Contra β * - Continuous Functions in Topological Spaces

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Abstract

The notion of contra continuous functions was introduced and investigated by Dontchev. In this paper, we apply the notion of β -closed sets in topological space to present and study a new class of functions called contra β* -continuous and almost contra β* -continuous functions as a new generalization of contra continuity.*

Keywords: *contra β* -continuous and almost contra β* -continuous functions*

I. Introduction

Generalized open sets play a very important role in General Topology and they are now the research topics of many topologist worldwide. Indeed a significant theme in General Topology and Real Analysis concerns the variously modified forms of continuity, separation axioms etc, by utilizing generalized closed sets. Recently, as generalization of closed sets, the notion of β* -closed sets were introduced and studied by [13]. Dontchev [6] introduced the notions of contra continuity and strong S-closedness in topological spaces. He defined a function f : X → Y is contra continuous if the preimage of every open set of Y is closed in X. A new weaker form of this class of functions called contra semicontinuous function is introduced and investigated by Dontchev and Noiri [7]. Caldas and Jafari [5] have introduced and studied contra β-continuous function. Jafari and Noiri [9, 10] introduced and investigated the notions of contra super continuous, contra precontinuous and contra α-continuous functions. Almost contra precontinuous functions were introduced by [8] and recently have been investigated further by Noiri and Popa [12]. Nasef [11] has introduced and studied contra γ-continuous function. In this direction, we will introduce the concept of contra β* -continuous and almost contra β*-continuous functions via the notion of β* -open set and study some properties of contra β* -continuous and almost contra β* -continuous functions.

II. Preliminaries

Throughout this paper (X, τ), (Y, σ) and (Z, η) or X , Y , Z represent non-empty topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X, τ), cl(A) and int(A) denote the closure and the interior of A respectively. The power set of X is denoted by P(X).

Definition 2.1: A subset A of a topological space X is said to be a β*- open [2] if A $\subseteq$ cl (int* (cl(A))).

Definition 2.2: A function f: (X, τ) ⟶ (Y, σ) is called a β* - continuous [1] if f^{-1} (O) is a β *- open set of (X, τ) for every open set O of (Y, σ).

Definition 2.3: A map f: (X, τ) ⟶ (Y, σ) is said to be perfectly β*- continuous [3] if the inverse image of every β*- open set in (Y, σ) is both open and closed in (X, τ) .

Definition 2.4: A function f: (X, τ) ⟶(Y, σ) is said to be β* - Irresolute [1] if f^{-1} (O) is a β*- open in (X, τ) for every β*open set O in (Y, σ).

Definition 2.5: A function f: (X, τ) ⟶ (Y, σ) is called a contra g continuous if f^{-1} (O) is a g - closed set of (X, τ) for every open set O of (Y, σ).

Definition 2.6: A function f: (X, τ) ⟶ (Y, σ) is called a contra continuous [3]if f^{-1} (O) is a closed set of (X,τ) for every open set O of (Y, σ).

Definition 2.7: A function f: (X, τ) ⟶ (Y, σ) is called a contra β - continuous [4] if f^{-1} (O) is a β - closed set of (X, τ) for every open set O of (Y, σ).

Definition 2.8: A function f: (X, τ) ⟶ (Y, σ) is called a contra semi continuous [4] if f^{-1} (O) is a semi closed set of (X, τ) for every open set O of (Y, σ).

Definition 2.9: A Topological space X is said to be $\beta^*T_{1/2}$ space or β*space [2] if every β*- open set of X is open in X.

Definition 2.10: A Topological space X is said to be a locally indiscrete if each open subset of X is closed in X.

Definition 2.11: Let A be a subset of a topological space (X, τ). The set ∩{U ∈ τ | A ⊂ U} is called the Kernel of A and is denoted by ker(A).

Lemma 2.12: The following properties hold for subsets A,B of a space X:

1. x ∈ ker(A) if and only if A ∩ F $\neq \phi$,for any F ∈ C(X, x);

2. A ⊂ ker(A) and A = ker(A) if A is open in X;

3. If A ⊂ B, then ker(A) ⊂ ker(B).

Theorem 2.13: [1] Let f: (X, τ) ⟶(Y, σ) be a function. Then the following are equivalent:

1. f is β*- continuous
2. The inverse image of closed set in Y is β*- closed in X.
3. f (β* cl (A)) $\subseteq$ f(cl(A)) for every subset A in X.
4. β* cl(f^{-1} (G)) $\subseteq$ f^{-1} (cl(G)) for every subset G of Y.
5. f^{-1} (int (G)) $\subseteq$ β* int (f^{-1} (G)) for every subset G of Y.

Theorem 2.14*[2]:*

(i) Every open set is β*- open and every closed set is β*-closed set

(ii) Every β -open set is β*-open and every β -closed set is β*-closed.

(iii) Every g-open set is β* -open and every g-closed set is β*-closed.

III. Contra β*- continuous functions

Definition 3.1: A function f: (X, τ) ⟶ (Y, σ) is called Contra β*- continuous functions if f^{-1} (O) is β*- closed in (X, τ) for every open set O in (Y, σ).

Example 3.2: Let X = Y= {a, b, c}, τ = {ϕ,{a},{a, b},{a, c}, X} and σ = {ϕ, {a, b},Y}. β*C(X, τ) = {ϕ ,{b},{c},{b, c}, X}. Let f: *(X, τ) ⟶(Y, σ)* be defined by f(a) = c, f(b) = a, f(c) =b. clearly, f is contra β*- continuous .

Theorem 3.3: Every contra continuous function is a contra β*- continuous.

Proof: Let f: (X, τ) ⟶ (Y, σ) be a function. Let O be an open set in (Y, σ). Since, f is contra continuous , then f^{-1} (O) is closed in (X, τ) . Hence by theorem [2.14] , f^{-1} (O) is β*- closed in (X, τ) . Therefore, f is contra β*- continuous.

Remark 3.4 : The converse of the above theorem need not be true.

Example 3.5: Let X = Y= {a, b, c}, τ = {ϕ,{a, b}, X}, $τ^c$ = {ϕ,{c}, X} and σ = {ϕ, {a}, {a, b},{a, c},Y}. β*C(X, τ) = {ϕ , {a} ,{b}, {c} , {a, c} , {b, c} , X }. Let f: (X, τ) ⟶(Y, σ) be defined by f(a) = b, f(b) = c, f(c) = a. clearly, f is contra β*- continuous , but f^{-1} ({a, b}) = {a, c} f^{-1} ({a, c}) = {b, c} is not closed in X. Therefore f is not contra continuous.

Theorem 3.6: Every contra g - continuous map is contra β*- continuous.

Proof: Let f: (X, τ) ⟶ (Y, σ) be a contra g -continuous map and O be any open set in Y. Since, f is contra g-continuous, then f^{-1} (O) is g - closed in (X, τ) . Hence by theorem [2.14] , f^{-1} (O) is β*- closed in (X, τ) .Therefore, f is contra β*- continuous.

Remark 3.7: The converse of the above theorem need not be true.

Example 3.8: Let X = Y= {a, b, c}, τ = {ϕ,{a, b}, X}, g-closed (X, τ) = {ϕ,{c},{a, c} , {b, c} ,X} and σ = {ϕ, {a}, Y}. β*C(X, τ) = {ϕ , {a} ,{b} , {c} , {a, c} ,{b, c}, X }. Let f: (X, τ) ⟶(Y, σ) be defined by f(a) = a, f(b) =b , f(c) = c. clearly, f is contra β*- continuous , but f^{-1} ({a}) = {a} is not g-closed in X. Therefore f is not contra g-continuous.

Theorem 3.9: Every contra β - continuous map is contra β*- continuous.

Proof: Let f: (X, τ) ⟶ (Y, σ) be a contra β - continuous map and O be any open set in Y. Since, f is contra β - continuous, then f^{-1} (O) is β - closed in (X, τ) . Hence by theorem [2.14] , f^{-1} (O) is β*- closed in (X, τ) .Therefore, f is contra β*- continuous.

Remark 3.10: The converse of the above theorem need not be true.

Example 3.11: Let X = Y= {a, b, c, d}, τ = {ϕ,{a}, {b},{a, b}, {a, b, c}, X}, and σ = {ϕ, {a, b, c},Y}. β*C(X, τ) = {ϕ , {a},{b},{c} , {d} , {a, b},{a, c},{a, d} ,{b, c}, {b, d}, {c, d} , {a, b, d} , {a, c, d} , {b, c, d} , X }. β C(X, τ) = {ϕ , {a},{b},{c} , {d} , {a, c},{a, d} ,{b, c}, {b, d}, {c, d} , {a, c, d}, {b, c, d},X}. Let f: (X, τ) ⟶(Y, σ) be defined by f(a) = a , f(b) = b , f(c) = d , f(d) = c. clearly, f is contra β*- continuous , but f^{-1} ({a, b, c}) = {a, b, d} is not β - closed in X. Therefore f is not contra β - continuous.

Theorem 3.12: Every contra semi - continuous map is contra β*- continuous.

Proof: Let f: (X, τ) ⟶ (Y, σ) be a contra semi - continuous map and O be any open set in Y. Since, f is contra semi - continuous, then f^{-1} (O) is semi - closed in (X, τ) . Hence by theorem [2.14] , f^{-1} (O) is β*- closed in (X, τ) .Therefore, f is contra β*- continuous.

Remark 3.13: The converse of the above theorem need not be true.

Example 3.14: Let X = Y= {a, b, c, d}, τ = {ϕ,{a, b}, {a, b, c}, X}, and σ = {ϕ, {a},{b},{c},{a, b},{a, c},{b, c},{a, b, c},Y}. β*C(X, τ) = {ϕ , {a},{b},{c} , {d} , {a, b},{a, c},{a, d} ,{b, c}, {b, d}, {c, d} , {a, b, d} , {a, c, d} , {b, c, d} , X }. S C(X, τ) = {ϕ , {c} , {d} , {c, d} , X}. Let f: (X, τ) ⟶(Y, σ) be defined by f(a) = a , f(b) = b , f(c) = d , f(d) = c. clearly, f is contra β*- continuous , but f^{-1} ({a}) = {a} , f^{-1} ({b}) = {b}, f^{-1} ({a, b}) = {a, b} , f^{-1} ({a, c}) = {a, d} , f^{-1} ({b, c}) = {b, d} , f^{-1} ({a, b, c}) = {a, b, d} is not semi -closed in X. Therefore f is not contra semi - continuous.

Theorem 3.15: Every β* - continuous map is contra β*- continuous.

Proof: Let f: (X, τ) ⟶ (Y, σ) be β*- continuous map and O be any open set in Y. Since, f is β*- continuous, then f^{-1} (O) is β* - closed in (X, τ) .Therefore, f is contra β*- continuous.

Remark 3.16: The converse of the above theorem need not be true.

Example 3.17: Let X = Y= {a, b, c, d}, τ = {ϕ, {a},{b},{c},{a, b}, {a, c},{b, c}, {a, b, c}, X}, and σ = {ϕ, {a},{b},{a, b},{a, b, c},Y}. β*C(X, τ) = {ϕ , {a},{b},{c} ,{d}, {a, b},{a, c},{a, d} ,{b, c}, {b, d}, {c, d} , {a, b, d}, {a, c, d}, {b, c, d}, X}, β*O(X, τ) = {ϕ , {a},{b},{c}, {a, b},{a, c},{a, d} ,{b, c}, {b, d}, {c, d} , {a, b, d}, {a, c, d}, {b, c, d}, X}.}. Let f: (X, τ) ⟶(Y, σ) be defined by f(a) = d , f(b) = b , f(c) = c , f(d) = a. clearly, f is contra β*- continuous , but f^{-1} ({a}) = {d} , is not β* - open in X. Therefore f is not β*- continuous.

Remark 3.18: The concept of contra β*- continuous and contra g*- continuous are independent.

Example 3.19: Let X = Y= {a, b, c, d}, τ = { ϕ , {a} ,{a, b, c} , X } and σ = { ϕ , {a} , {b},{a, b},{a, b, c},Y}, β* C (X,τ)={ϕ, {a} , {b} , {c} , {d}, {a, c} ,{a, d}, {b, c} , {b, d},{c, d},{a, c, d} , {b, c, d} , X} g *C (X, τ) = {ϕ , {d} ,{b, d} , {c, d} , {a, d}, {a, b, d} ,{a, c, d} , {b, c, d}, X} . Let f: (X, τ) ⟶(Y, σ) be a map defined by f (a) =c, f(b) = d, f(c) = a, f(d) = b , clearly, f is contra β*- continuous but f is not g* - continuous because f^{-1} ({a}) ={c} is not g* - closed in X.

Example 3.20: Let X = Y = {a, b, c, d}, τ = {ϕ,{a}, {b},{c},{a, b}, {a, c},{b, c} ,{a, b, c}, X}, and σ = {ϕ,{a, b, c},Y}. β* C (X,τ)={ϕ, {a} , {b} , {c} , {d},{a, b}, {a, c} ,{a, d}, {b, c} , {b, d},{c, d}, {a, b, d},{a, c, d} , {b, c, d} , X} g *C (X, τ) = {ϕ , {d} ,{a, d}, {b, d},{c, d},{a, b, c},{ {a, b, d} ,{a, c, d} , {b, c, d}, X} . Let f: (X, τ) ⟶(Y, σ) be a map defined by f (a) =a, f(b) = b, f(c) = c, f(d) = d , clearly, f is contra g*- continuous but f is not contra β* - continuous because f^{-1} ({a, b, c}) ={a, b, c} is not g* - closed in X.

Remark 3.21: The composition of two contra β*- continuous need not be contra β*- continuous.

Example 3.22: Consider X = Y = Z = {a, b, c }, τ = {ϕ,{a},{a, b},{a, c}, X} and σ = {ϕ, {a, b},Y}, η ={ϕ,{a}, Z}, β*C(X, τ) = {ϕ,{b},{c},{b, c},X}, β*C (Y, σ) = {ϕ ,{a},{b}, {c},{a, c},{b, c}, Y}.Let f: (X, τ) ⟶(Y, σ) be defined by f(a) = c , f(b) = a, f(c) = b. Clearly, f is contra β*-continuous. Consider the map g: Y ⟶ Z defined g(a) = b, g(b) = c, g(c) = a , clearly g is contra β*-continuous. But g ∘f : X ⟶ Z is not a contra β*-continuous, $(g \circ f)^{-1}$ ({a}) = g^{-1} (f^{-1}{a}) = g^{-1} (b) = a which is not a β*- closed in X.

Theorem 3.23: Let f: (X, τ) ⟶ (Y, σ) be a map. The following are equivalent.

1. f is contra β*- continuous.
2. The inverse image of a closed set F of Y is β*- open in X

Proof: let F be a closed set in Y . Then Y\F is an open set in Y. By the assumption of (1), f^{-1} (Y\F) = X \ f^{-1} (F) is β*- closed in X . It implies that f^{-1} (F) is β*- open in X. Converse is similar.

Theorem 3.24: The following are equivalent for a function f: (X, τ) ⟶ (Y, σ). Assume that β*O(X, τ) (resp β*C (X, τ)) is closed under any union (resp; intersection).

1. f is contra β*- continuous.
2. The inverse image of a closed set F of Y is β*- open in X

3. For each x ⊂ X and each closed set B in Y with f(x) ⊂ B , there exists an β*- open set A in X such that x ⊂ A and f(A) ⊂ B

4. f(β*cl (A)) ⊂ ker (f(A)) for every subset A of X.

5. β*cl (f^{-1} (B)) ⊂ f^{-1} (ker B) for every subset B of Y.

Proof:

(1) ⇒ (3) Let x ∈ X and B be a closed set in Y with f(x) ∈ B. By (1), it follows that f^{-1} (Y\B) = X\ f^{-1} (B) is β*-closed and so f^{-1} (B) is β*- open. Take A = f^{-1} (B). We obtain that x ∈ A and f(A) ⊂ B.

(3) ⇒ (2) Let B be a closed set in Y with x ∈ f^{-1} (B). Since, f(x) ∈ B, by (3), there exist an β*- open set A in X containing x such that f(A) ⊂ B. It follows that x ∈ A ⊂ f^{-1} (B). Hence, f^{-1} (B) is β*- open.

(2) ⇒ (1) Follows from the previous theorem

(2) ⇒ (4) Let A be any subset of X. Let y ∉ ker (f(A)). Then there exists a closed set F containing y such that f(A) ∩ F = ϕ . Hence, we have A ∩ f^{-1} (F) = ϕ and β*cl (A) ∩ f^{-1} (F) = ϕ . Hence, we obtain f(β*cl (A)) ∩ F = ϕ and y ∉ f(β*cl (A)). Thus, f(β*cl (A)) ⊂ ker (f(A)).

(4) ⇒ (5) Let B be any subset of Y. By (4) and Theorem [2.13] f(β*cl (f^{-1} (B))) ⊂ (ker B) and β*cl (f^{-1} (B)) ⊂ f^{-1} (ker B)

(5) ⇒ (1) Let B be any open set in Y. By (5) and Theorem [2.13] β*cl (f^{-1} (B)) ⊂ f^{-1} (ker B) = f^{-1} (B) and β*cl (f^{-1} (B)) = f^{-1} (B). We obtain that f^{-1} (B) is β* - closed in X.

Theorem 3.25: If f: (X, τ) ⟶ (Y, σ) is β*- irresolute and g: (Y, σ) ⟶ (Z, η) is contra β*- continuous , then their composition g ∘ f: (X, τ) ⟶ (Z, η) is contra β*- continuous.

Proof: Let O be any open set in (Z, η). Since, g is contra β*- continuous, then g^{-1} (O) is β*- closed in (Y, σ) and since f is β*- irresolute , then f^{-1} (g^{-1} (O)) is β*- closed in (X, τ) . Therefore, g ∘ f is contra β*-continuous.

Theorem 3.26: If f: (X, τ) ⟶ (Y, σ) is contra β*- continuous g: (Y, σ) ⟶ (Z, η) is continuous, then their composition g ∘ f: (X, τ) ⟶ (Z, η) is contra β*- continuous.

Proof: Let O be any open set in (Z, η). Since, g is continuous, then g^{-1} (O) is open in (Y, σ) and since f is contra β*- continuous, then f^{-1} (g^{-1} (O)) is β*- closed in (X, τ). Therefore, g ∘ f is contra β*- continuous.

Theorem 3.27: If f: (X, τ) ⟶ (Y, σ) is contra β - continuous g: (Y, σ) ⟶ (Z, η) is continuous, then their composition g ∘ f: (X, τ) ⟶ (Z, η) is contra β*- continuous.

Proof: Let O be any open set in (Z, η). Since, g is continuous, then g^{-1} (O) is open in (Y, σ) and since f is contra β- continuous, then f^{-1} (g^{-1} (O)) is β- closed in (X, τ).Hence by theorem [2.15], every β - closed set is β*- closed. We have f^{-1} (g^{-1} (O)) is β*- closed in (X, τ). Therefore, g ∘ f is contra β*- continuous.

Theorem 3.28: If f: (X, τ) ⟶ (Y, σ) is contra β*- continuous g: (Y, σ) ⟶ (Z, η) is g-continuous, then their composition g ∘ f: (X, τ) ⟶ (Z, η) is contra β*- continuous.

Proof: Let O be any open set in (Z, η). Since, g is g- continuous, then g^{-1} (O) is g- open in (Y, σ) and since f is contra β*- continuous, then f^{-1} (g^{-1} (O)) is β*- closed in (X, τ). Therefore, g ∘ f is contra β*- continuous.

Theorem 3.29: If f: (X, τ) ⟶ (Y, σ) is strongly β*- continuous and g: (Y, σ) ⟶ (Z, η) is contra β*- continuous , then their composition g ∘ f: (X, τ) ⟶ (Z, η) is contra continuous.

Proof: Let O any open set in (Z, η). Since, g is contra β*- continuous, then g^{-1} (O) is β*- closed in (Y, σ) and since f is strongly β*- continuous, f^{-1} (g^{-1} (O)) is closed in (X, τ). Therefore, g ∘ f is contra continuous.

Theorem 3.30: If f: (X, τ) ⟶ (Y, σ) is perfectly β*- continuous, and g: (Y, σ) ⟶ (Z, η) is contra β*-continuous, then their composition g ∘ f: (X, τ) ⟶ (Z, η) is perfectly β*- continuous.

Proof: Let O any open set in (Z, η). By theorem [2.14] every open set is β*- open set which implies O is β*-open in (Z, η). Since, g is contra β*- continuous, then g^{-1} (O) is β*- closed in (Y, σ) and since f is perfectly β*- continuous , then f^{-1} (g^{-1} (O)) is both open and closed in X, which implies $(g \circ f)^{-1}$ (O) is both open and closed in X . Therefore, g ∘ f is perfectly β*- continuous.

Theorem 3.31: Let f: (X, τ) ⟶ (Y, σ) is surjective β* - irresolute and β*- open and g: (Y, σ) ⟶ (Z, η) be any function. Then g ∘ f: (X, τ) ⟶ (Z, η) is contra β*- continuous if and only if g is contra β*- continuous.

Proof: The if part is easy to prove. To prove the only if part , let F be any closed set in (Z, η). Since g ∘ f is contra β*- continuous, then f^{-1} (g^{-1} (F)) is β*- open in (X, τ) and since f is β*- open surjection,then f(f^{-1} (g^{-1} (F))) = g^{-1} (F) is β*- open in (Y, σ). Therefore, g is contra β*- continuous.

Theorem 3.32: Let f: (X, τ) ⟶ (Y, σ) be a function and X a β*- $T_{1/2}$ space. Then the following are equivalent:

1. f is contra continuous
2. f is contra β*- continuous

Proof:

(1) ⇒ (2) Let O be any open set in (Y, σ). Since f is contra continuous, f^{-1} (O) is closed in (X, τ) and since every closed set is β*-closed, f^{-1} (O) is β*-closed in (X, τ). Therefore, f is contra β*- continuous.

(2) ⇒ (1) Let O be any open set in (Y, σ). Since, f is contra β*- continuous, f^{-1} (O) is β*-closed in (X, τ) and since X is β*- $T_{1/2}$ space, f^{-1} (O) is closed in (X, τ). Therefore, f is contra continuous.

Theorem 3.33: If f: (X, τ) ⟶ (Y, σ) is contra β*- continuous and (Y, σ) is regular , then f is β**-continuous.

Proof: Let x be an arbitrary point of X and O be any open set of Y containing f(x). Since Y is regular, there exists an open set U in Y containing f(x) such that cl(U) ⊂ O. Since, f is contra β*- continuous, so by theorem[3.24] , there exists N ∈ β*O (X, τ) , such that f(N) ⊂ cl(U) . Then, f(N) ⊂ O. Hence by theorem[2.14], f is β* -continuous.

Theorem 3.34: If f is β*-continuous and if Y is locally indiscrete, then f is contra β*-continuous.

Proof: Let O be an open set of Y. Since Y is locally discrete, O is closed. Since, f is β*- continuous, f^{-1} (O) is β*- closed in X. Therefore, f is contra β* - continuous.

Theorem 3.35: If a function f : (X, τ) ⟶(Y, σ) is continuous and X is a locally indiscrete space, then f is contra β* - continuous.

Proof: Let O be any open set in (Y, σ). Since f is continuous f^{-1} (O) is open in X. and since X is locally discrete, f^{-1} (O) is closed in X. Every closed set is β*- closed. f^{-1} (O) is β*- closed in X. Therefore, f is contra β*- continuous

Theorem 3.36: Let f: (X, τ) ⟶ (Y, σ) be a function and g: X ⟶ X × Y the graph function, given by g(x) = (x, f(x)) for every x ∈X . Then f is contra β*- continuous if g is contra β*- continuous.

Proof: Let F be a closed subset of Y. Then X × F is a closed subset of X × Y. Since g is a contra β*-continuous, then g^{-1} (X × F) is a β*-open subset of X. Also, g^{-1} (X × F) = f^{-1} (F). Hence, f is contra β*- continuous.

Theorem 3.37: Let {X_i / i ∈ I}be any family of topological spaces. If f : X → Π X_i is a contra β*- continuous function. then π_i∘ f : X → X_i is contra β*- continuous for each i ∈ I, where π_i is the projection of Π X_i onto X_i .

Proof: Suppose U_i is an arbitrary open sets in X_i for i ∈ I. Then π_i^{-1} (U_i)is open in Π X_i. Since f is contra β*-continuous, f^{-1} (π_i^{-1} (U_i)) =(π_i∘ f$)^{-1}$ (U_i) is β*- closed in X. Therefore, π_i∘ f is contra β*- continuous. For a map f: (X, τ) ⟶ (Y, σ), the subset {(x , f(x)): x ∈X} ⊂ X × Y is called the graph of f and is denoted by G(f).

IV. Contra β*- closed graph

Definition 4.1: The graph G(f) of a function f: (X, τ) ⟶ (Y, σ) is said to be Contra β*- closed graph in X× Y if for each (x,y) ∈ (X × Y) - G(f) there exists U ∈ β* O (X, τ) and V∈ C (Y, y) such that (U × V) ∩ G(f) = ϕ.

Lemma 4.2: The graph G(f) of a function f: (X, τ) ⟶ (Y, σ) is β*- closed in (X × Y) if and only if for each (x,y) ∈ (X × Y) - G(f) , there exists an U∈ β*O (X, x) and an open set V in Y containing y such that f(U) ∩ V = ϕ.

Proof: We shall prove that f(U) ∩ V = ϕ ⇔ (U × V) ∩ G(f) = ϕ. Let (U × V) ∩ G(f) ≠ ϕ. Then there exists (x,y) ∈ (X × Y) and (x,y) ∈ G(f). This implies that x ∈ U, y ∈ V and y = f(x) ∈ V. Therefore, f(U) ∩ V ≠ ϕ. Hence the result follows.

Theorem 4.3: If f: (X, τ) ⟶ (Y, σ) is contra β*- continuous and Y is Urysohn, G(f) is a contra β*- closed graph in X × Y.

Proof: Let (x,y) ∈ (X × Y) - G(f), then y ≠ f(x) and there exist open sets A and B such that f(x) ∈A , y∈B and cl(A) ∩ cl(B) = ϕ. Since f is contra β*- continuous, there exist O∈ β*O (X, x) such that f(O) ⊂ cl(A). Therefore, we obtain f(O) ∩ cl(B) = ϕ. Hence by lemma [4.2], G(f) is contra β*- closed graph in X × Y.

Theorem 4.4: If f: (X, τ) ⟶ (Y, σ) is β*- continuous and Y is T_1, then G(f) is Contra β*- closed graph in X× Y.

Proof: Let (x,y) ∈ (X × Y) - G(f), then y≠ f(x) and since Y is T_1 there exists open set V of Y, such that f(x) ∈ V, y∉ V. Since f is β*- continuous, there exist β*- open set U of X containing x such that f(U) ⊂ V.Therefore, f(U) ∩ (Y – V) = ϕ and Y –V is a closed set in Y containing y. Hence by lemma [4.2], G(f) is contra β*- closed graph in X × Y.

Definition 4.5: A space X is said to be locally β*- indiscrete if every β*- open set of X is closed in X.

Theorem 4.6: A contra β*- continuous function f : (X, τ) ⟶ (Y, σ) is continuous when X is locally β*- indiscrete.

Proof: Let O be an open set in Y. Since, f is contra β*- continuous then f^{-1} (O) is β*- closed in X. Since, X is locally β*- indiscrete which implies f^{-1} (O) is open in X. Therefore, f is continuous.

Theorem 4.7: Let f : (X, τ) ⟶ (Y, σ) is β*-irresolute map with Y as locally β*- indiscrete space and g: (Y, σ) ⟶ (Z, η) is contra β*-continuous, then g ∘ f is β*- continuous.

Proof: Let B be any closed set in Z. Since g is contra β*- continuous, g^{-1} (B) is β*- open in Y. But Y is locally β*- indiscrete, g^{-1} (B) is closed in Y. Hence, g^{-1} (B) is β*- closed in Y. Since, f is β*-irresolute,

f^{-1} (g^{-1} (B)) = (g ∘ f)$^{-1}$ (B) is β*-closed in X. Therefore, g ∘ f is β*-continuous.

Definition 4.8: A map f : (X, τ) ⟶ (Y, σ) is said to be pre β* - open if the image of every β*open set of X is β*-open in Y.

Theorem 4.9: Let f : (X, τ) ⟶ (Y, σ) be surjective β*- irresolute pre β*-open and g: (Y, σ) ⟶ (Z, η) be any map. Then g ∘ f: (X, τ) ⟶ (Z, η) is contra β*-continuous if and only if g is contra β*-continuous.

Proof: The if part is easy to prove. To prove the "only if " part , let g ∘ f: (X, τ) ⟶ (Z, η) be contra β*-continuous and let B be a closed subset of Z. Then (g ∘ f)$^{-1}$ (B) is β*-open in X which implies f^{-1} (g^{-1}(B)) is β*-open in X. Since, f is pre β*-open, f(f^{-1} (g^{-1} (B))) is β*-open of X. So, g^{-1} (B) is β*-open in Y. Therefore, g is contra β*-continuous.

REFERENCE

[1] P.Anbarasi Rodrigo and K.Rajendra Suba, More functions associated with β *-Continuous, International Journal Of Advancement In Engineering Technology Management & Applied Science, Volume 5, (47- 56).

[2] Anbarasi Rodrigo and Rajendra Suba, A new notion of β*- closed sets, IMRF.

[3]Anbarasi Rodrigo and Rajendra Suba, Functions related to β*- closed sets in topological space, International Conference On Recent Trends In Mathematics And Information Technology(ICRMIT-2018)

[4] M. Caldas, S. Jafari, T. Noiri, M. Simeos, A new generalization of contra-continuity via Levines g-closed sets,Chaos Solitons Fractals 42 (2007), 1595–1603.

[5] M. Caldas and S. Jafari, Some properties of contra- continuous functions, Mem. Fac. Sci. Kochi Univ. Series A. Math., 22 (2001),pp. 19-28.70 A. VADIVEL, R. RAMESH, AND D. SIVAKUMAR

[6] J. Dontchev, Contra-continuous functions and strongly S-closed spaces, Internat. J.Math. Math. Sci. 19 (1996),303–310.

[7] J. Dontchev, T. Noiri, Contra-semicontinuous functions, Math. Pannon. 10 (1999),159–168.

[8] E. Ekici, Almost contra-precontinuous functions, Bull. Malays. Math. Sci. Soc. , 27 (2004), pp. 53-65.

[9] S. Jafari and T. Noiri, On contra-precontinuous functions, Bull.Malays. Math. Sci. Soc. , 25 (2002), pp. 115-128.

[10] S. Jafari and T. Noiri, Contra-continuous functions between topological spaces, Iran. Int. J. Sci., 2 (2001), pp. 153-167.

[11] A.A. Nasef, Some properties of contra--continuous functions,Chaos Solitons Fractals, 24 (2005), pp. 471-477.

[12] T. Noiri and V. Popa, Some properties of almost contra pre continuous functions, Bull. Malays. Math. Sci. Soc., 28 (2005),pp. 107-116.

[13] P.G. Palanimani and R. Parimelazhagan, closed sets in topologicalspaces, ROSR Journal of Mathematics, 5 (2013), pp. 47-50.

[14] Pious Missier .S and P.Anbarasi Rodrigo, Some Notions of nearly open sets in Topological Spaces, International Journal of Mathematical Archive 4(12), 2013, 1-7