Semi by Compact Sets and Semi Regularity Axioms in a Space

Md.Mamun Ar Rashid

Santipur College , Santipur , Nadia ,W.B.India.

Abstract : *In this paper semi regularity and in a semi normality axioms are introduced space defined by A . D. Alexandroff and some of their properties are investigated. Also semi bi compact and semi Housdorff spaces are defined and a few results are expressed.*

Key words: *semi bi compact set, semi regular space, semi normal space.*

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I INTRODUCTION:

Topological spaces have been generalized in several ways. For example Mashhour et. al. [4] omitted the intersection condition and then Das and Samanta [3] investigated a space without any structural conditions. Perhaps the first to introduce such a generalisation was Alexandroff [1], who weakened the union requirements of a topological space. In this paper semi bi compact sets and semi regularity axioms are introduced in a space defined by A.D.Alexandroff and some of their properties are investigated.

II.PRELIMINARIES:

Definition 1 [1]. A set X is called a space if in it is chosen a system of subsets F satisfying the following axioms

(i) The intersection of a countable number of sets from F is a set in F.

(ii) The union of a finite number of sets from F is a set in F.

(iii) The void set is a set in F.

(iv) The whole set X is a set in F.

Sets of F are called closed sets. Their complementary sets are called open. It is clear that instead of closed sets in the definition of a space, one may put open sets with subject to the conditions of countable summability , finite intersectability and the condition that X and the void set should be open. The collection of such open sets will sometimes be denoted by τ and the space by (X, τ). In general τ is not a topology. By a space we shall always mean an Alexandroff space.

Definition 2[1] . With every $M \subset X$ we associate its closure cl(M), the intersection of all closed sets containing M and scl(M), the intersection of all semi closed sets containing M.

Note that cl(M) and scl(M) is not necessarily closed and semi closed respectively in a space.

Definition 3[7]. Two sets A, B in X are said weakly separated if there are two open sets U,V such that $A \subset U, B \subset V$ and $A \cap V = B \cap U = \Phi$.

Definition 4[1]. A set N, a subset of X is said to be a semi neighborhood of a point x of X if and only if there exist a semi open set O containing x such that $O \subset N$.

Definition 5[7]. The semi interior of a set A in a space X is define as the union of all semi open sets contained in A and is denoted by s-int (A).

III. SEMI REGULAR AND SEMI NORMAL SPACE:

Definition 6. A space (X,τ) is said to be semi regular space if for any $x \in X$ and any semi closed set F such that x does not belongs to F ,there exist semi open sets U,V such that $x \in U, F \subset V$ and $U \cap V = \Phi$.

Definition 7 . A space (X,τ) is said to be semi normal space if for any two disjoint semi closed sets F and T ,there exist semi open sets U,V such that $F \subset U, T \subset V$ and $U \cap V = \Phi$.

Definition 8. A space (X,τ) is said to be semi R0-space if for each semi open set G and $x \in G$ implies $s\ cl\{x\} \subset G$.

Definition 9. A space (X,τ) is said to be semi R1-space if for $x,y \in X$ such that x does not belong to s cl {y}, there exist semi open sets U,V such that $x \in U$,$y \in V$ and $U \cap V = \Phi$.

Definition 10. A space (X,τ) is said to be semi Housdorff space if for $x,y \in X$ such that $x \neq y$, there exist semi open sets U,V such that $x \in U$,$y \in V$ and $U \cap V = \Phi$.

We know that a topological space (X,τ) is semi regular if and only if for any point $x \in X$ and any semi open set U containing x there is a semi open set V such that $x \in V \subset \tau\text{-}scl(V) \subset U$.

But this is not true in a space.

Example1. Let X=R-Q and $\tau = \{X, ,G_i\}$ where G_i runs over all countable subsets of R-Q .Then (X,τ) is a space but not a topological space .Clearly in this space for any $\alpha \in X$, $scl\{\alpha\} = \{\alpha\}$.Thus for $\alpha \in X$ and semi open set U such that $\alpha \in U$ then $\alpha \in \{\alpha\} \subset s\ cl\{\alpha\} \subset U$. But (X,τ) is not semi regular.

Theorem 1. A space X is semi regular if and only if for any $x \in X$ and any semi open set U containing x ,there is a semi open set V and a semi closed set F such that $x \in V \subset F \subset U$.

The proof is simple and omitted.

The similar characterization of semi normal space in a space also does not hold.

Theorem 2 . A space X is semi normal if and only if for any semi closed set F and any semi open set U containing F, there is semi open set V and semi closed set F' such that $F \subset V \subset F' \subset U$.

Theorem 3. Every semi regular space is semi R1.

The proofs are straight forward and so omitted.

IV. SEMI BI COMPACT AND LOCALLY SEMI BI COMPACT SPACE:

Definition 10. A space (X,τ) is said to be semi bi compact if every semi open cover of it has a finite sub cover

Definition 11. A space X is said to be locally semi bi compact if for every point x϶X there is semi open neighborhood U of x such that s cl(U) is semi bi compact where s-cl(U) is semi closure of U.

Theorem 4. If a space (X,τ) is semi R1 and semi bi compact ,then it is both semi regular and semi normal.

Proof. Let x϶X and F be a semi closed set such that x does not belongs to F. Let y϶F. Then x does not belong to s cl{y}. Since (X,τ) is semi R1 ,there are semi open sets U y , V y such that x϶ Uy, y϶ Vy and Uy ∩ Vy =Φ. Now [Vy : y϶F} along with X-F forms an semi open cover of X. Since X is semi bi compact , there are finite number of points y1,……yn in F such that F ⊂ U{ $V_{yi.}$:i=1,2,..n}. V= U{ V_{yi} :i=1,2,..n}, U= ∩{ U_{yi} :i= 1,2,..n}

Then x϶U, F ⊂ V and U ∩ V=Φ. So X is semi regular. To show that (X,τ) is semi normal , let F_1,F_2 are two disjoint semi closed sets in X. Let x϶ F_1, then x does not belongs to F_2. .Since X is semi regular there are semi open sets U'x, V'x such that x϶ U'x , F ⊂ V'x and U'x, ∩ V'x =Φ. Then {U' x : x϶F_1} along with X-F_1 forms an open cover of X. Then proceeding as above we can find open sets U',V' such that F_1 ⊂ U',F_2 ⊂ V' and U' ∩ V'= Φ.

Note 1. A semi Housdorff semi bi compact space is both semi regular and semi normal.

Definition 12 . A space X is said to be locally semi bi compact if for every point x϶ X, there is a semi open neighborhood U of x such that scl {U} is semi bi compact.

Remark 1. It is well known that a Housdorff locally semi compact topological space is semi regular . But this is not true in a space.

Example 2. Consider the above example , then X is Housdorff . Since for any for α϶X , {α} is an open set that is semi open set containing α such that {α} = cl{α}=scl{α} is semi bi compact , so X is locally semi bi compact .But X is not semi regular.

Remark 2 .In a semi Housdorff topological space semi compact subsets are semi closed. But this is not true in a space.

Example 3. Consider the above example 1 . (X,τ) is semi Housdorff . Let α϶X, then {α} is semi bi compact but not semi closed.

Theorem 5. In a semi Housdorff space semi bi compact sets are semi closed if and only if arbitrary union of semi open sets whose complement is semi bi compact is semi open.

Proof. Let (X,τ) is a semi Housdorff space in which semi bi compact sets are semi closed . Let {Gi} be a collection of semi open sets and G= U Gi such that X-G is semi bi compact . Then X-G is semi closed so G is semi open.

Conversely let the condition be hold. Let A be a semi bicompact set. We shall show that A is semi closed .Let x϶X-A. Then x does not belongs to A .Let y϶A. Then x ≠ y. Since (X,τ) is semi Housdorff , there exist semi open sets Uy ,Vy such that x϶ Uy, y϶ Vy and Uy ∩ Vy =Φ.Now {Vy : y϶A} is a semi open cover of A . Since A is semi bi compact there exist finite number of points y_1,……yn in A such that A ⊂ U{ $V_{yi.}$:i=1,2,..n}=V x(say). Let U x = ∩{ U_{yi} :i= 1,2,..n}. Then x϶ Ux , A ⊂ V x and U x ∩ V x =Φ .Now Ux ⊂ X-V x ⊂ X-A. Thus U Ux ⊂ X-A ⊂ U{{x}: x϶X-A} ⊂ U{Ux: x϶X-A} , which implies that X-A= U{Ux: x϶X-A} . So X-A is the union of semi open sets whose complement A is semi bicompact. So X-A is semi open. So A is semi closed.

Theorem 6 . In a semi R_1 topological space (X,τ) ,if A is semi compact then U{s-cl({a}):a϶A} is semi closed.

Proof .Let (X,τ) be a semi R_1 topological space .Let A be semi compact .Let x does not belongs to U{s-cl ({a}):a϶A} .Then x does not belongs to s-cl({a}),for all a϶A .Since (X,τ) is semi R_1 there exist semi open sets Ua,Va such that x϶ Ua, a϶ Va and Ua ∩ Va=Φ.Now {Va: a϶A} is an semi open cover of A . Since A is semi compact there exist finite number of points a_1,….,.an€A such that A ⊂ U{ V $_{ai}$:i=1,2,..n}=V., Let U= ∩{ U $_{ai}$:i= 1,2,..n}. Then x∊U, A ⊂ V and U ∩ V=Φ.Now a∊A ⊂ V ⊂ X-U , where X-U is semi closed .So s-cl({a}) ⊂ X-U, for all a belongs to A .Hence U{s-cl({a}):a∊A} ⊂ X-U . This implies that U ∩ U{s-cl({a}):a∊A} =Φ.Thus x is not a semi limiting point of U{τ-cl({a}):a∊A} and this shows that U{s-cl({a}):a∊A} is semi closed.

Remark 3. The result of above theorem is not true in a space . If we take A={a} the previous example then result will follow.

Theorem 7. In a semi R1 space (X,τ) ,if A is semi bi compact then U{s-cl({a}):a∊A} is semi closed iff union of an arbitrary number of semi open sets whose complement is the semi closure of a semi bicompact set is semi open.

Proof .Let G be an arbitrary union of semi open sets such that X-G=s-cl(A), where A is semi bi compact .Let A*= U{s-cl({a}):a∊A} .Then A* is semi closed. Also since A ⊂ A*, s-cl(A) ⊂ A*.Clearly A* ⊂s- cl(A).Hence X-G=s-cl(A)=A*.Since A* is semi closed ,G is semi open.

Conversely let the condition be hold .Let A be a semi bi compact set .Let y does not belongs to A*,so that y does not belongs to s- cl{a}: for all a∊A.

Since (X,τ) is semi R1 there exist semi open sets Ua , Va such that y∊ Ua , a∊ Va and Ua ∩ Va =Φ.Now {Va : a∊A} is an semi open cover of A. Since A is semi bi compact there exist finite number of points a1,…..an∊A such that A ⊂ U{ V $_{ai}$:i=1,2,..n}=Vy, Let Uy = ∩{ U $_{ai}$:i= 1,2,..n}. Then y∊ Uy , A ⊂ Vy and Uy ∩ Vy =Φ. Now A ⊂ Vy ⊂ X-Uy, where X - Uy is semi closed .So s cl (A) ⊂ X-Uy and y does not belongs to X-Uy .Therefore y ∉ scl (A). So s cl (A) ⊂ A*. Again clearly A* ⊂ s-cl (A).Hence s-cl (A)= A*.So X-A*=X-s cl (A)= U{X-F:F is a semi closed set containing A} .Thus X - A* is arbitrary union of semi open sets whose complement A*=s- cl (A) is the closure of a semi bi compact set A . Hence X - A* is semi open and so A* is semi closed.

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