to Modern Methods and Applications, John Wiley and Sons, New York, (2009).
[6] Juan Zhang, Zhen Jin Gui- Quan Sun, Xiang-Dong Sun and Shigui Ruan Modeling Seasonal Rabies Epidemic in China, Bull Math Biol 74 (2012), pp. 1226-1251.
[7] M. Braun, Differential equations and their applications: An introduction to applied mathematics, Springer-Verlag, New York (1983).
[8] O.N. Bjornstad, B.F. Finkenstadt and B.T. Grenfell, Dynamics of measles epidemics: Estimating scaling of transmission rates using a time series SIR model, Ecol Monogr., 72(2) (2002), pp. 169-184.
[9] R.M. Anderson and R.M. May, Infectious Disease of humans, Oxford University Press Inc, New York, (1991).
[10] S. A. A. Karim and R. Razali, A proposed mathematical model of influenza A, H1N1 for Malaysia, Journal of Appl. Sci,11(8) (2011), pp. 1457-1460.
[11] S.F. Dowell, Seasonal variation in host susceptibility and cycles of certain infectious diseases, Emerg. Infect. Dis.7(3) (2001), pp. 369-374.
[12] http://www.who.int/denguecontrol/arbo_viral/other_arboviral_chikungunya/en/.
[13] http://www.who.int/csr/sarsarchive/2003_05_07a/en/.
[14] http://www.who.int/mediacentre/factsheets/fs103/en/.
[15] http://www.who.int/mediacentre/factsheets/mers-cov/en/.
[16] W. O. Kermack and A.G. McKendrick, A contribution to the mathematical theory of Epidemics. Proc. R. Soc. Lond. A 115(772) (1927) 700-721.
[17] W. London and J.A. Yorke, Recurrent outbreaks of measles, chickenpox and mumps. i. Seasonal variation in contact rates. Am. J. Epidemiol., 98 (6) (1973), pp. 453-468.
[18] Y.Li Michael, J.S. Muldowney and P.V.N. Driessche, Global stability of SEIRS model in epidemiology,Canadian Applied Mathematics Quarterly 7 (1999), pp. 409-425.
[19] Y.Li Michael, John R. Graef, Liancheng Wang and Janos Karsai, Global dynamics of a SEIR model with varying total population size, Math. Biosci. 160 (1999), pp. 191-213.