A comparative Study of Hermite, Lagranges and Birkhoff Interpolation and Regularity.

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Abstract— *The object of the present paper to compare the different type of interpolation and their regularity.*

Keywords— *Interpolation, polynomials, regularity, Interpolation matrix, nodes.*

I. Introduction

In recent years, the emergence of computer and computational capabilities has greatly encouraged the researchers to focus their study in the area of approximation theory. Interpolation is an area in which we consider the problem of approximating a given function by a class of simpler functions mainly polynomials. Roughly saying: A polynomial p(x) is called an interpolating polynomial if the value of p(x) and/or its certain derivatives coincide with those of given function f(x) and/or its same order derivatives at one or more points called nodes. In case of Lagranges interpolation matching at derivatives is not required whereas the Hermite interpolation involves matching of the interpolating polynomial and the given function at consecutive order of derivatives.

II. Interpolation and Regularity

Birkhoff interpolation considers the case of approximation with possible gaps in the order of derivatives. Birkhoff interpolation is a generalization of the Hermite case, obtained by relaxing the requirement of consecutive derivatives at the nodes when this is done, even the existence of an interpolant becomes questionable. For example, there is no quadratic p(x), satisfying p(-1)= p(1)=0, p'(0)=1 on the other hand, there is a unique quadratic satisfying $p(-1)=y_1$, $p(0)=y_2$, $p(1)=y_3$ for any reals y_1,y_2,y_3.The basic problem of Birkhoff interpolation then, is to determine those configuration of derivatives which always admit unique interpolants. This is of course a very general question and probably may not be solved in such generality. Mathematically, the case of interpolation of derivates is formulated as follows:

Let $x_1,x_2,\ldots,x_r$ be given distinct real points which are associated with multiplicities m_j (positive integers), j=1,2….,r such that $m_1+m_2+\ldots+m_r=n+1$. Under Hermite interpolation by n-th degree polynomial p_n, we expect it to satisfy the following conditions:

$$P_n^{(l)}(x_j)= c_{j,l};\ l=0,1\ldots,m_j-1\ ,j=1\ldots,r \quad \textbf{……….(1)}$$

If $c_{j,l}=f^{(l)}(x_j)$, we say that the polynomial $p_n = p_n(f,.)$ is the Hermite interpolation of the function f. If $m_j=1$ for every j,r = n+1, we call p_n the Lagrange interpolant of f at $\{x_1,x_2{----},x_{n+1}\}$. In the case of Lagrange and Hermite interpolation there is a unique polynomial satisfying the interpolating condition (1). It was in 1966, that Schoenberg formalized the problem in terms of interpolation matrices.

The definition of interpolation matrix is for the n-th degree polynomial p_n, the m x (n+1) interpolation matrix $E=(e_{i,k})$, i=1,2,---, m; k=0,….n; with elements $e_{i,k}$ that are zeros or ones with exactly n+1 ones is viewed as connected with the following interpolatory conditions:

$$p_n^{(k)}(x_i)= C_{i,k} \ \text{ if } e_{i,k} = 1 \quad \textbf{……….(2)}$$

Here $C_{i,k}$ are given prescribed values and $e_{i,k}=0$ indicates that the k-th derivative of the polynomial p_n at x_i is not being interpolated. Thus the interpolation matrix for the Lagrange interpolation problem has $e_{i,0}=1$, $e_{i,k}=0$, k>0, for all i. Similarly the interpolation matrix $E=(e_{j,l})$, j=1,2….r; l=0,….m_{j-1} ; for Hermite interpolation (1) is defined as follows: $e_{j,l}=1$ l=0,….m_{j-1}, $e_{j,l}=0$ otherwise. An interpolation matrix is said to be regular (almost regular) if a unique polynomial exists satisfying the corresponding interpolatory conditions for all choices of distinct $x'_j{}^{s}$. Lagrange and Hermite interpolation matrices are regular. However this important property does not hold for Birkhoff's interpolation matrices, in general. In view of this the study of regularity or almost regularity of Birkhoff interpolation matrices is of considerable interest.

One important property for ascertaining regularity of Birkhoff interpolation matrices is in numbers M_r (the total numbers of ones in the first r+1 column of the matrix). That is, for r=0,1,…n;

$M_r=\sum\sum e_{i,k.}$ where k=0,…..r i=1,….m.

$M_n=n+1$. The set of conditions:

$r+1\leq M_r$,r=0,1,….,n

is called Polya Condition. Any interpolation matrix satisfying the Polya condition is called a Polya matrix. If an interpolation matrix satisfies the Polya condition then it called regular.

III. CONCLUSIONS

Object is to study the various types of cases pertaining to Birkhoff interpolation with the main aim to find a condition like a polya condition in a more general setting.

REFERENCES

[1] J.F Steffensen, Interpolation, 2nd e'd. Chelsea, Newyork 1950.
[2] J,L walsh, Interpolation and approximation by rational functions in the comptex domain, 5th d. Amer, Math, Soc, Colloq, Publ. , no. 20, Amer, math. Soc. Providence, R.I. 1969.
[3] J.M. Whittaker, Interpolatory function theory, Stechert-Hafner, Newyork, 1964.
[4] E.W. Cheney, Introduction to approximation theory, McGraw-Hill, New York, 1966.
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