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An optimization model for buyer-supplier co-ordination under limited warehouse space and incremental price discount

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Abstract

This paper considers incremental price discount, limited warehouse space and payments delay. It is often seen that when a retailer purchases products for which supplier offers incremental price discounts. In Goyel [1] investigated the inventory replenishment problem under condition of payments delay, where he assumed that: (i) The unit selling price and unit purchasing price are equal (ii) The retailer starts paying higher interest charges on the items in stocks and returns money of the remaining balance immediately when the items are sold (iii) Retailer warehouse space is unlimited. Here we consider that unit purchasing price are not constant but buyer has taken advantage of offering incremental price discount given by supplier. It is assumed that the buyer will borrow total purchasing cost from the bank to pay off the account, buyer must pay the amount of purchasing cost to the supplier at the end of trade credit period. This also assumed that the buyer's storage space is limited. Numerical examples are given to illustrate results obtained in of the present study.

Keywords: *Inventory, Production, Optimization, Price setting, Quantity discount, Limited Warehouse space, Payments delay.*

1 Introduction

Offering quantity discounts is a common industrial practice. Some of the reasons so far offering quantity discounts are as follows:

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(1) to stimulate the demand for the product;

(2) to reduce per-unit set-up cost for the supplier;

(3) to meet competition.

Although all unit discount schedule are more common, many suppliers offer incremental discounts. In an incremental discount schedule, lower unit-price is applicable only to the incremental quantity.

In the classical incremental discount economic order quantity model, demand for the product is considered to be constant (cf. Hadley and Whitin [2], Max and Candea [3], Tersine and Toelle [4], Jackson and Munson [5], Tamjidzad and MirMuhammai [6]). A related paper considers all-unit type quantity discounts schedule (cf. Abad [7]-[8]). In Kunreuther and Richard [9], the pricing and lot-sizing problem for the linear demand and no quantity discounts is considered. Tajbakhsh [10] also considered an inventory model with random discount. Down the years, a number of reseachers have appeared in the literature that treat inventory problem with varying conditions under payments delay indeed to link financing marketing as well as operations concerned. Some of the prominent papers are discussed here. Goyel [1] established a single item inventory model under trade credit. Chen and Chuang [11] investigated light buyer's inventory policy under trade credit by the concept of discounted cash flow. Sarkar et al. [12] investigated this topic with inflation. Jamal et al. [13] addressed the optimal payment time under permissible delay in payments and deterioration. Chung and Huang [14] developed an efficient procedure to determine the retailer's optimal ordering policy. Supplier uses favorable trade credit policy to encourage retailer to order large quantities because a delay in payments indirectly reduces inventory cost (cf. Ting [15]). Hence, the retailer may purchase more goods than that can be stored in his/her own warehouse. These excess quantities are stored in a rented warehouse. This proposed model is applicable for the business of small and medium sized firm since their storage capacity are small and limited. In general, the inventory holding charges in rented warehouse is greater than that for own warehouse. When the demand occurs, it first is replenished from the rented warehouse. This is done to reduce inventory holding costs. It is assumed that the transportation costs between warehouses are negligible. This viewpoint can be found in Sarma [16], Pakkala and Achary[17], Goswami and Chaudhuri [18], Benkherouf [19], Bhunia and Maiti [20], and Önal [21].

2 Model formulation

The integrated production inventory model proposed in this paper is based on the following notations and assumptions:

(1) n raw materials are required to manufacture a single final product.

(2) The production rate of the final item is P (constant).

(3) The demand rate of the final item is price sensitive and is given by

$$D(p) = Kp^{-e}, \ e > 1, \ \ K > 0,$$

where p is the selling price of unit final item together with $P > D(p)$.

(4) The raw materials are received from out-supplier at an infinite rate of replenishment.

(5) The supplier of raw materials offers an incremental price discount to the manufacturer. Manufacturer's purchase per unit item is of the following pattern (cf. Abad [8], Naddor [22]):

$$v(q) = \begin{cases} v_1, & \text{for } q_0 \leq q < q_1, \\ v_2, & \text{for } q_1 \leq q < q_2, \\ v_3, & \text{for } q_2 \leq q < q_3, \\ \dots\dots\dots\dots\dots\dots\dots\dots & \\ \dots\dots\dots\dots\dots\dots\dots\dots & \\ v_{n-1}, & \text{for } q_{n-2} \leq q < q_{n-1}, \\ v_n, & \text{for } q \geq q_n, \end{cases}$$

where q is the quantity of raw materials to be purchased and v_n is a strictly monotonic decreasing sequence.

(6) Supplier of raw materials also offers a trade off credit period M. If the credit period is shorter than the cycle length, the manufacturer sale them and accumulate sales revenue and earned interest throughout the cycle.

(7) Shortages of raw materials and final product are not allowed.

(8) T is the inventory cycle time and T_1 is the production run of the final product.

(9) Let $I(t)$ is the inventory level of the final product and $I_r(t)$ is the inventory level of the raw materials at time t.

(10) Let Q_r is the amount of raw materials to be purchased by the manufacturer and its corresponding cost is $V(Q_r)$

(11) Manufacturer must pay the amount of the purchasing raw materials to the supplier at time M.

(12) Manufacturer has finite storage capacity W. If the ordered quantity is larger than the manufacturer's storage space then he will rent a warehouse to store the exceeding items. It is to be noted that, when manufacturing starts it start production by using raw materials from the rented warehouse.

2.1 Some notations:

A_m = cost of placing an order.

D_r = demand of raw materials, $D_r = nP$.

p = unit selling price of the final item.

C_m = production cost of unit final item excluding the raw material cost.

h_r = unit raw material holding cost of own warehouse per year.

k_r = unit raw material holding cost of rented warehouse per year.

h = unit finished good holding cost.

C_h = total finished good holding cost.

I_e = annual interest earned rate.

$I_p(> I_e)$ = annual interest charged rate.

M = trade credit period (constant).

T_1 = supplying duration of raw materials.

Q_r = lot size of raw materials.

Q = lot size of final items.

T = the cycle length.

W = manufacturer's own storage capacity.

t_W = the rented house time given by the following equations:

$$t_W = \begin{cases} \frac{D_r T_1 - W}{D_r}, & \text{if} \quad T_1 > \frac{W}{D_r} \\ 0, & \text{if} \quad T_1 \leq \frac{W}{D_r} \end{cases} .$$

B = capital amount invested by the manufacturer.

2.2 The model and optimality condition

Insert **Figures** 1 and 2 here

Let us consider that the consumption rate of raw materials is D_r and the consumption continue up to $T_1(< T)$. At the beginning of each cycle, the required raw material are received and the production stage of the final item starts. In this stage the variation of the inventory level of the final product $I(t)$ and raw materials $I_r(t)$ are governed by the following differential equations:

$$\left.\begin{array}{rcl}\frac{dI}{dt} & = & P-D(p),\ 0\leq t\leq T_1\\ & = & -D(p),\qquad T_1<t\leq T\end{array}\right\} \tag{2.1}$$

with initial conditions $I(0)=0,\ I(T)=0$, and

$$\left.\begin{array}{rcl}\frac{dI_r}{dt} & = & -D_r,\ 0\leq t\leq T_1\\ & = & 0,\qquad T_1<t\leq T\end{array}\right\} \tag{2.2}$$

with $I_r(T_1)=0$.

The solutions of the differential equations (2.1) and (2.2) are given by

$$\left.\begin{array}{rcl}I(t) & = & (P-D(p))t,\qquad 0\leq t\leq T_1\\ & = & -D(p)(T-t),\ T_1<t\leq T\end{array}\right\}, \tag{2.3}$$

and

$$\left.\begin{array}{rcl}I_r(t) & = & D_r(T_1-t),\ 0\leq t\leq T_1\\ & = & 0,\qquad T_1<t\leq T\end{array}\right\}. \tag{2.4}$$

Since n raw materials are required to produce unit final element, therefore

$$nP=D_r. \tag{2.5}$$

Again the total production of final item during T_1 equal to the total demand during T. Hence,

$$T_1P=D(p)T. \tag{2.6}$$

Let Q_r be the amount of raw materials purchased by the manufacturer and assume that $q_{j-1}\leq Q_r\leq q_j$, then the purchasing cost $V(Q_r)$ of raw materials is given by the following equation

$$\begin{aligned}V(Q_r) &= v_1(q_1-q_0)+v_2(q_2-q_1)+\ldots+v_{j-1}(q_{j-1}-q_{j-2})+v_j(Q_r-q_{j-1})\\ &= v_jQ_r-v_1q_0+\sum_{k=1}^{j-1}q_k(v_k-v_{k-1}).\end{aligned} \tag{2.7}$$

Manufacturer objective is to set the selling price of the final product and cycle length in such a way so that his net profit will be maximized. Manufacturer profit consists the following components:

(1) Sales revenue per cycle $=pQ=pD(p)T$, i.e., sales revenue per year $=\frac{pQ}{T}=pD(p)$.

(2) Purchase cost of raw materials per year $= \frac{V(Q_r)}{T}$.

(3) Ordering cost per year $= \frac{A_m}{T}$.

(4) The total manufacturing cost of the final item per year $= C_m D(p)$ (excluding the raw materials cost).

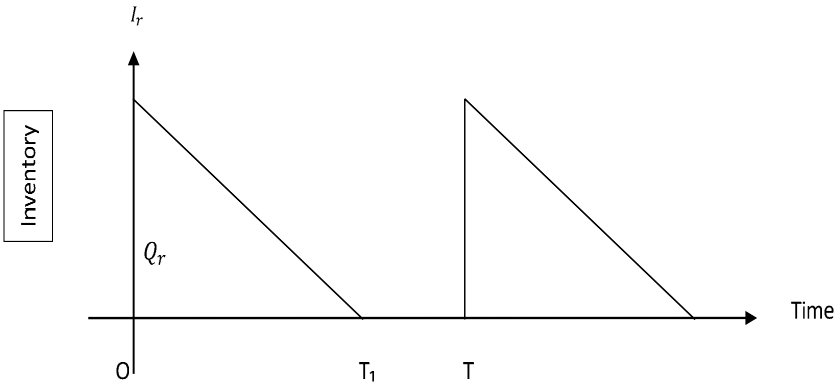

Figure 1: Inventory of the raw materials throughout a periodic cycle T.

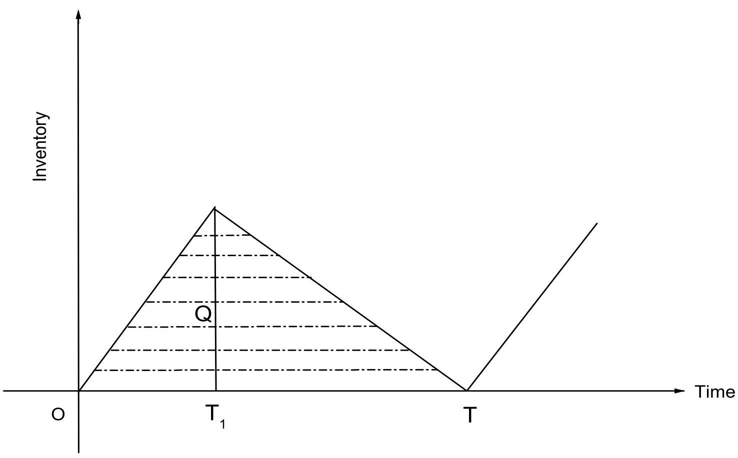

Figure 2: Graphical representation of Inventory vs Time.

(5) The total holding cost for the manufacturer can be considered with respect to two situations (i) $M > \frac{W}{D_r}$ and (ii) $M < \frac{W}{D_r}$. We see that there are two types of holding costs arise in this model namely (a) raw material holding cost and (b) finished good holding cost. It is clear that for the above cases have no effect on finished good holding cost. The

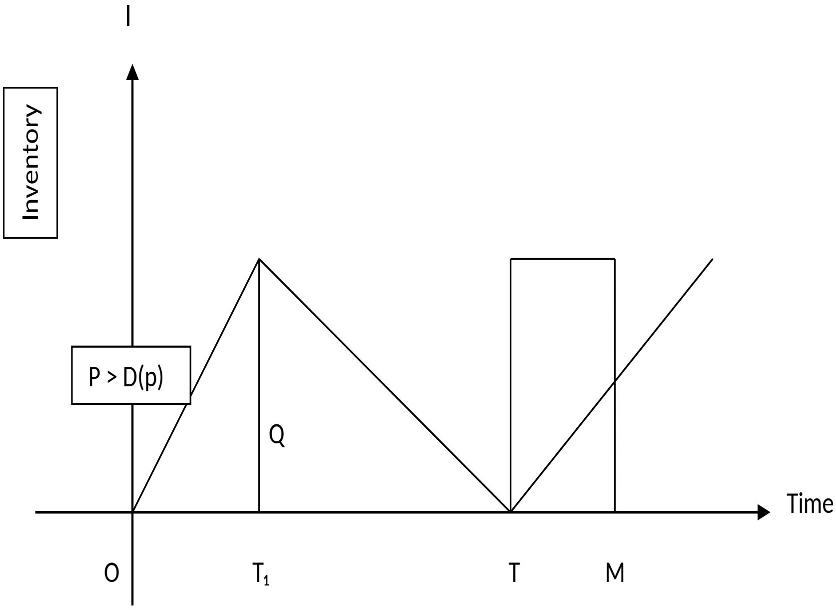

Figure 3: Inventory of the final product throughout a periodic cycle T.

finished good holding cost per cycle is given by (cf. Figure 2)

$$\begin{aligned} C_h &= h[\text{ average production over time period T}] \\ &= h[\text{ area of the shaded region}] \\ &= h\frac{1}{2}D(p)T(T-T_1). \end{aligned}$$

Therefore, the finished good holding cost per year $= \frac{hD(p)(T-T_1)}{2}$.

2.3 Calculation of raw materials holding cost in different situations

There are three sub cases to be occurred in annual stock holding cost:

Subcase I: $0 \le T_1 < \frac{W}{D_r} < M < T$.

In this case the order size is not larger than manufacturer own storage capacity so that manufacturer will not rent the warehouse to store raw materials.

$$\begin{aligned} \text{Hence, the annual stock holding cost} &= \frac{1}{T}\int_0^{T_1} h_r D_r t dt \\ &= \frac{h_r D_r T_1^2}{2T}. \end{aligned}$$

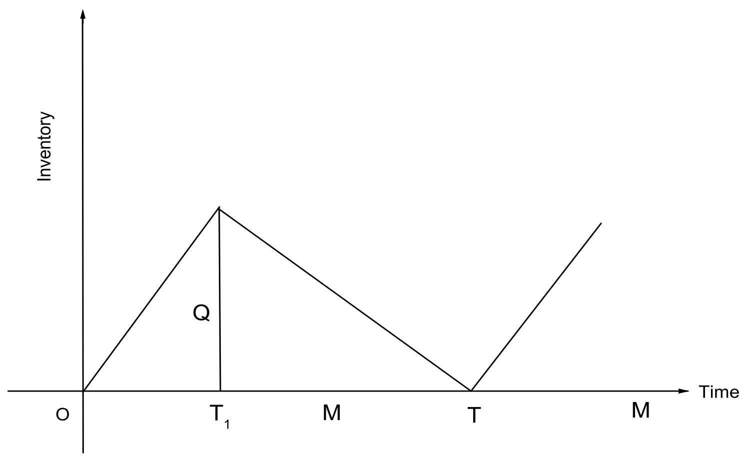

Figure 4: Graphical representation of Inventory vs Time with credit period M.

Subcase II: $0 \leq \frac{W}{D_r} < T_1 \leq T \leq M$.

In this case the order quantity is larger than the manufacturer storage capacity. Hence, the annual stock holding cost = (annual stock holding cost for the rented house + annual stock holding cost for own warehouse), i.e.,

$$\begin{aligned}\text{the annual stock holding cost} &= \frac{k_r t_W (D_r T_1 - W)^2}{2T} + \frac{h_r\left(t_W W + \frac{W(T_1 - t_W)}{2}\right)}{T} \\ &= \frac{k_r (D_r T_1 - W)^2}{2D_r T} + \frac{h_r W (2D_r T_1 - W)}{2D_r T}.\end{aligned}$$

Subcase III: $0 < \frac{W}{D_r} < T_1 < M < T$.

Using the similar technique used in subcase II, we have the annual raw material holding cost as follows:

$$= \frac{k_r (D_r T_1 - W)^2}{2D_r T} + \frac{h_r W (2D_r T_1 - W)}{2D_r T}.$$

2.4 Calculation of the cost of interest charged in different situations

There are three cases occur in cost of interest charges for the item kept in stock per year.

Case I: $0 \leq T_1 < M < T$.

In this case the credit period is larger than the cycle length. Hence no interest is charged for the item kept in the stock.

Case II: $T_1 < T < M$.

Similar as in case I, no interest is charged for the item kept in the stock.

Case III: $T > M$.

In this case cost of interest charged for the items kept in the stock per year is

$$= \frac{I_p V(Q_r)(T-M)}{T}.$$

2.5 Calculation of the cost of interest earned per year in different situations

There are three cases occur in interest earned per year.

Case I: $0 \leq T_1 \leq \frac{W}{D_r} < M < T$.

In this case the manufacturer sell the items and earned interest until the end of the credit period. Hence in this case the total interest earned per year is

$$\begin{aligned} &= pI_e \frac{\left[\frac{1}{2}D(p)T^2 + D(p)T(M-T)\right]}{T} \\ &= D(p)I_e p\left(M - \frac{T}{2}\right). \end{aligned}$$

Case II: $0 < \frac{W}{D_r} < T_1 < T < M$.

Similar as in case I, total interest earned per year is

$$= D(p)I_e p\left(M - \frac{T}{2}\right).$$

Case III: $T > M$.

In this case the manufacturer can sell the items and earned interest throughout the inventory cycle. Hence the total interest earned per year is

$$\begin{aligned} &= pI_e\frac{1}{T}\int_0^T D(p)tdt \\ &= \frac{I_e pD(p)T}{2}. \end{aligned}$$

From the above argument the total annual profit for the manufacturer can expressed as

$$\begin{aligned} \max \Pi(p,T,T_1) &= \text{sales revenue - manufacturing cost - ordering cost - inventory} \\ &\quad \text{holding cost - interest charged cost + interest earned cost.} \end{aligned}$$

Thus, the total annual profit $\Pi(p,T,T_1)$ takes the following form:

$$\Pi(p,T,T_1) = \begin{cases} \Pi_1(p,T,T_1), & \text{if } 0 \le T_1 < T \le \frac{W}{D_r} < M, \\ \Pi_2(p,T,T_1), & \text{if } 0 < \frac{W}{D_r} < T_1 \le T \le M, \\ \Pi_3(p,T,T_1), & \text{if } T > M, \end{cases} \tag{2.1}$$

where

$$\begin{aligned} \Pi_1(p,T,T_1) &= (p-C_m)D(p) - \frac{V(Q_r)}{T} - \frac{A_m}{T} - \frac{hD(p)(T-T_1)}{2} - \frac{h_r D_r T_1^2}{2T} \\ &\quad + I_e D(p)\left(M - \frac{T}{2}\right), \end{aligned} \tag{2.2}$$

$$\begin{aligned} \Pi_2(p,T,T_1) &= (p-C_m)D(p) - \frac{A_m}{T} - \frac{hD(p)(T-T_1)}{2} + I_e D(p)\left(M - \frac{T}{2}\right) \\ &\quad - \frac{V(Q_r)}{T} - \left(\frac{k_r(D_r T_1 - W)^2}{2D_r T} + \frac{h_r W(2D_r T_1 - W)}{2D_r T}\right), \end{aligned} \tag{2.3}$$

and

$$\begin{aligned} \Pi_3(p,T,T_1) &= (p-C_m)D(p) - \frac{V(Q_r)}{T} - \frac{A_m}{T} - \frac{I_p V(Q_r)(T-M)}{T} + \frac{pI_e D(p)T}{2} \\ &\quad - \frac{hD(p)(T-T_1)}{2} - \left(\frac{k_r(D_r T_1 - W)^2}{2D_r T} + \frac{h_r W(2D_r T_1 - W)}{2D_r T}\right). \end{aligned} \tag{2.4}$$

If n raw materials are required to produce the final item. Therefore, $Q_r = nQ = nD(p)T$, also we have

$$\begin{aligned} \frac{V(Q_r)}{T} &= \frac{v_j Q_r - V_0}{T}, \text{ where } V_0 = v_1 q_0 - \textstyle\sum_{k=1}^{j-1} q_k(v_k - v_{k-1}) \\ &= n v_j D(p) - \frac{V_0}{T}. \end{aligned} \tag{2.5}$$

Substituting the value of T_1 from (2.6) and using the equation (2.5), manufacturer's annual profit function defined in (2.1) is reduced to the following one:

$$\Pi(p,T) = \begin{cases} \Pi_1(p,T), & \text{if } 0 \le T_1 < T \le \frac{W}{D_r} < M, \\ \Pi_2(p,T), & \text{if } 0 < \frac{W}{D_r} < T_1 \le T \le M, \\ \Pi_3(p,T), & \text{if } T > M, \end{cases} \tag{2.6}$$

where

$$\begin{aligned} \Pi_1(p,T) &= (p - C_m - n v_j)D(p) - \frac{A_m - V_0}{T} - \frac{hD(p)T}{2}\left(1 - \frac{D(p)}{P}\right) - \frac{h_r D_r D^2(p)T}{2P^2} \\ &\quad + I_e D(p)\left(M - \frac{T}{2}\right), \end{aligned} \tag{2.7}$$

$$\begin{aligned} \Pi_2(p,T) &= (p - C_m - n v_j)D(p) - \frac{A_m - V_0}{T} - \frac{hD(p)T}{2}\left(1 - \frac{D(p)}{P}\right) + I_e D(p)\left(M - \frac{T}{2}\right) \\ &\quad - \left[\frac{k_r\Big(nD(p)T - W\Big)^2}{2D_r T} + \frac{h_r W\Big(2nD(p)T - W\Big)}{2D_r T}\right], \end{aligned} \tag{2.8}$$

and

$$\begin{aligned} \Pi_3(p,T) &= (p - C_m - n v_j)D(p) - \frac{A_m - V_0}{T} - \frac{hD(p)T}{2}\left(1 - \frac{D(p)}{P}\right) \\ &\quad - \left[\frac{k_r\Big(nD(p)T - W\Big)^2}{2D_r T} + \frac{h_r W\Big(2nD(p)T - W\Big)}{2D_r T}\right] \\ &\quad + \frac{I_e p D(p) T}{2} - I_p\left(n v_j D(p) - \frac{V_0}{T}\right)(T - M). \end{aligned} \tag{2.9}$$

Thus, our problem is to maximize the objective function

$$\Pi(p,T) = \begin{cases} \Pi_1(p,T), & \text{if } 0 \le T_1 < T \le \frac{W}{D_r} < M, \\ \Pi_2(p,T), & \text{if } 0 < \frac{W}{D_r} < T_1 \le T \le M, \\ \Pi_3(p,T), & \text{if } T > M, \end{cases} \tag{2.10}$$

subject to the following conditions:

$$\begin{aligned} (i) \;\; P &> D(p), \\ (ii) \;\; B &\ge V(Q_r) + C_m Q, \\ (iii) \;\; p &> \frac{nV(Q_r)}{Q_r} + C_m. \end{aligned}$$

As it is tough to undertake the above constrained maximization problem analytically, we must directed to solve this problem numerically.

3 Numerical simulation

In this section we consider two types of demand rates of final item Case (i): constant price elasticity demand function: $D(p) = Kp^{-e}$, where $K(>0)$ is the scaling factor and e is (constant) price elasticity. This demand function is applicable for those commodities whose price elasticity $e > 1$. Case (ii): the linear demand function: $D(p) = a - bp$, where a, b are positive constants and $p < \frac{a}{b}$. For the first case, if we choose the following hypothetical values of the parameters $P = 5000$; $C_m = 1$; $n = 5$; $h = 1$; $h_r = 0.25$; $k_r = 0.5$; $K = 2$; $e = 1.5$; $M = 0.2$; $D_r = 35000$; $I_e = 0.05$; $I_p = 0.09$; $W = 2000$; $A_m = 500$; $j = 4$; $T = 0.6$; $q_0 = 0$; $q_1 = 100$; $q_2 = 500$; $q_3 = 1000$; $q_4 = 1500$; $v_1 = 10$; $v_2 = 9.5$; $v_3 = 9.0$; $v_4 = 8.5$; $v_5 = 7.0$ then we have the optimal value of selling price $p \to 122.145$ and corresponding maximum profit is found from $\max(\Pi_1(p,T)) \to 2166.66$. It is to be noted that these optimal decision values also satisfy all the constraints for this objective function.

If we consider the linear trend demand function $D(p) = a - bp$, for the following set of values of the parameters: $P = 5000$; $C_m = 0.2$; $n = 5$; $h = 0.5$; $h_r = 0.8$; $k_r = 0.5$; $M = 0.5$; $D_r = 25000$; $I_e = 0.05$; $I_p = 0.08$; $W = 8000$; $A_m = 500$; $j = 4$; $q_0 = 0$; $q_1 = 130$; $q_2 = 500$; $q_3 = 1000$; $q_4 = 1500$; $v_1 = 10$; $v_2 = 9.5$; $v_3 = 9.0$; $v_4 = 8.5$; $v_5 = 7.0$; $T = 0.4 < M = 0.5$; $a = 4000$; $b = 15$, then we have the optimal value of selling price $p \to 155.1$ and corresponding maximum profit is obtained by $\max(\Pi_2(p,T)) \to 184604$.

These parameter values satisfy the restrictions given in Case (ii). To find the numerical solution I have taken the help of the built-in function 'FindMaximum' in Mathematica software [23].

4 Concluding remarks

We have considered in this paper the joint price and cycle length determination problem faced by retailer when he purchase products for which the supplier offers incremental quantity discounts. Incremental discounts are offered in practice in many realistic situation. It is reported that the Figures: 1–4 are given for different inventory levels with respect to time. Our observations have shown that linear trend demand is more profitable than that of constant price elasticity demand.

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