Effect of Charge on the Gravitational Collapse of Vaidya Space Time

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ABSTRACT
It has been shown in many earlier papers that the end state of gravitational collapse depends on initial data (density,pressure etc;). Considering this fact we have altered the input of mass function in Generalised Vaidya space time and obtained the comparative results in the gravitational collapse of radiating star.

Keywords – *Strange quark matter, Cosmic Censorship, Gravitational Collapse, Naked Singularity*
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INTRODUCTION

In the history of general relativity, the issue of space time singularities arises earlier. The physical theory ceases to make sense when basic quantities became infinite a singularity is a sign that the theory has been applied beyond its domain of validity. [1, 15] In theory of general relativity naked singularities occurs in variety of ways. In one of the versions, it is clear that the Cosmic censorship conjecture states that the strong curvature singularities are never naked. So, it is the most important issue which is to be cleared. So, we observe the curvature along the non space like geodesic joining a naked singularity before the same is the best example of counter example to the cosmic censorship curvature. This is achieved by verifying the growth of curvature along

both null geodesics and time like geodesics joining the naked singularity not necessary the radial ones. Dynamical solutions are always of interest to

people in gravity theories. Einstein gravity, the generalized Vaidya solution can describe absorbing or shining of the stars[2]. Always it is known that the end state of the gravitational collapse is whether a black hole or a naked singularity by depending upon the different mass function we have choosen [3].

Naked singularities are existed in some examples of spherically symmetric gravitational collapse. One of the earlier examples given by Papapetrou [15] explains about collapse of null radiation in Vaidya space times. Wang theory introduces a general family of Vaidya space times which covers anti-de-sitter, de-sitter, monopole and generalized vaidya solutions [7,11]. However, despite of this activity over the years, about the validity of this conjecture is still an open question. These singularities have the property that all the objects approaching them are collapsed with zero volume [9,12]. We assume that all singularities occurring in the space time of strong curvature type for finding the proof of CCH. Certain exact solutions of Einsteins equations follows the fact that visible singularities are occurred in the exact solutions. It is thus possible that the existence of naked singularities of the strong curvature type will always be accomplanished by space time singularities and so one can hope to prove a formulation of CCH which is a suitable criterion of generality or stability. From the earlier results of P.S.Joshi and Dwiwedi,[14] the occurrence of naked singularity in generalized vaidya space time was proved. In the conclusion naked singularities were formed whether the space time is asymptotically flat or not in the case of collapsing radiation shells [10,13].

Our investigation is focused on the nature of the singularity which may possibly form after the gravitational collapse.

In this paper our aim is to investigate the nature of singularities formed in Vaidya space time and these are compared with the same values when the charge is added for the space time.

This paper shows the Vaidya space time in section 2 followed by Charged Vaidya space time in section 3. In section 4 we show the nature of the singularities as well as we compare how they are varying when the charge was given. We end the section 5 with concluding remarks.

2. Generalized Vaidya space time

Generalized Vaidya space time is given by [1,5,6]

$$ds^2 = -\left[1 - \frac{2m(w,r)}{r}\right] dw^2 + 2\,dwdr + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right) \qquad (1)$$

Where w is the advanced Eddington time and m is a positive value and r is radial coordinate with the condition $0 < r < \infty$

And m (w,r) is mass function.

The energy momentum tensor can be written in the form [20, 24]

$$T_{\mu\nu}^{(m)} = (P+\rho)\left(l_\nu n_\mu + l_\mu n_\nu\right) + Pg_{\mu\nu} + \mu\, l_\mu l_\nu \qquad (2)$$

We have consider the two null vectors n_ν, l_μ such that

$$l_\mu = \lambda_\mu^0 , \qquad (3)$$

$$n_\mu = \frac{1}{2}\left[1 - \frac{2m(w,r)}{r}\right]\lambda_\mu^0 - \lambda_\mu^1 ,$$

$$l_\upsilon n^\upsilon = -1, \qquad l_\upsilon l^\upsilon = n_\upsilon n^\upsilon = 0$$

And $\mu = \dfrac{2\dot{m}(w,r)}{kr^2}$,

$$\rho = \frac{2m'(w,r)}{kr^2}, \qquad P = \frac{m''(w,r)}{kr},$$

Where k is a gravitational constant, dash and dot denotes the partial derivatives with respect to r and w respectively.

Einstein field equations is given by

$$G_{\delta\gamma} = KT_{\delta\gamma}$$

Where $G_{\delta\gamma}$ is Einstein tensor, K is Gravitational constant.

The energy conditions for the above will be as follows:

1. The dominant energy conditions are
   $$\mu > 0, \quad P \ge 0, \quad \rho \ge 0 \qquad (4)$$
2. The weak and strong energy conditions are
   $$\mu > 0, \quad \rho \ge P \ge 0, \qquad (5)$$

When $\rho = P = 0$, the space times reduce to the Vaidya space time with m=m (w) and the energy conditions reduces to $\mu > 0$. Then the above space times are regarded as generalized Vaidya space times. The radiation here discussed is focused into a central singularity at $r = 0,\ w = 0$, of growing mass $m(w, r)$. This $m(w, r)$ is an arbitrary non negative increasing function of w and r. If the source of radiation is turned off after a long time then it settles into a Schwarzschild space time.

From the behavior of the radial null geodesics we can able to examine whether the singularity is naked or not. The geodesics equations are given as follows:

$$\frac{d}{dk}\left(K^{w}\right) - \left(\frac{m'}{r} - \frac{m}{r^{2}}\right)\left(K^{w}\right)^{2} - \left(\left(K^{\theta}\right)^{2} + \sin^{2}\theta\left(K^{\theta}\right)^{2}\right) = 0, \tag{6}$$

$$\frac{d}{dk}\left(K^{r}\right) + \left(\frac{m}{r} - \frac{m'}{r} + \frac{m}{r^{2}} + \frac{2mm'}{r^{2}} - \frac{2m^{2}}{r^{3}}\right)\left(K^{w}\right)^{2} - \left(1 - \frac{2m}{r}\right) r\left(\left(K^{\theta}\right)^{2} + \sin^{2}\theta\left(K^{\phi}\right)^{2}\right) + 2\left(\frac{m'}{r} - \frac{m}{r^{2}}\right) K^{w} K^{r} = 0 \tag{7}$$

$$\frac{d}{dk}\left(K^{\theta}\right) + \frac{2}{r} K^{r} K^{\theta} - \sin\theta\cos\theta\left(K^{\phi}\right)^{2} = 0 \tag{8}$$

$$\frac{d}{dk}\left(r^{2}\sin^{2}\theta\, K^{\phi}\right) = 0 \tag{9}$$

Where $(K^{w}, K^{r}, K^{\theta}, K^{\phi})$ i.e $\left(\frac{dw}{dk}, \frac{dr}{dk}, \frac{d\theta}{dk}, \frac{d\phi}{dk}\right)$ are components of a tangent vector and k is he affine parameter. On solving the above two equations we get,

$$K^{\phi} = \frac{c}{r^{2}\sin^{2}\theta} \quad \text{and} \quad K^{\theta} = \frac{1}{r^{2}}\left(c_{1} - \frac{c^{2}}{\sin^{2}\theta}\right)^{1/2} \tag{10}$$

Where c and c_1 are the constant of integration. By substituting $c_1 = l^2,\ c = l\cos\beta$ we get

$$K^{\phi} = \frac{l\cos\beta}{r^{2}\sin^{2}\theta} \quad \text{and} \quad K^{\theta} = \frac{1}{r^{2}\sin\theta}\left(\sin^{2}\theta - \cos^{2}\beta\right)^{1/2} \tag{11}$$

Where l is the impact parameter and β isotropy parameter. We also have

$$(K^{\theta})^{2} + \sin^{2}\theta\left(K^{\phi}\right)^{2} = \frac{l^{2}}{r^{4}} \tag{12}$$

If K^{a} is a null vector, we have $K_{a}K^{a} = 0$. Using this equations 3,4,9 we get

$$\left(1 - \frac{2m}{r}\right)\left(K^{w}\right)^{2} - 2K^{w}K^{r} = \frac{l^{2}}{r^{2}} \tag{13}$$

Let $K^{w} = \frac{dw}{dk} = R(w, r)/r$ (14)

Then 10 becomes $K^{r} = \left(1 - \frac{2m}{r}\right)\frac{R}{2r} - \frac{l^{2}}{2rR'}$ (15)

Where R satisfies the differential equation

$$\frac{dR}{dk}-\frac{R^2}{2r^2}\left(1-\frac{4m}{r}\right)-\frac{l^2}{2r^2}-\frac{R^2}{r^2}m'=0$$

If we find the explicit expression for R then we get the behaviour of a non space like curve near the singularity. However in general, complete integration of the above geodesic equation is a difficult task. Hence to analyze the nature of singularity we restrict ourselves to the study of behavior of radial null geodesics $l=0$, and some special nature of the mass function $m(w,r)$.

The mass function is taken as $m(w)=\lambda w$ that means it is taken as follows:

$m(w)=\lambda w$ where λ is constant. The formation of naked singularities at $r=0$, for a certain range of values of λ and the behavior of radial null geodesics is shown below

For outgoing null radial geodesic $ds^2=0, d\theta=0, d\varphi=0$

$$0=\left(1-\frac{2m(w,r)}{r}\right)dw^2+2\,dwdr$$

$$2\,dw\,dr=\left(1-\frac{2m(w,r)}{r}\right)dw^2 \qquad (16)$$

$$2\,dw\,dr=\left(1-2\lambda\frac{u}{r}\right)dw^2 \qquad (17)$$

The metric obtained for the outgoing radial null geodesics is given by

$$\frac{dr}{dw}=\frac{1}{2}\left[1-\frac{2\lambda w}{r}\right]$$

The radial null geodesics for the metric should satisfy the null condition

$$\frac{dw}{dr}=\frac{2}{1-2\lambda\frac{u}{r}}$$

The coordinate w is an advanced time coordinate. The above equation has a singularity at $r\to 0$, $w\to 0$

Here we are considering the limiting value as $X=\frac{w}{r}$ along the singular geodesic to approach a singularity

Hence, for the geodesic tangent to exist at this point uniquely we should have the following

$$X_0=\lim_{\substack{w\to 0\\ r\to 0}}\frac{w}{r}=\lim_{\substack{w\to 0\\ r\to 0}}\frac{dw}{dr}$$

$$X_0=\lim_{\substack{w\to 0\\ r\to 0}}\frac{2}{1-2\lambda\frac{w}{r}} \qquad (18)$$

$$X_0=\frac{2}{1-2\lambda X_0}$$

$$X_0-2\lambda X_0^2=2$$

$$X_0 - 2\lambda X_0^2 - 2 = 0 \qquad (19)$$

From this Eq. we decide the nature of the singularity. The singularity will be naked if there exists at least one real and positive root. If there is no real positive root then it ends into a black hole.

To investigate, whether naked singularities will arise or not, we take some different values of λ .

3. Charged Vaidya space time

Another case of Generalised Vaidya space time is Charged Vaidya space time. Here we choose our mass function as [1, 8]

(i) $$2m(w,r) = 2\lambda w - \frac{\lambda}{r} w^2 \qquad (20)$$

Where $\sqrt{\lambda w}$ denotes the electric charge. Normally electric charge is denoted by $q(w)$ but in this case we take $2m(w,r) = 2\lambda w - \frac{q^2(w)}{r}$. There admits a homothetic killing field with the above mass function. Hence X_0 is the radial null geodesic at the singularity given by

Hence, for the geodesic tangent to exist at this point uniquely we should have the following

$$X_0 = \lim_{\substack{w \to 0 \\ r \to 0}} \frac{w}{r} = \lim_{\substack{w \to 0 \\ r \to 0}} \frac{dw}{dr}$$

$$X_0 = \frac{2}{1 - 2\lambda X_0 + \lambda X_0^2}$$

$$X_0 - 2\lambda X_0^2 + \lambda X_0^3 = 2$$

$$X_0 - 2\lambda X_0^2 + \lambda X_0^3 - 2 = 0$$

$$\lambda X_0^3 - 2\lambda X_0^2 + X_0 - 2 = 0 \qquad (21)$$

(ii)

$$2m(w,r) = 2\lambda w - \frac{\lambda/2}{r} w^2 \qquad (22)$$

Where $\sqrt{\lambda w/2}$ denotes the electric charge. Normally electric charge is denoted by $q(w)$ but in this case we take $2m(w,r) = 2\lambda w - \frac{q^2(w)}{2r}$. There admits a homothetic killing field with the above mass function. Hence X_0 is the radial null geodesic at the singularity given by

Hence, for the geodesic tangent to exist at this point uniquely we should have the following

$$X_0 = \lim_{\substack{w \to 0 \\ r \to 0}} \frac{w}{r} = \lim_{\substack{w \to 0 \\ r \to 0}} \frac{dw}{dr}$$

$$X_0 = \frac{2}{1 - 2\lambda X_0 + \lambda X_0^2/2}$$

$$X_0 - 2\lambda X_0^2 + \frac{\lambda}{2} X_0^3 = 2$$

$$X_0 - 2\lambda X_0^2 + \frac{\lambda}{2} X_0^3 - 2 = 0$$

$$\lambda X_0^3 - 4\lambda X_0^2 + 2X_0 - 4 = 0 \tag{23}$$

(iii)

$$2m(w,r) = 2\lambda w - \frac{\lambda/3}{r} w^2 \tag{24}$$

Where $\sqrt{\lambda w/3}$ denotes the electric charge. Normally electric charge is denoted by $q(w)$ but in this case we take $2m(w,r) = 2\lambda w - \frac{q^2(w)}{3r}$. There admits a homothetic killing field with the above mass function. Hence X_0 is the radial null geodesic at the singularity given by

Hence, for the geodesic tangent to exist at this point uniquely we should have the following

$$X_0 = \lim_{\substack{w \to 0 \\ r \to 0}} \frac{w}{r} = \lim_{\substack{w \to 0 \\ r \to 0}} \frac{dw}{dr}$$

$$X_0 = \frac{2}{1 - 2\lambda X_0 + \lambda X_0^2/3}$$

$$X_0 - 2\lambda X_0^2 + \frac{\lambda}{3} X_0^3 = 2$$

$$X_0 - 2\lambda X_0^2 + \frac{\lambda}{3} X_0^3 - 2 = 0$$

$$\lambda X_0^3 - 6\lambda X_0^2 + 3X_0 - 6 = 0 \tag{25}$$

4. Comparing the singularities in Generalized Vaidya space time and in Charged Vaidya space time.

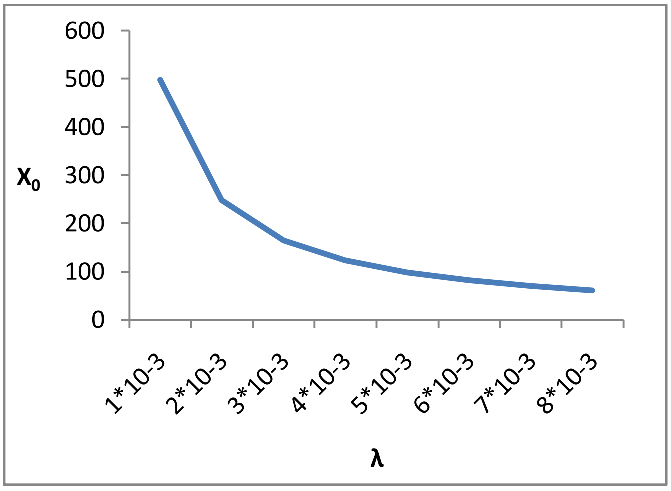
Now by taking the different values of λ we verify whether the roots appearing are positive or not. If at least one root is real and positive then the final state of the singularity is naked. So, we also compare the values in generalized and charged Vaidya space time by taking same values of λ in both the cases.

First consider the equation (19) of generalized Vaidya space time we take the different of λ then we get different values for X_0

Table.1. Values of X_0 for different values of λ

λ	X_0
$1*10^{-3}$	497.99
$2*10^{-3}$	247.98
$3*10^{-3}$	164.64
$4*10^{-3}$	122.97

$5*10^{-3}$	97.958
$6*10^{-3}$	81.283
$7*10^{-3}$	69.369
$8*10^{-3}$	60.432



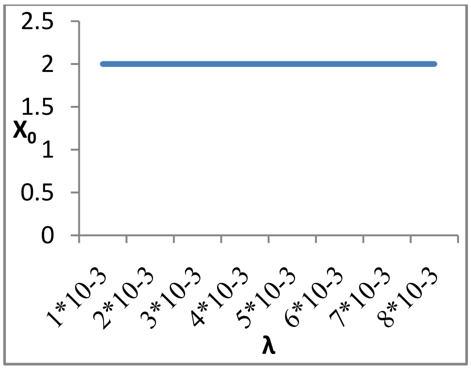
Graph.1. Graph of the Values of X_0 against the values of λ

Here we observe from the graph, that the values of X_0 are decreasing as the values of λ are increasing. We also noticed that at least one of the values obtained after solving the equation are positive root. So, the singularity obtained will be naked.

consider the equation (21) of generalized Vaidya space time we take the different of λ then we get different values for X_0

Table.2. Values of X_0 for different values of λ

λ	X_0
$1*10^{-3}$	2
$2*10^{-3}$	2
$3*10^{-3}$	2
$4*10^{-3}$	2
$5*10^{-3}$	2
$6*10^{-3}$	2
$7*10^{-3}$	2
$8*10^{-3}$	2



Graph.2. Graph of the Values of X_0 against the values of λ

Here from the graph we noticed that all the values of X_0 are having equal values and are also of positive roots even after increasing the values of λ.

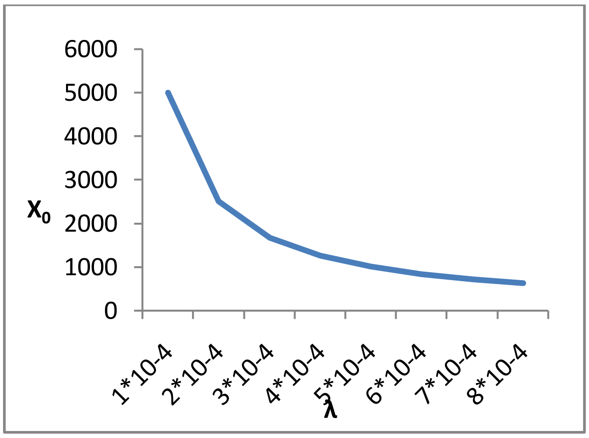
So, by comparing the graphs 1 and 2 we can conclude that in Vaidya space time the values of X_0 are decreasing where as for the same mass function the values of X_0 are remaining unchanged in Charged Vaidya space time.

Hence because of the small electric charge there is difference in the values even though the values of λ are increasing in both the space times.

consider the equation (19) of generalized Vaidya space time we take the different of λ then we get different values for X_0

Table.3. Values of X_0 for different values of λ

λ	X_0
$1*10^{-4}$	4998
$2*10^{-4}$	2498
$3*10^{-4}$	1664.7
$4*10^{-4}$	1248
$5*10^{-4}$	998
$6*10^{-4}$	831.33
$7*10^{-4}$	712.28
$8*10^{-4}$	622.99



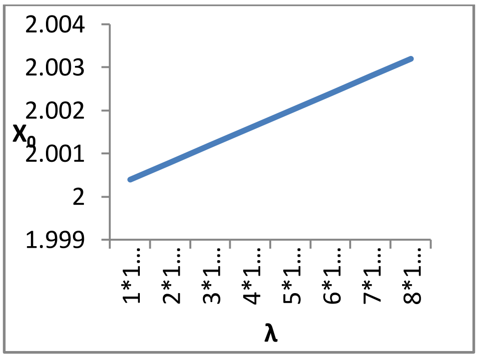
Graph.3. Graph of the Values of X_0 against the values of λ

Here we observe from the graph that the values of X_0 are decresaing as the values of λ are increasing. We also noticed that at least one of the values obtained after solving the equation are positive root. So, the singularity obtained will be naked.

consider the equation (23) of generalized Vaidya space time we take the different of λ then we get different values for X_0

Table.4. Values of X_0 for different values of λ

λ	X_0
$1*10^{-4}$	2.0004
$2*10^{-4}$	2.0008
$3*10^{-4}$	2.0012
$4*10^{-4}$	2.0016
$5*10^{-4}$	2.002
$6*10^{-4}$	2.0024
$7*10^{-4}$	2.0028
$8*10^{-4}$	2.0032



Graph.4. Graph of the Values of X_0 against the values of λ

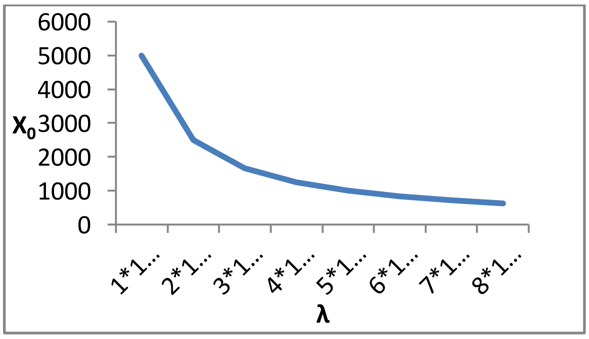
Here from the graph we noticed that the values of X_0 are increasing and are also of positive roots with the increase of λ.

So, by comparing the graphs 3 and 4 we can conclude that in Vaidya space time the values of X_0 are decreasing where as for the same mass function the values of X_0 are increasing in Charged Vaidya space time.
Hence because of the small electric charge there is difference in the values even though the values of λ are increasing in both the space times.

consider the equation (19) of generalized Vaidya space time we take the different of λ then we get different values for X_0

Table.5. Values of X_0 for different values of λ

λ	X_0
$1*10^{-5}$	4998
$2*10^{-5}$	2498
$3*10^{-5}$	1664.7
$4*10^{-5}$	1248
$5*10^{-5}$	998
$6*10^{-5}$	831.33
$7*10^{-5}$	712.28
$8*10^{-5}$	622.99



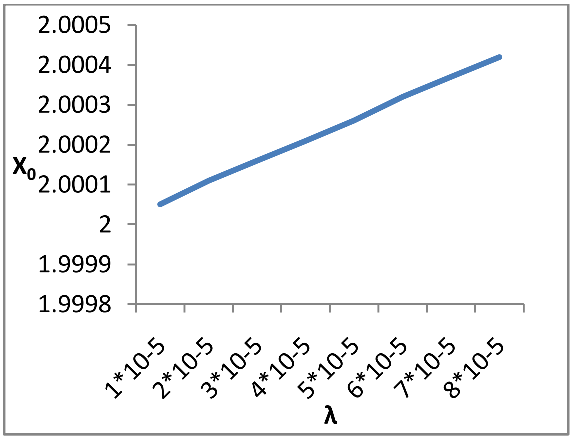
Graph.5. Graph of the Values of X_0 against the values of λ

Here we observe from the graph that the values of X_0 are decresaing as the values of λ are increasing. We also noticed that at least one of the values obtained after solving the equation are positive root. So, the singularity obtained will be naked.

consider the equation (25) of generalized Vaidya space time we take the different of λ then we get different values for X_0

Table.6. Values of X_0 for different values of λ

λ	X_0
$1*10^{-5}$	2.00005
$2*10^{-5}$	2.00011
$3*10^{-5}$	2.00016
$4*10^{-5}$	2.00021
$5*10^{-5}$	2.00026
$6*10^{-5}$	2.00032
$7*10^{-5}$	2.00037
$8*10^{-5}$	2.00042



Graph.6. Graph of the Values of X_0 against the values of λ

Here from the graph we noticed that the values of X_0 are increasing and are also of positive roots with the increase of λ.

So, by comparing the graphs 5 and 6 we can conclude that in Vaidya space time the values of X_0 are decreasing where as for the same mass function the values of X_0 are increasing in Charged Vaidya space time.
Hence because of the small electric charge there is difference in the values even though the values of λ are increasing in both the space times.

V CONCLUSION

The aim of this manuscript is to expand the work done in the paper on the generalized Vaidya space times and Charged Vaidya space times.[ ]. In this paper we have shown the nature of radial null geodesics obtained in generalized Vaidya space times and also in charged Vaidya space time are naked. The possible occurrence of naked singularity has been shown that the singularity is visible by the outside observer. So the existence of positive root from the above equations is a counter example to the strong version of Cosmic Censorship hypothesis. We also have compared these two space times by giving different values for λ . By comparing we observed that the values of X_0 are decreasing in generalized Vaidya space time where as in chaged Vaidya space time the values of X_0 are constant with the increase in the same values of λ .In other case the values of X0 are decreasing in generalized Vaidya space time s well as in the charged Vaidya space time. So, by taking same values of λ we have different graphs in one case and for the other two cases the values of X0 are decreasing with the increasing of the values of λ . Finally in both the space times we got the singularities are naked and the nature of their graphs is different for fixed value of λ .

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